Self-consistent gaussian model of nonperturbative QCD vacuum

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Cracow School of Theoretical Physics

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- Nonperturbative QCD vacuum

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- Pion DA

Introducing NLC in QCD calculations

$$T\left(\bar\psi\psi\right) = \overbrace{\bar\psi\psi} + :\bar\psi\psi: \text{(Wick theorem)}$$

$$\langle T\left(\bar\psi\psi\right)\rangle = i^{-1}\hat S_0(x) + ?$$

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[**SVZ'79**]

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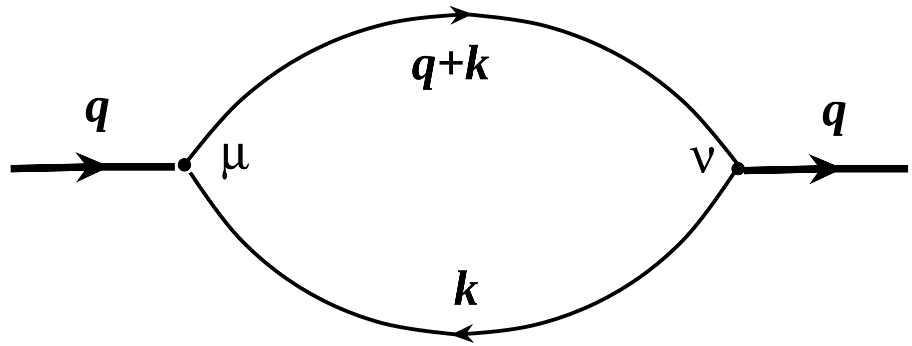
Distribution
Amplitudes,
Form Factors

Diagrams for $\langle T\left(J_\mu(z)J_\nu(0)\right)\rangle$



PT

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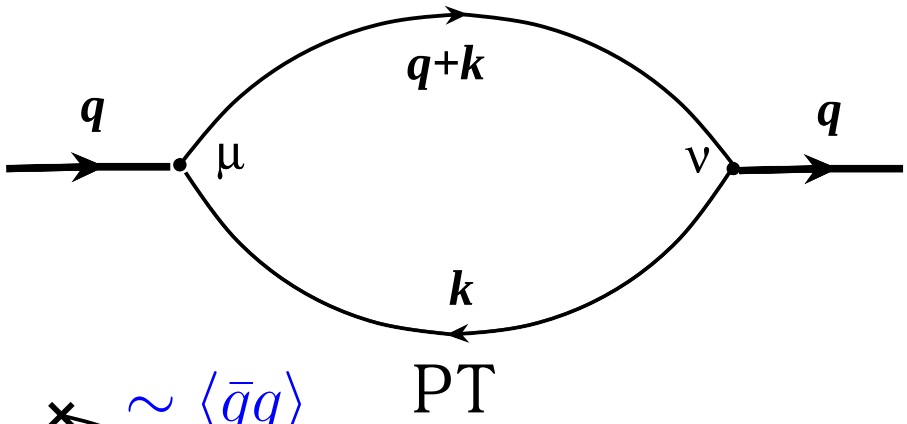

