Multi-particle production in QCD at high energies

Raju Venugopalan

Brookhaven National Laboratory

Riverhead, NY


KRAKOW
3786 MI
237
POLONAISE PARK
POLISH TOWN, U.S.A.
POLISH TOWN
CIVIC ASSOCIATION
Euro Deli
IMPORTED SPECIALTIES
HOME MADE
FRESH
&
NATURALY SMOKED
POLISH KIELBASA
SMOKED HAMS
ALL COLD CUTS
HEAD CHEESE
KISZKA
STUFFED CABBAGE
PIEROGIS
POLISH BREAD
CHEESE BABKA
CHRUSCIKI
PACZKI

Outline of Lectures

- **Lecture I:** EFT approach to high energy QCD-The Color Glass Condensate;

  multi-particle production in the CGC

- **Lecture II:** Hadronic scattering in the CGC-multiple scattering & quantum evolution effects

  in limiting fragmentation & quark pair production

- **Lecture III:** Plasma instabilities & thermalization

  in the CGC; computing particle production in

  Heavy Ion collisions to next-to-leading order (NLO)

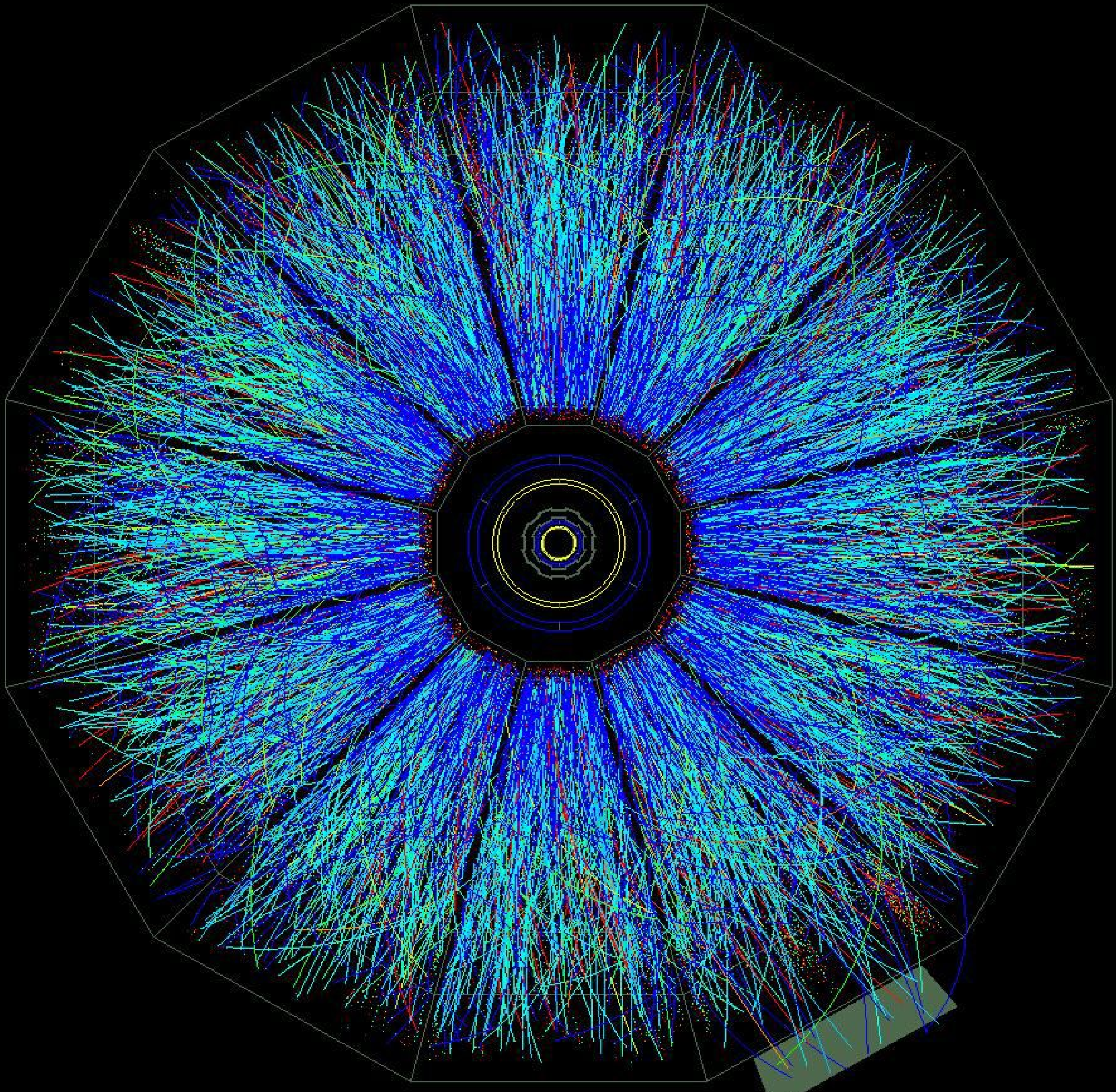
Lectures based on:

- *Plasma Instabilities:* P. Romatschke & RV,
  PRL 96: 062302 (2006); hep-ph/0605045

- *Multiparticle production to NLO:* F. Gelis & RV,
  hep-ph/0601209; hep-ph/0605246

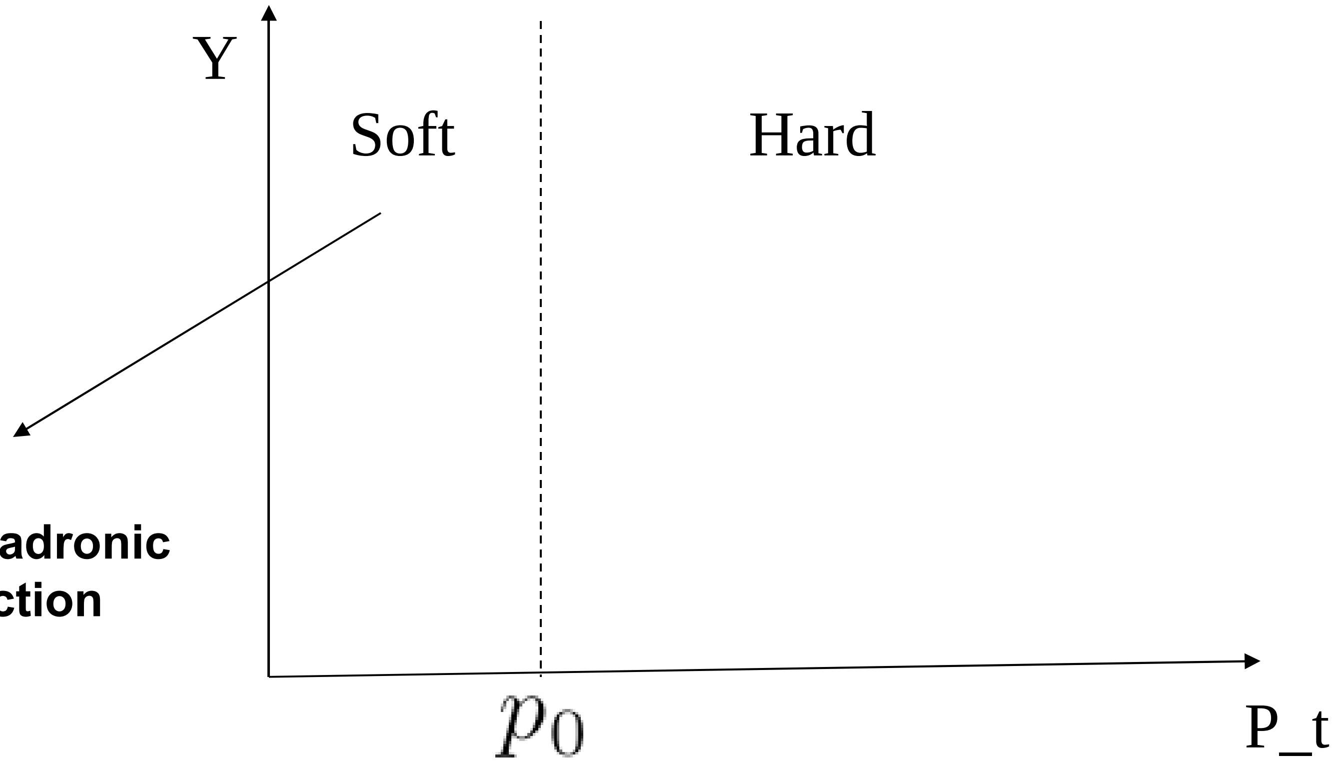
- *Limiting Fragmentation:* F. Gelis, A. Stasto & RV,
  hep-ph/0605087

- *Quark Pair Production:* H. Fujii, F. Gelis & RV,
  PRL 95: 162002 (2005); hep-ph/0603099



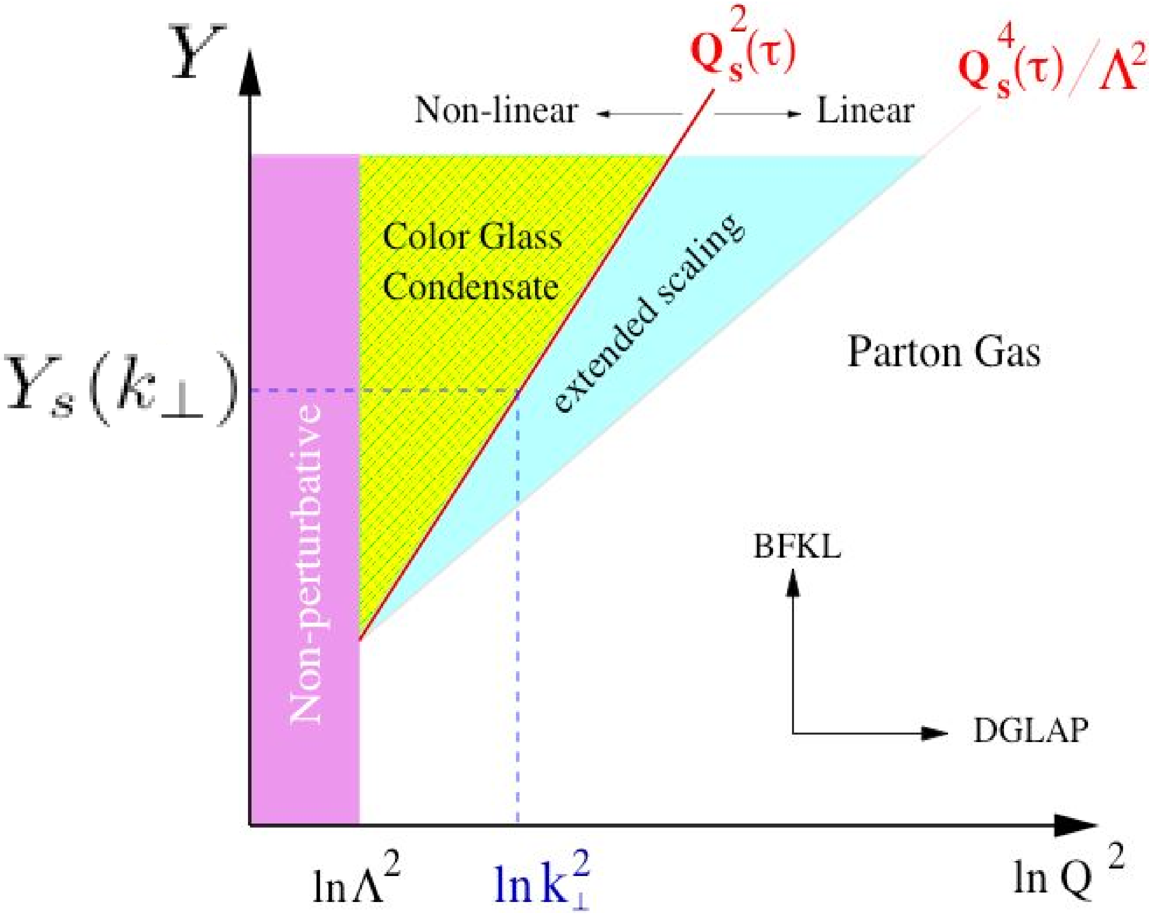
STAR

Traditional view of hadronic collisions



Bulk of hadronic cross-section

- An alternative perspective…

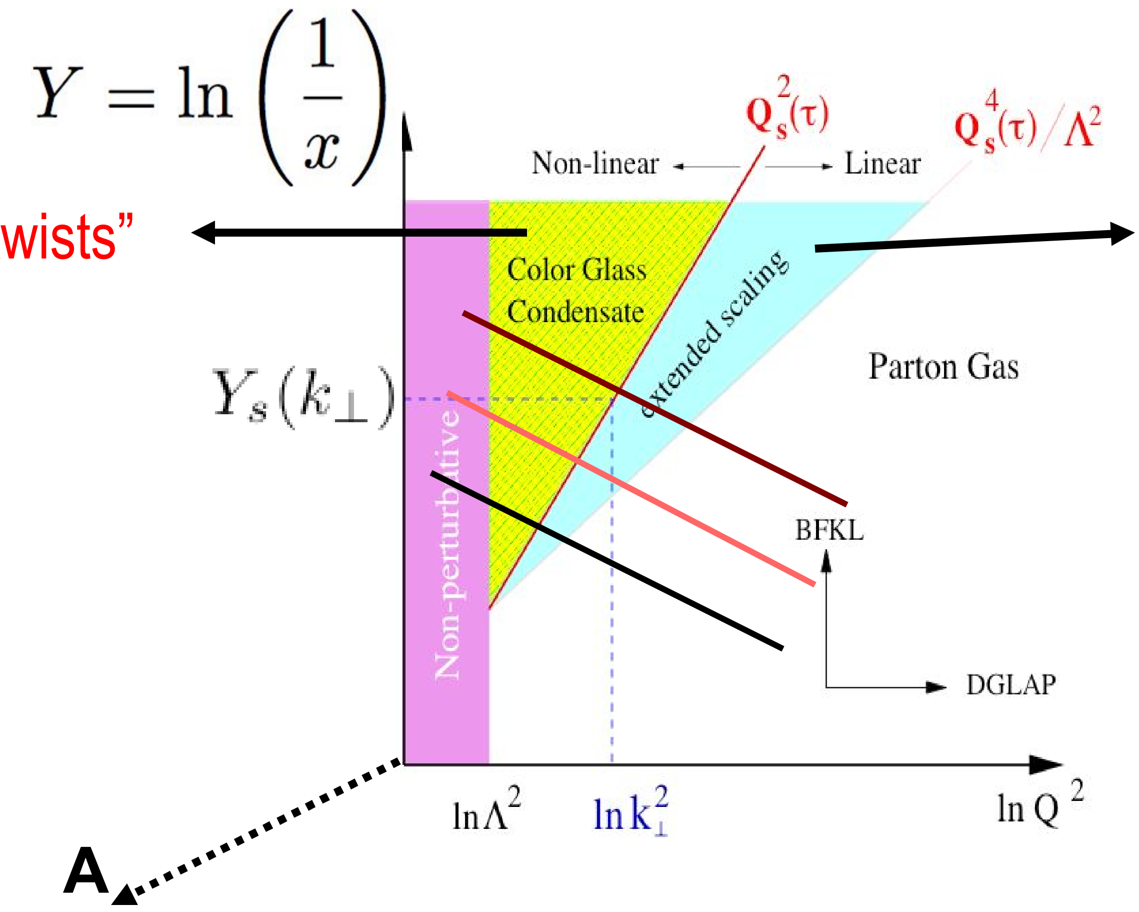


Review:
Iancu & RV:
hep-ph/0303204

$$\alpha_S(Q_s^2) << 1 \text{ for } Q_s^2 >> \Lambda_{\text{QCD}}^2$$

Weak coupling techniques may be applicable…

Novel regime of QCD at high energies



"Higher twists"

Leading twist shadowing

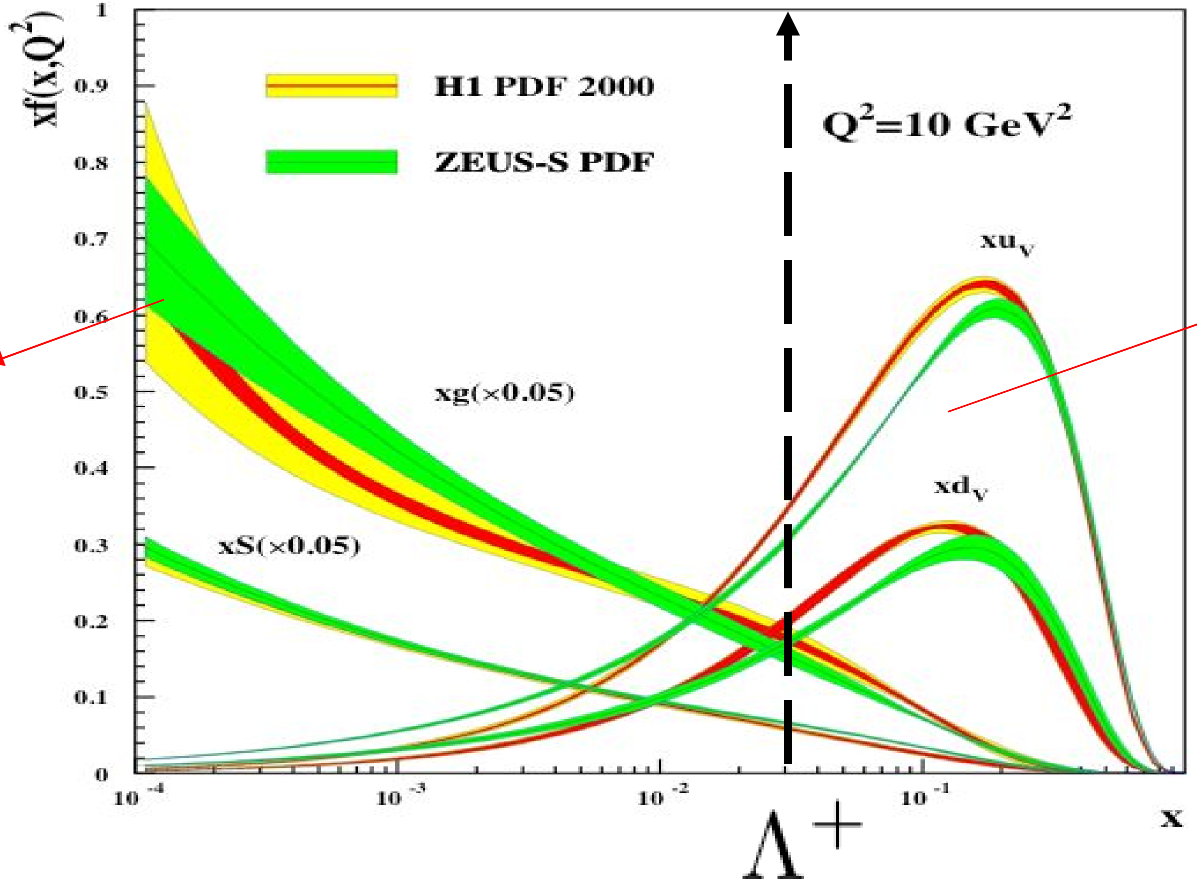
$$Y_{\mathrm{RHIC-central}} = 0 \ (x = 10^{-2}, Q_s = 1.4\,\mathrm{GeV})$$

$$Y_{\mathrm{RHIC}} = 3, Y_{\mathrm{LHC-central}} = 0 \ (x = 5 \cdot 10^{-4}, Q_s = 2.2\,\mathrm{GeV})$$

$$Y_{\mathrm{LHC}} = 3 \ (x = 3 \cdot 10^{-5}, Q_s = 3.4\,\mathrm{GeV})$$

THE HADRON AT HIGH ENERGIES

McLerran, RV;
Kovchegov;
Jalilian-Marian,Kovner,McLerran, Weigert



Dynamical
Wee modes

Valence modes-
are static
sources for wee
modes

Born-Oppenheimer: separation of large x and small x modes

Weinberg, 1966
Susskind, 1968
Leutwyler-Stern, 1976

Q.F.T on the light cone

isomorphism

Two dimensional quantum mechanics

Light cone dispersion relation:

Momenta

$$P^- = \frac{(P_t^2 + M^2)}{2P^+}$$

Energy

Mass

Light cone pert. theory = Rayleigh-Schrodinger pert. theory

Large x = Large masses
Small x = Small masses > **Effective field theory**

HIGH ENERGY EFFECTIVE ACTION

Generating functional:

Scale separating sources and fields

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional describing distribution of the sources

$$S[A,\rho] = \frac{-1}{4} \int d^4x F^2_{\mu\nu} + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp) \ln\left(U_{-\infty,\infty}[A^-]\right)\right)$$