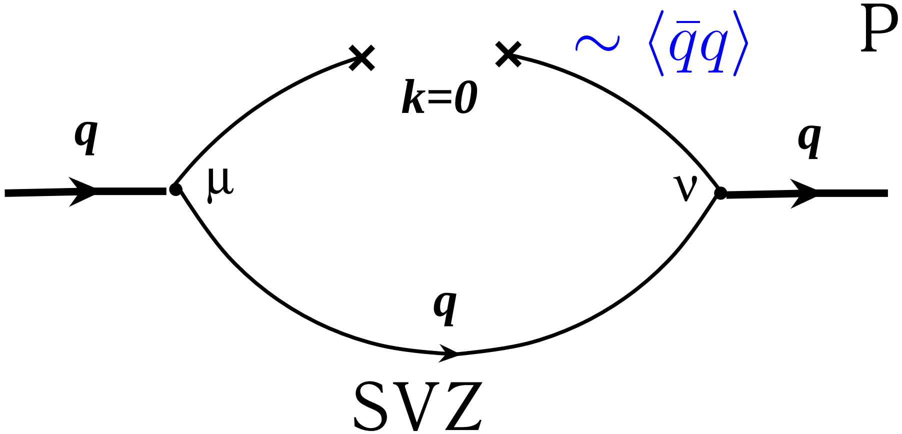

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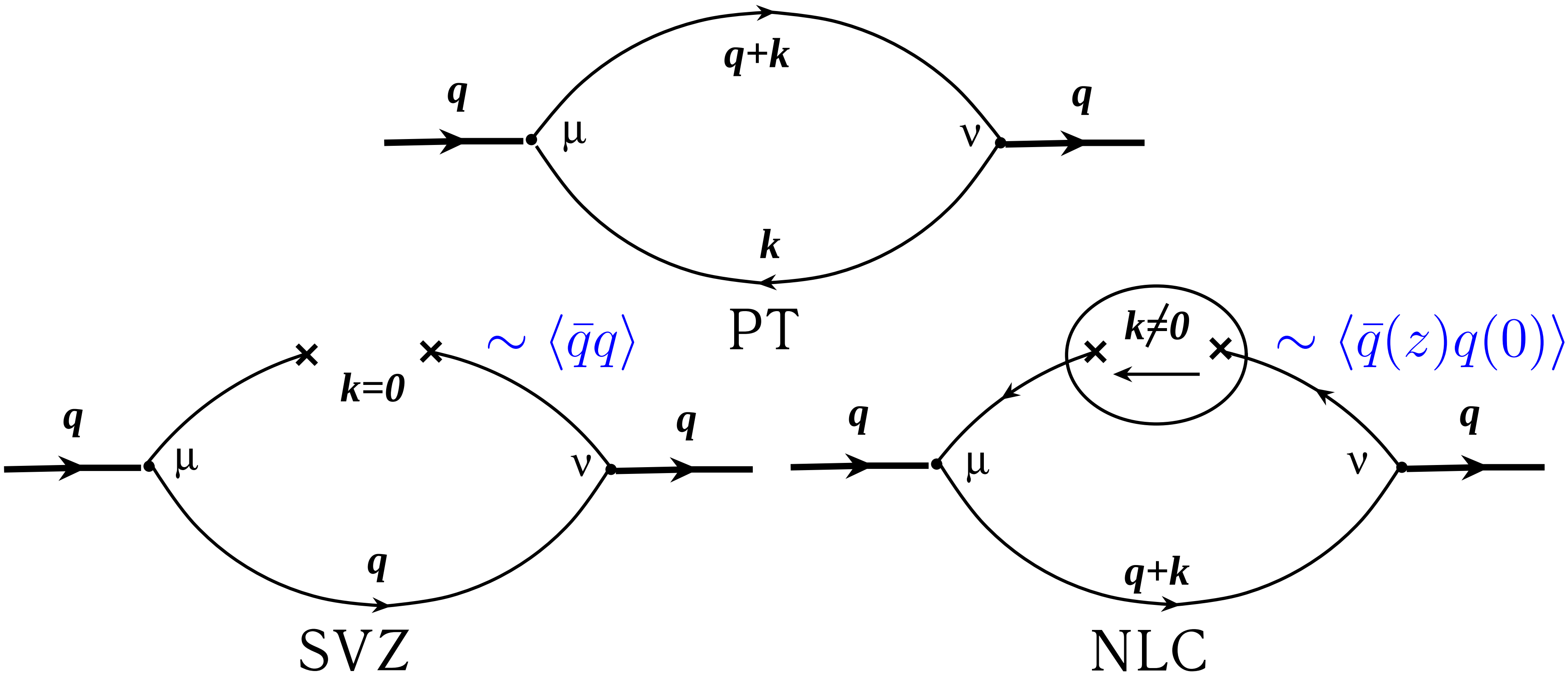
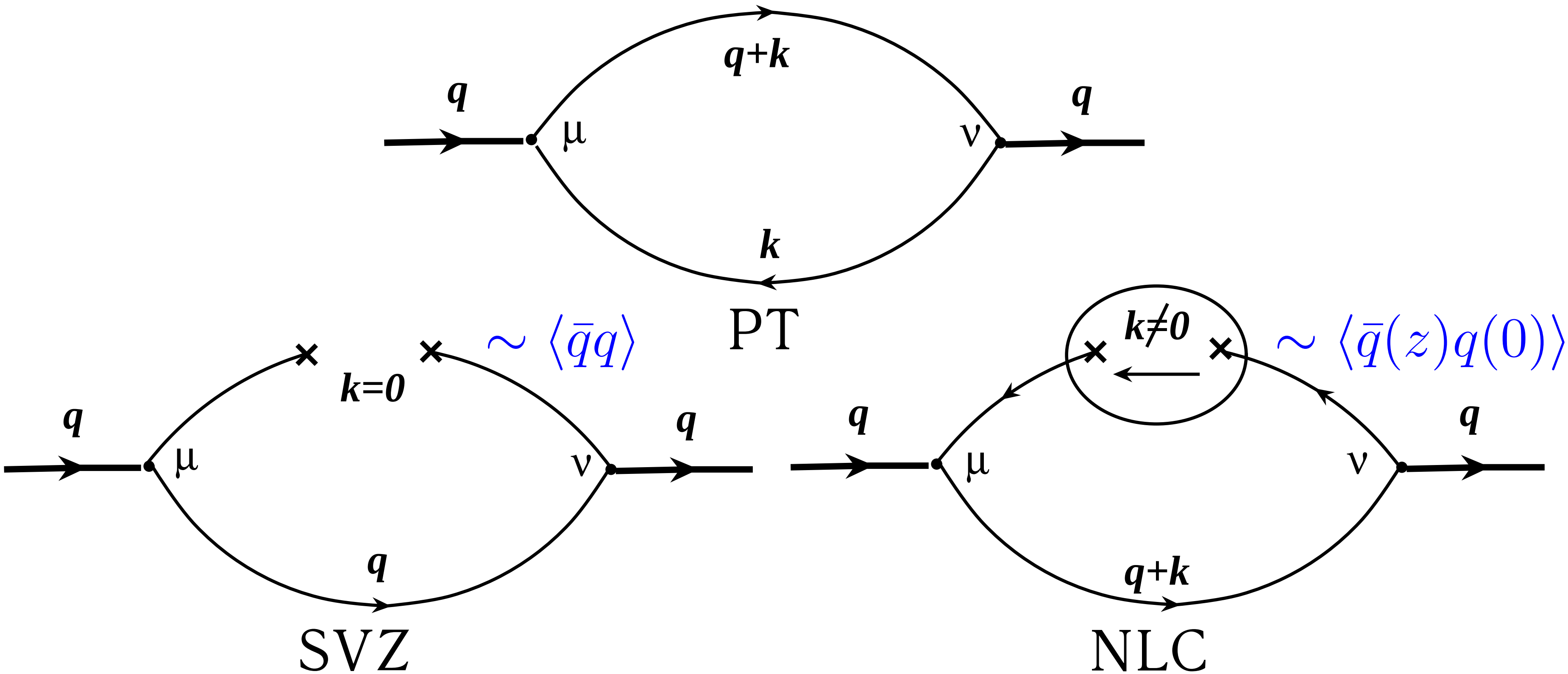