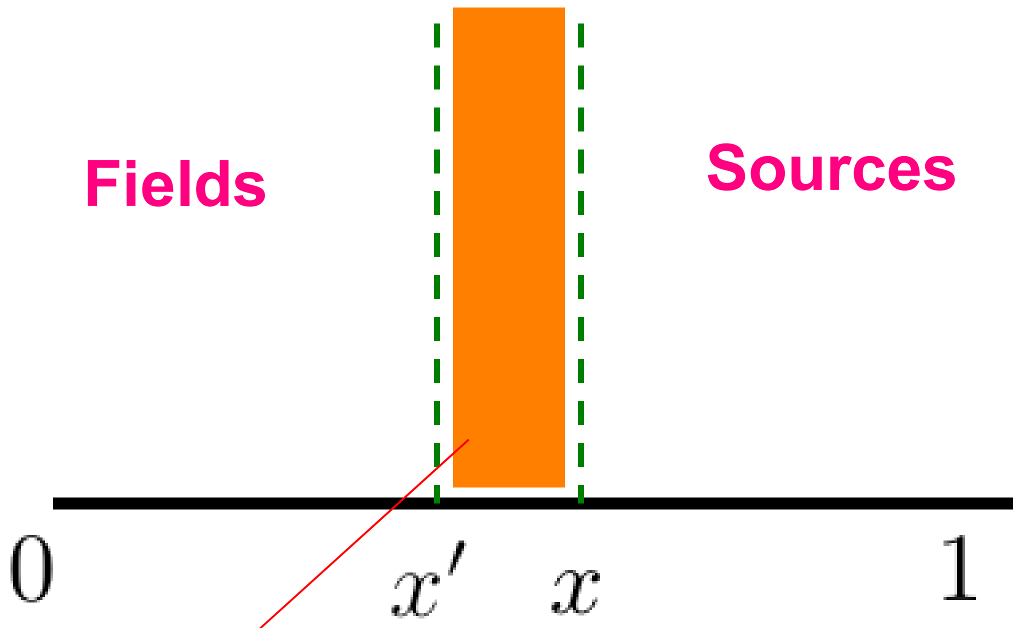
Dynamical wee fields

Coupling of wee fields to classical sources

where $U_{-\infty,\infty}[A^-] = \mathcal{P} \exp\left(ig \int dx^+ A^{-,a} T^a \right)$

Fukushima; Jalilian-Marian, Jeon, RV

Quantum evolution of classical theory: Wilsonian RG



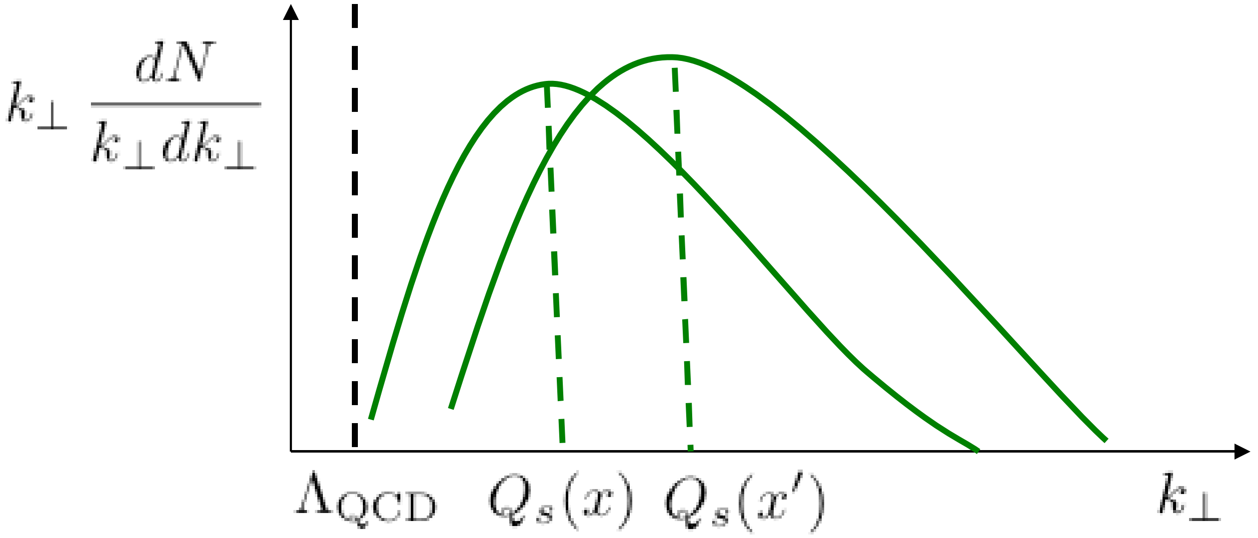
Integrate out

Small fluctuations => Increase color charge of sources

JIMWLK

(Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner)

Hadron at high energies is a Color Glass Condensate



- ✓ Gluons are colored
- ✓ Random sources evolving on time scales much larger than natural time scales-very similar to spin glasses
- ✓ Bosons with large occupation ~ $\frac{1}{\alpha_S}$ - form a condensate
- ✓ Typical momentum of gluons is Q_s

Hadron & Nuclear Scattering at high energies

$$\boxed{M \approx \sqrt{s} >> \Lambda_{\mathrm{QCD}}}$$

$$x_1 G(x_1, M^2)$$

$$x_2 G(x_2, M^2)$$

Collins,Soper,Sterman

$$\Lambda_{\mathrm{QCD}} << M << \sqrt{s}$$

$$\phi_A(x_1, k_{\perp 1})$$

$$\phi_B(x_2, k_{\perp 2})$$

Are these “un-integrated gluon distributions” universal?

In CGC, “Dipoles”- with evolution a la JIMWLK / BK RG equations

Catani, Ciafaloni, Hautmann
Collins, Ellis

HADRONIC COLLISIONS IN THE CGC FRAMEWORK

$W_{x_1}[\rho_{p1}]$

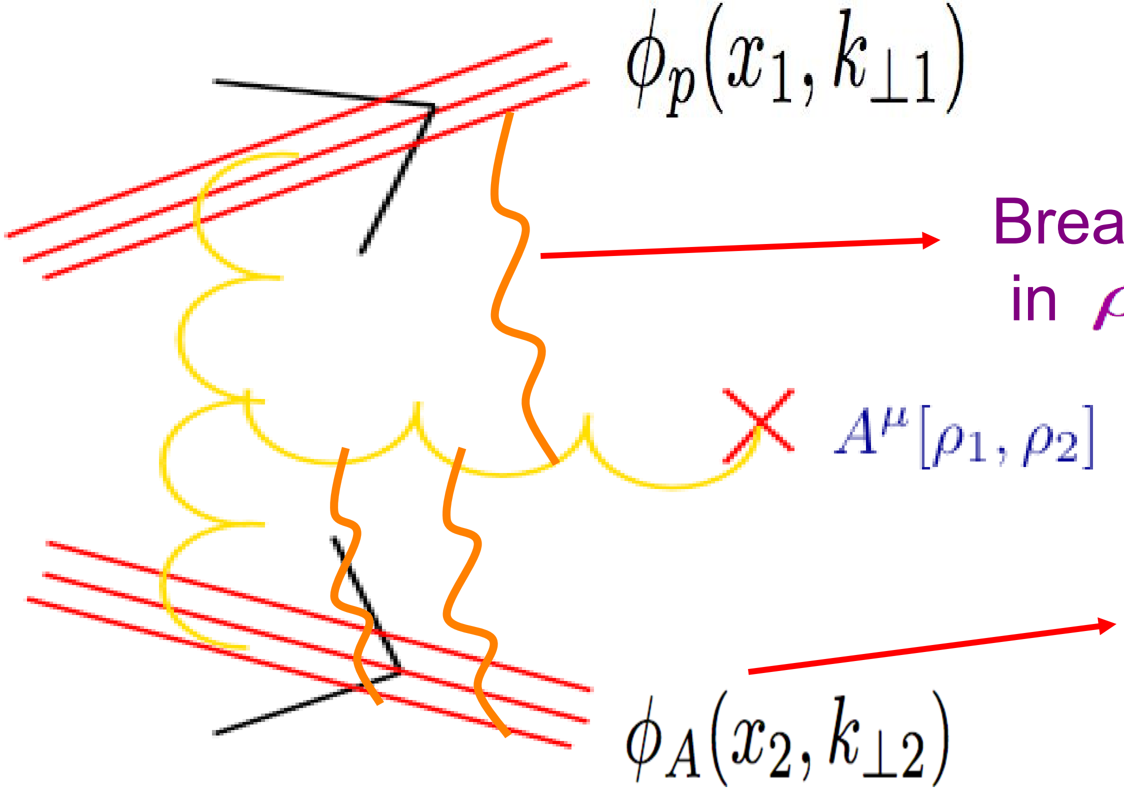
$A_\mu[\rho_{p1}, \rho_{p2}]$

$W_{x_2}[\rho_{p2}]$

Solve Yang-Mills equations for two light cone sources: ρ_{p1} & ρ_{p2}

For observables $O\left(A_\mu(\rho_{p1}, \rho_{p2})\right)$ average over $W_{x1}[\rho_{p1}]$ & $W_{x2}[\rho_{p2}]$

Inclusive gluon production in pp & p-A :



Breaks down at next order in $\rho_p / k_{1\perp}^2$

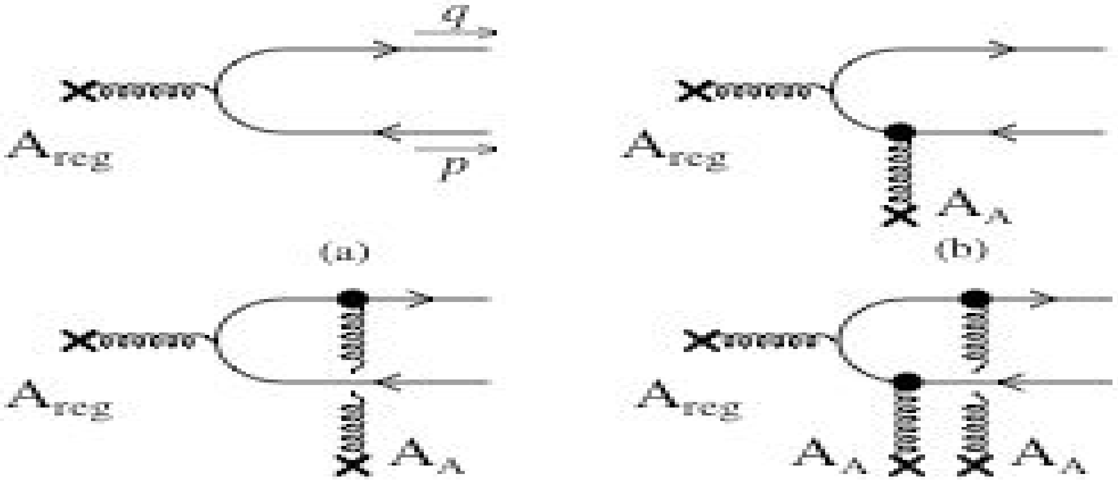
Krasnitz,RV;
Balitsky

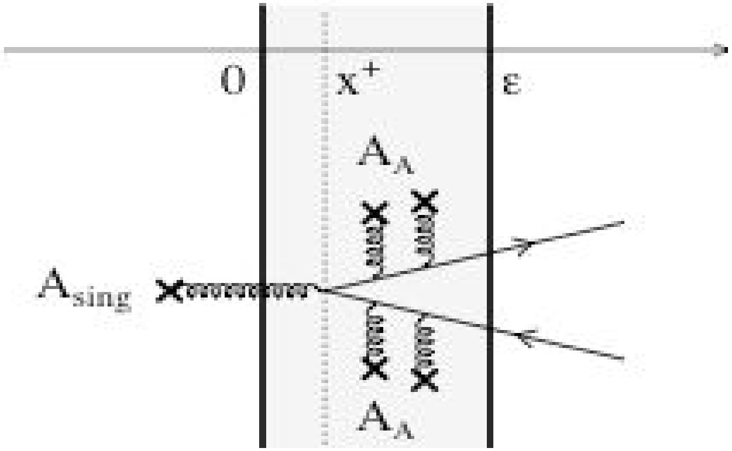
Adjoint dipole
-includes all twists

- K_t factorization seen "trivially" in p-p
- Also holds for inclusive gluon production lowest order in $\rho_p / k_{1\perp}^2$ but all orders in $\rho_A / k_{1\perp}^2$ (Kovchegov, Mueller)

Quark production to all orders in pA

Blaizot, Gelis, RV





- ☆ **Neither quark pair production nor single quark production is kt-factorizable**

Result can however still be "factorized" into novel multi-parton distributions

$$\frac{d\sigma^{pA\rightarrow q\bar{q}X}}{dy_p dy_A\, d^2p_\perp d^2q_\perp} \propto \phi_p \times \left[A\,\phi_{g,g} + \left(B\,\phi_{g;q\bar{q}} + h.c\right) + C\,\phi_{q\bar{q};q\bar{q}}\right]$$

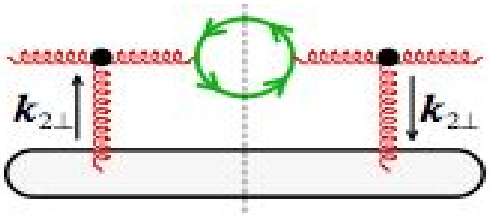
These multi-parton distributions can be computed in closed form in the Gaussian (MV) approximation

Quantum evolution of these distributions can be computed By solving the JIMWLK or BK renorm. group equations

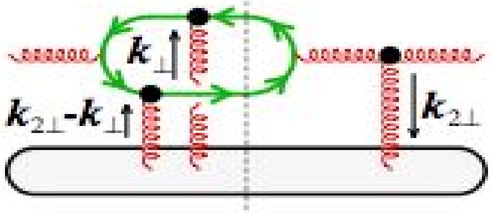
Interpretation:

Blaizot, Gelis, RV; Tuchin

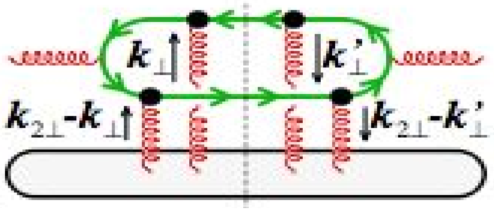
$$\phi_A^{g,g}(\vec{k}_{2\perp}) \quad \propto$$



$$\phi_A^{q\bar{q},g}(\vec{k}_{2\perp}|\vec{k}_\perp) \quad \propto$$



$$\phi_A^{q\bar{q},q\bar{q}}(\vec{k}_{2\perp}|\vec{k}_\perp,\vec{k}'_\perp) \quad \propto$$

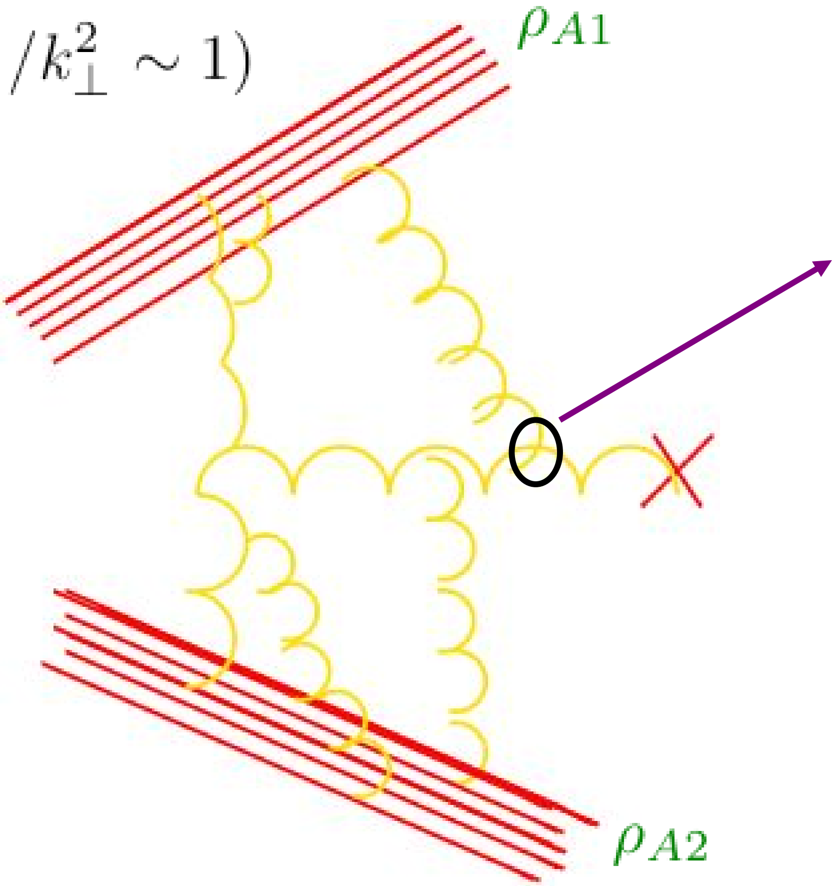


Wilson line correlators - the last appears in pair production only

<u>Simplify greatly</u> in large N_c limit
x-evolution can be computed with Balitsky-Kovchegov eqn.

Nucleus-Nucleus Collisions…leading order graphs

$(\rho_{A1}/k_\perp^2,\ \rho_{A2}/k_\perp^2 \sim 1)$



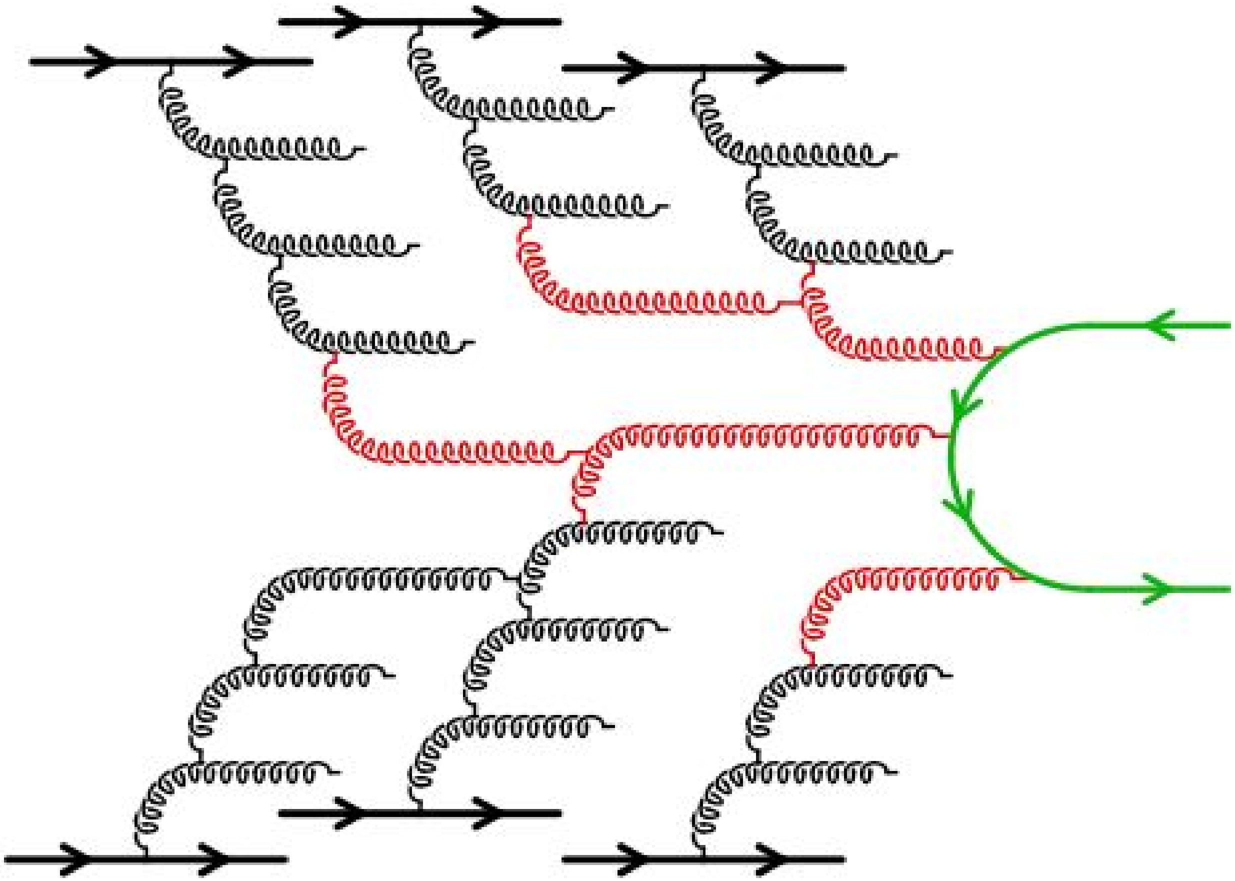
All such diagrams of Order O(1/g)

Inclusive multiplicity even to leading order requires 2 -> n Feynman amplitudes
- completely non-perturbative problem!

F. Gelis, RV
hep-ph/0601209

Solve Dirac equation in background field of two nuclei…

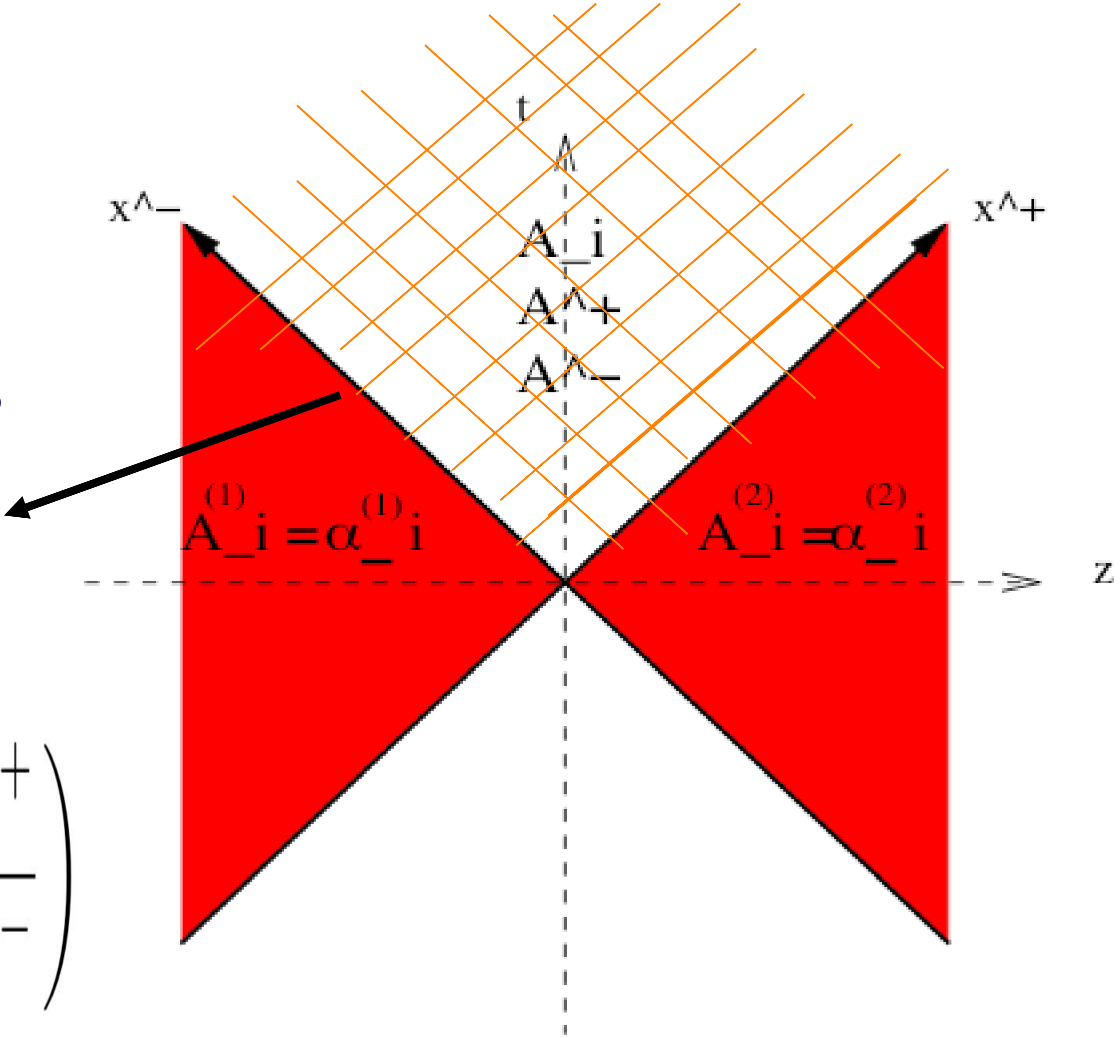
Gelis,Kajantie,Lappi PRL 2005



Yang-Mills Equations for two nuclei

$$D_\mu F^{\mu\nu,a} = \delta^{\nu+}\rho_1^a(x_\perp)\delta(x^-) + \delta^{\nu-}\rho_2^a(x_\perp)\delta(x^+)$$

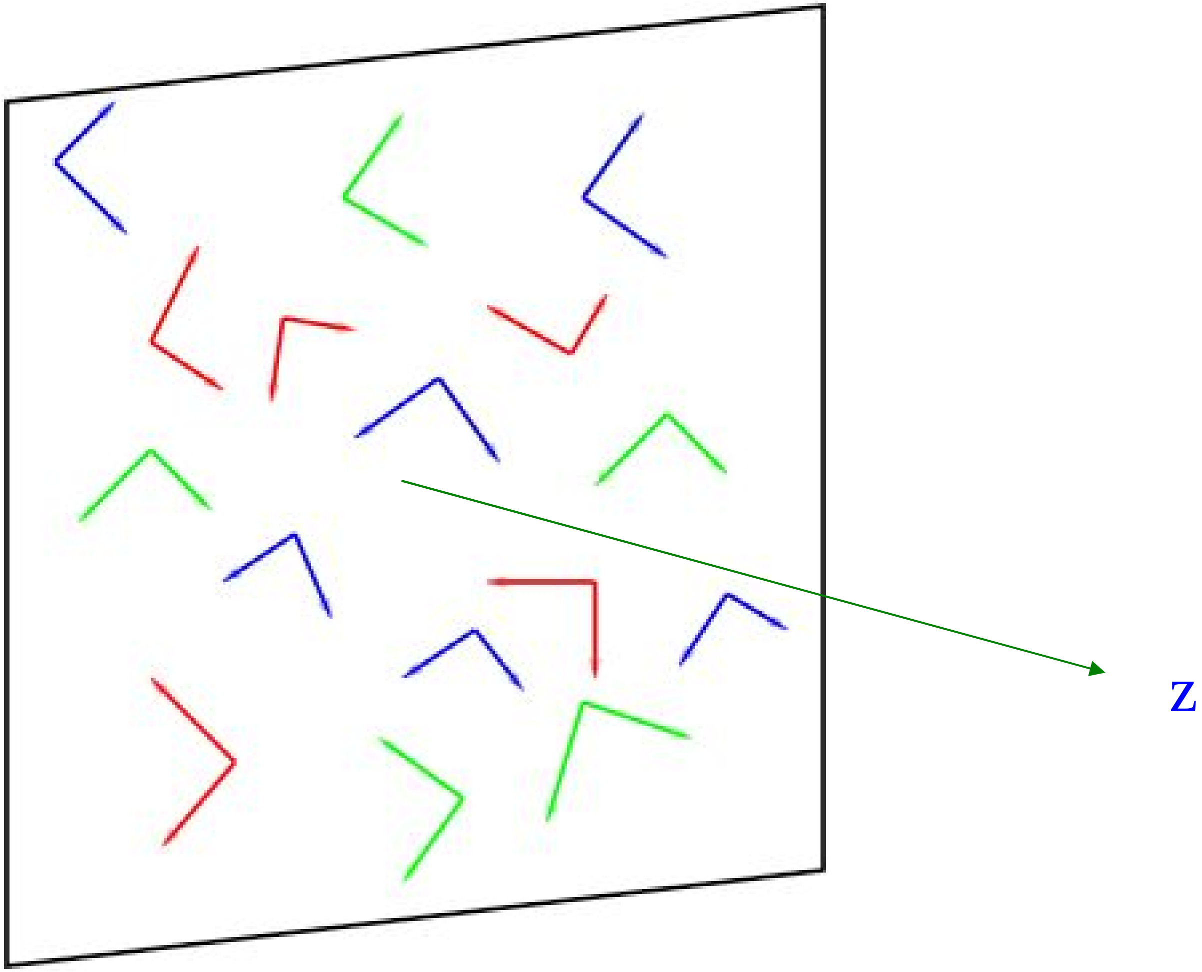
Kovner,McLerran,Weigert



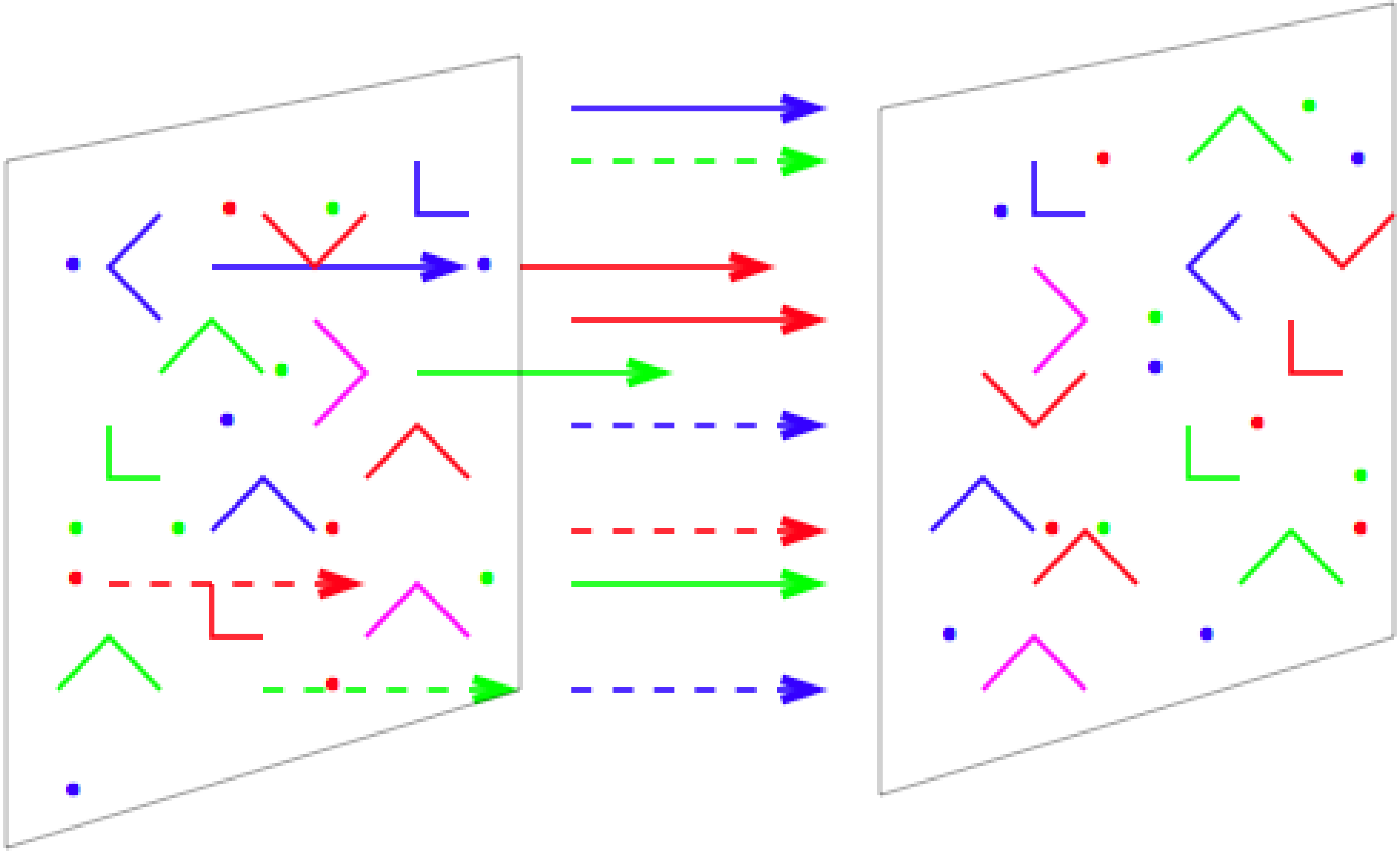
Initial conditions from matching eqns. of motion on light cone

$$\tau = \sqrt{2x^+x^-}\,;\ \eta = \frac{1}{2}\ln\left(\frac{x^+}{x^-}\right)$$

Random Electric & Magnetic fields in the plane of the fast moving nucleus



$x^- = 0$



Longitudinal E and B fields created right after the collision - non-zero Chern-Simons charge generated

$$\frac{d\nu}{d\tau} = \frac{g^2}{8\pi^2}\mathrm{Tr}(\vec{E}\cdot\vec{B})$$

Kharzeev,Krasnitz,RV; Lappi, McLerran

Lattice Formulation

Krasnitz, RV

- Hamiltonian in $A^\tau = 0$ gauge; per unit rapidity,

$$H = \frac{\tau}{2}\int d^2 r_\perp \left[p^\eta p^\eta + \frac{1}{\tau^2}E_r E_r + \frac{1}{\tau^2}(D_r\Phi)(D_r\Phi) + F_{xy}F_{xy}\right]$$

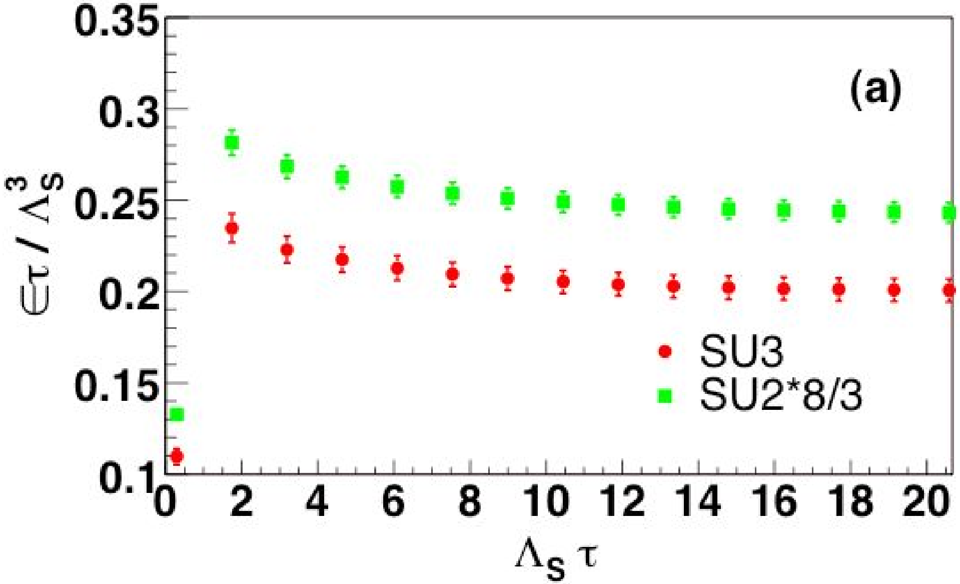
For ``perfect'' pancake nuclei, boost invariant configurations

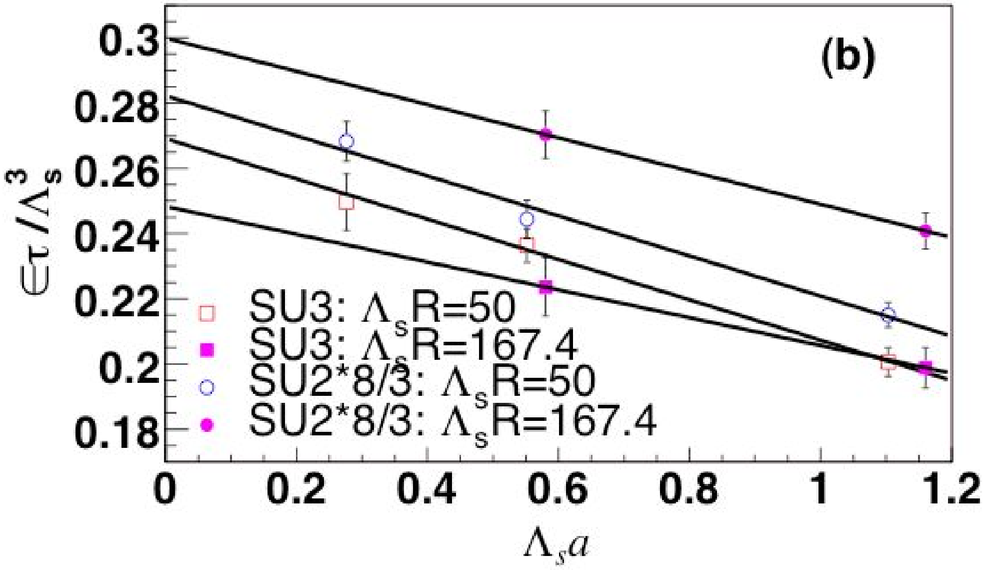
$$A_r(\tau,\eta,r_\perp) = A_r(\tau,r_\perp)\ ;\ A_\eta(\tau,\eta,r_\perp) = \Phi(\tau,r_\perp)$$

- Solve 2+1- D Hamilton's equations in real time for space-time evolution of glue in Heavy Ion collisions

Results

Krasnitz, Nara, RV
Lappi





Transverse Energy

$$\frac{1}{\pi R^2} \frac{dE_\perp}{d\eta}\Big|_{\eta=0} = \frac{f_E(\Lambda_s R)}{g^2} \Lambda_s^3$$

Proper time dependence

$$\varepsilon\tau = \alpha + \beta\, e^{-\gamma\tau}$$

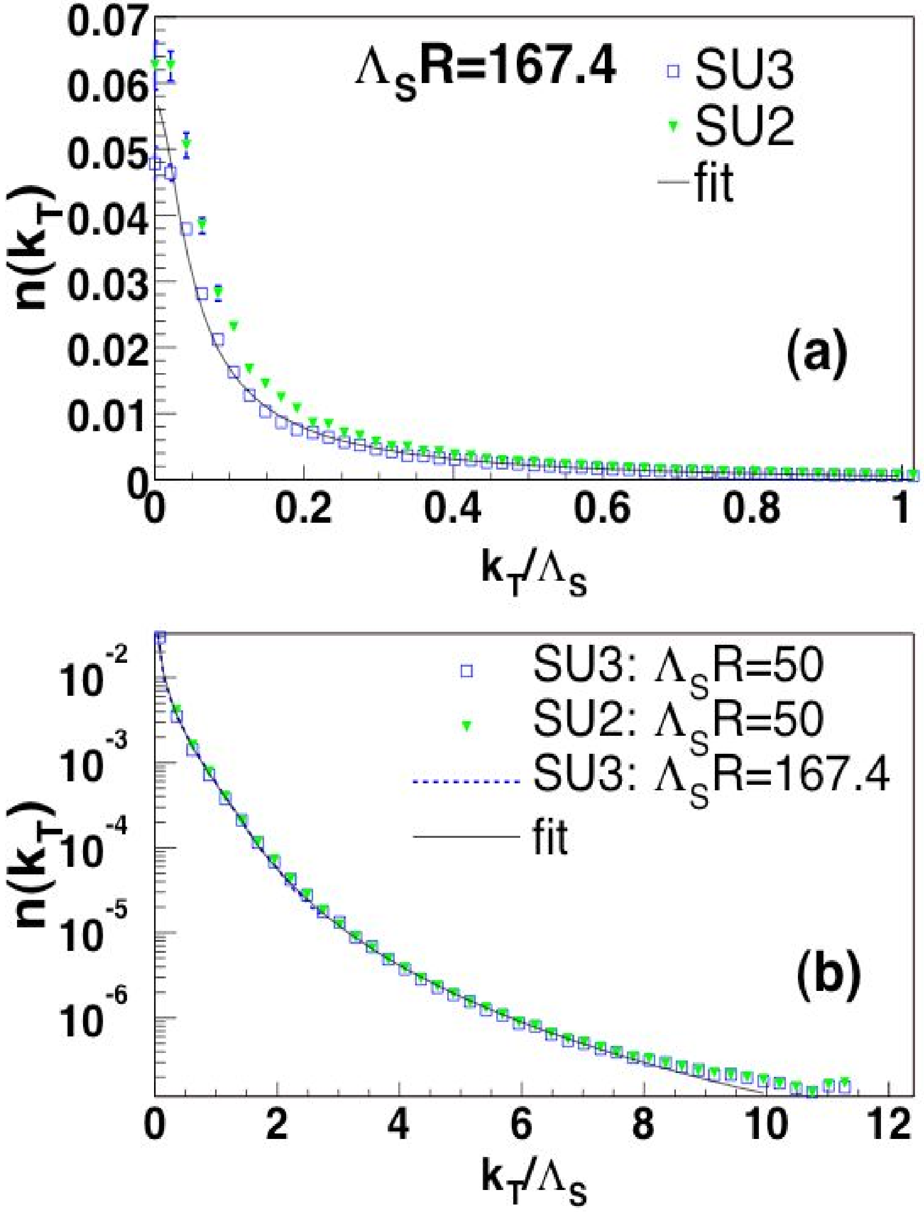
``Formation time”

$$\tau_F = 1/\gamma/\Lambda_s$$

~0.3 fm (RHIC) & 0.1 fm (LHC)

Energy Density

$$\varepsilon = \frac{0.08}{g^2} \Lambda_s^4$$



Gluon Multiplicity

$$\frac{1}{\pi R^2}\frac{dN}{d\eta}\Big|_{\eta=0} = \frac{1}{g^2}\frac{n(k_\perp)}{(N_c^2-1)}$$

with

$$n(k_\perp) = a_1 \frac{1}{\exp\left(\sqrt{k_\perp^2+m^2}/T_{\rm eff}\right)-1}$$

$(k_\perp/\Lambda_s \leq 1.5)$; $T_{\rm eff} = 0.47\,\Lambda_s$; $m = 0.03\,\Lambda_s$

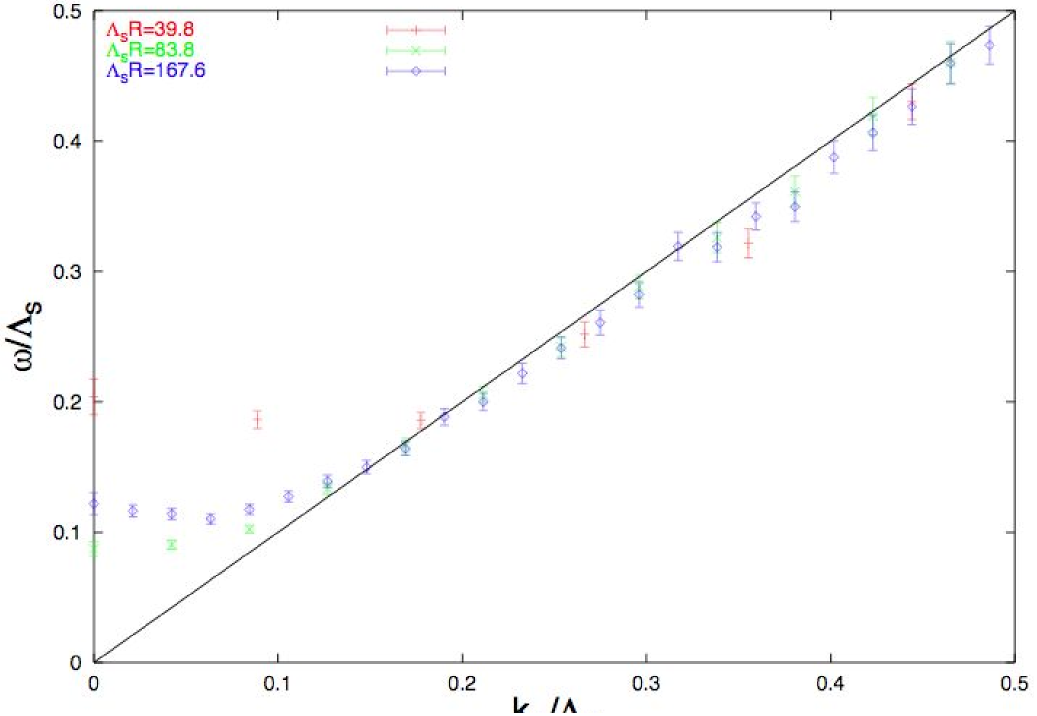
$$n(k_\perp) = a_2 \frac{\Lambda_s^4}{k_\perp^4}\ln(4\pi k_\perp/\Lambda_s)$$

$(k_\perp/\Lambda_s > 1.5)$

dists. are infrared finite

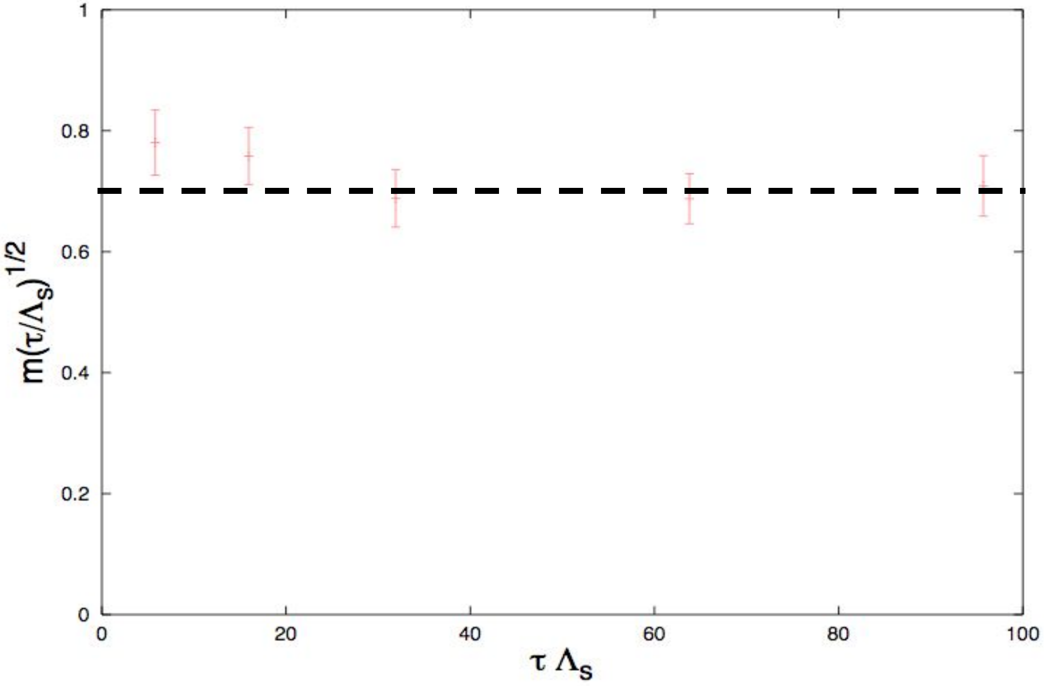
PRL 87, 192302 (2001)

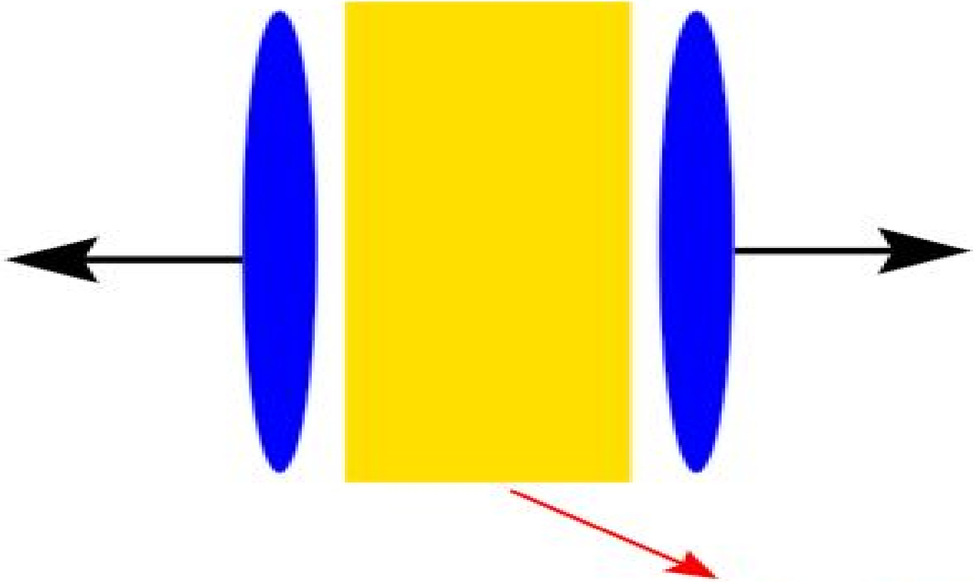
Dispersion relation:



$$m^2 \propto n(\tau) \sim \frac{1}{\tau}$$

Just as for a Debye screening mass





Classical Fields with occupation # f= $\frac{1}{\alpha_S}$

Initial energy & multiplicity of <u>produced</u> gluons depends on Q_s

$$\frac{1}{\pi R^2}\frac{dE}{d\eta} = \frac{0.25}{g^2} Q_s^3 \qquad \frac{1}{\pi R^2}\frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2$$

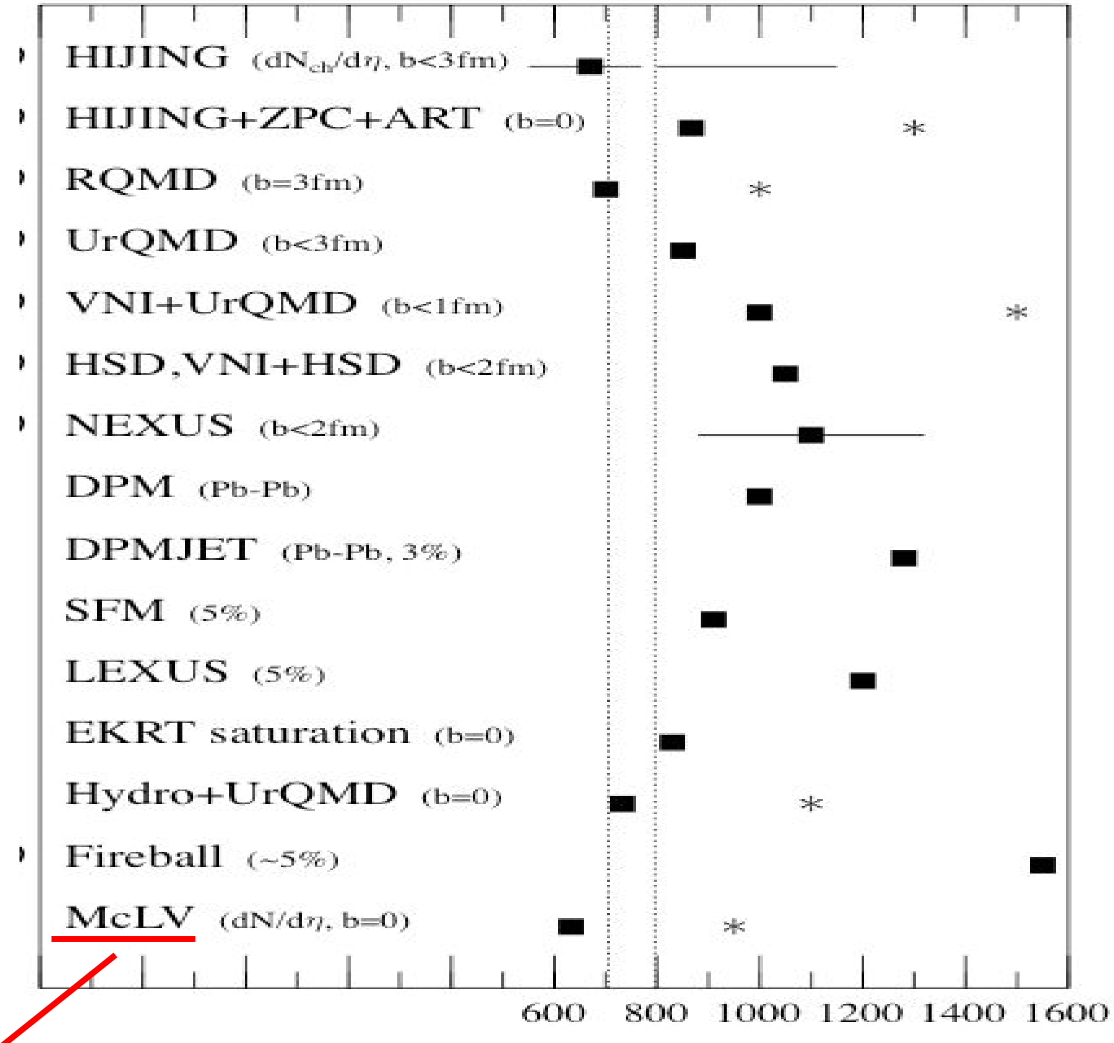
Straight forward extrapolation from fits of saturation models to HERA data

$$\boxed{(Q_s^2)^{\mathrm{RHIC}} = A^{1/3} \left(\frac{x_0}{x_{\mathrm{RHIC}}}\right)^{0.3} \mathrm{GeV}^2}$$

RHIC : $Q_s \approx 1.4\ \mathrm{GeV}$

LHC : $Q_s \approx 2.2\ \mathrm{GeV}$

Predictions for Au+Au multiplicity at RHIC

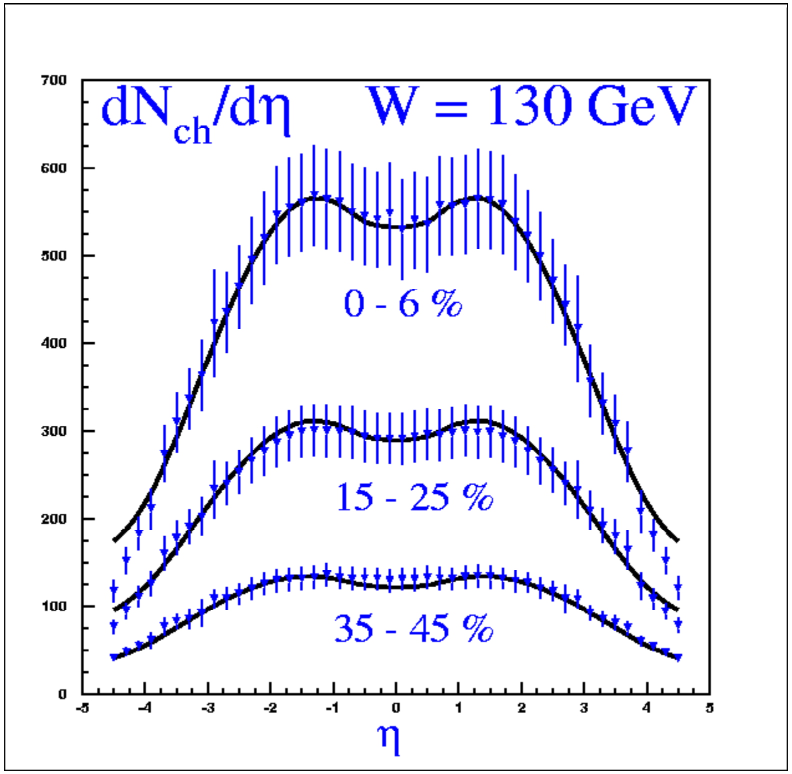


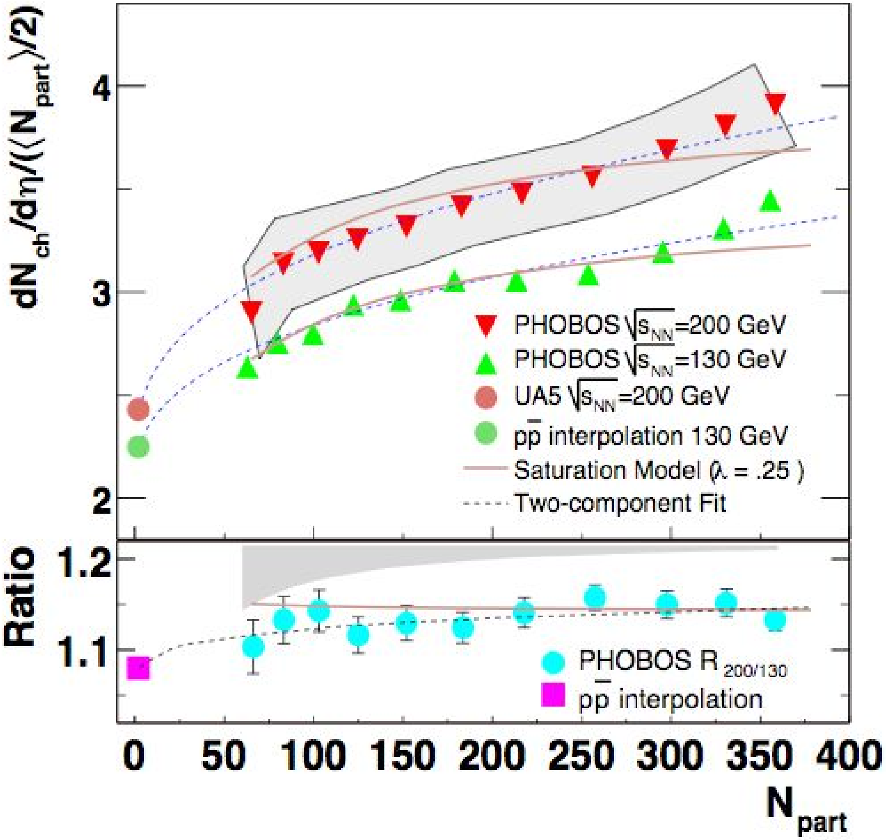
Eskola, QM 2001

Krasnitz, RV

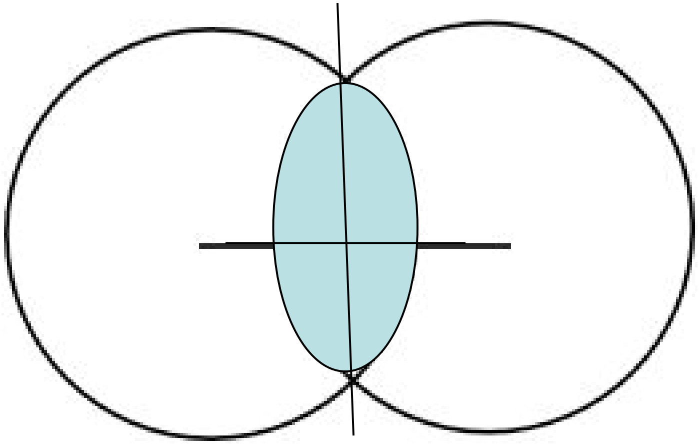
Successful KLN phenomenology for multiplicities at RHIC

Kharzeev,Levin,
Nardi





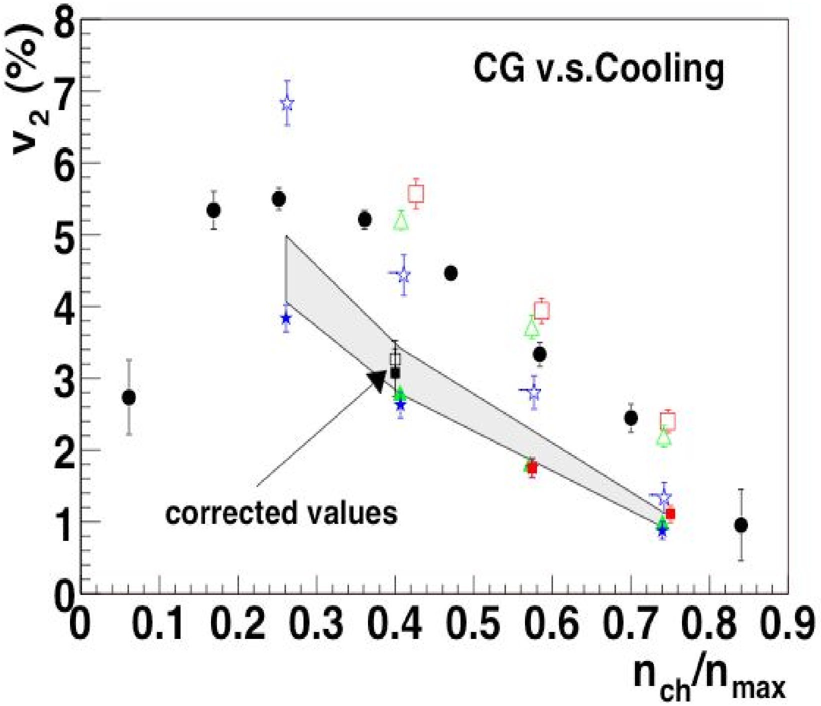
Elliptic flow of colored glass



$$v_2 N = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{dt}{\sqrt{t}} \left(T^{xx}(t) - T^{yy}(t)\right)$$

where

$$T_{xx} - T_{yy} = \int d^2x_\perp \left[E_y^2 - E_x^2 + (D_x\Phi)^2 - (D_y\Phi)^2\right]$$



Can compute with cooling (above) method and in Coulomb gauge- both methods converge at late times

Krasnitz, Nara, RV PLB (2003)

Also…

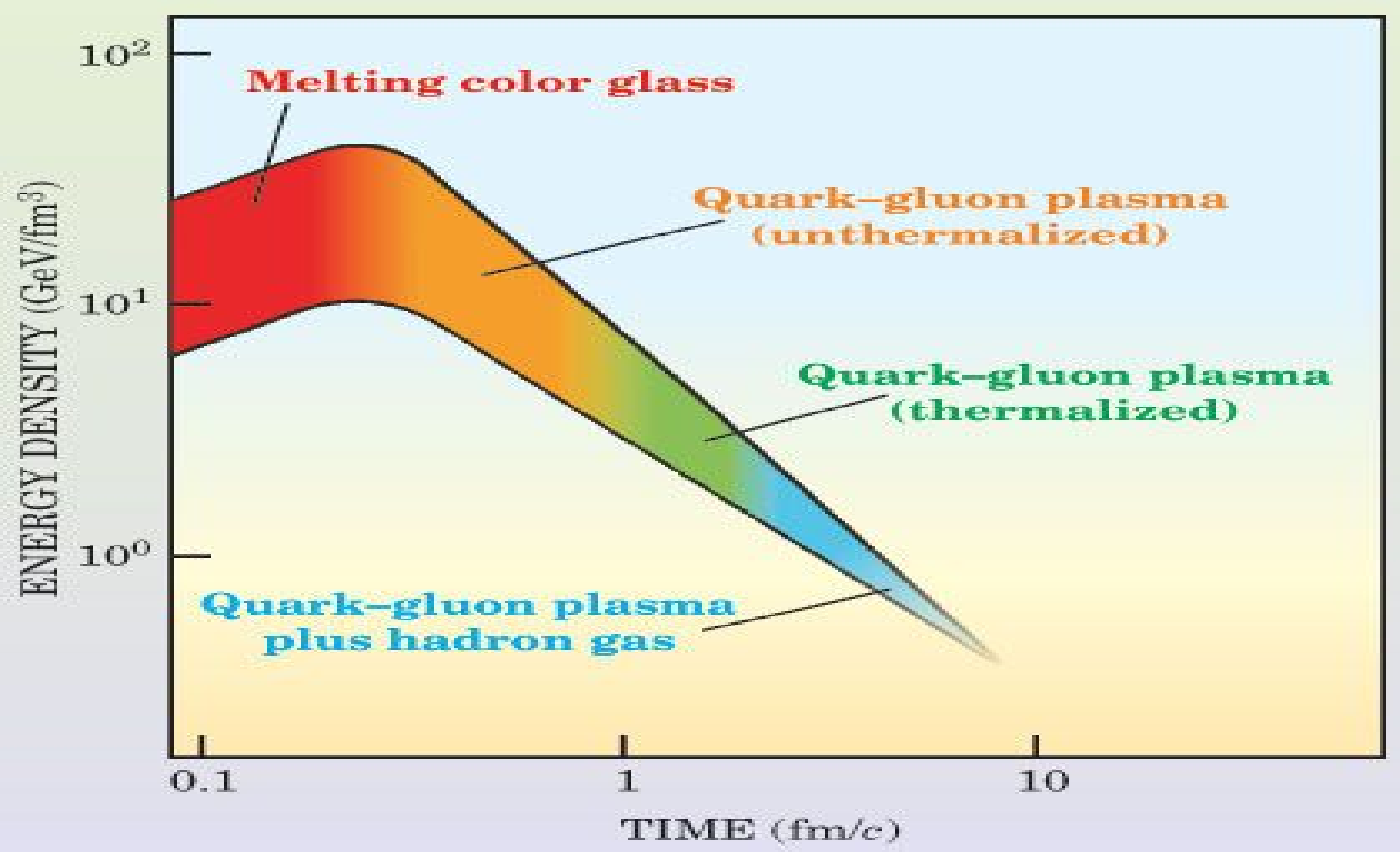
$$(E_\perp/N)^{\rm CGC} \approx Q_s$$

$$(E_\perp/N)^{\rm RHIC} \approx Q_s/3$$

OK, if system does P dV work - hydrodynamics…

Melting CGC to QGP

L. McLerran, T. Ludlam,
Physics Today



Glasma…

Classical field

Classical field / Particle

Particle

$$f \sim \frac{1}{\alpha_S}$$

$$1 \leq f < \frac{1}{\alpha_S}$$

$$f < \frac{1}{\alpha_S}$$

Mueller, Son;
Gelis, Jeon, RV, in preparation