Diagrams for $\langle T\,(J_\mu(z)J_\nu(0))\rangle$



Quarks run through vacuum with nonzero momentum $k \neq 0$:

$$\langle k^2 \rangle = \frac{\langle \bar{\psi} D^2 \psi \rangle}{\langle \bar{\psi}\psi \rangle} = \lambda_q^2 = 0.4 - 0.55\,\mathrm{GeV}^2$$

NLC parameterization

Parameterization for scalar condensate was suggested in works of Bakulev, Mikhailov and Radyushkin:

$$\langle : \bar{\psi}(0)\psi(x) : \rangle \; = \; \langle \bar{\psi}\psi \rangle \int_0^\infty \boxed{f_S(\alpha)} \, e^{\alpha x^2/4} \, d\alpha \, , \text{ where } x^2 < 0.$$

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Then, first approximation which take finite width of quark distribution in vacuum into account:

$$f_S(\alpha) = \delta\left(\alpha - \frac{\lambda_q^2}{2}\right), \qquad \lambda_q^2 = \frac{\langle \bar{\psi} D^2 \psi \rangle}{\langle \bar{\psi}\psi \rangle}.$$

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Such presentation correspond to Gaussian form $\boxed{\sim \exp\left(\lambda_q^2 x^2/8\right)}$ of NLC in coordinate representation.

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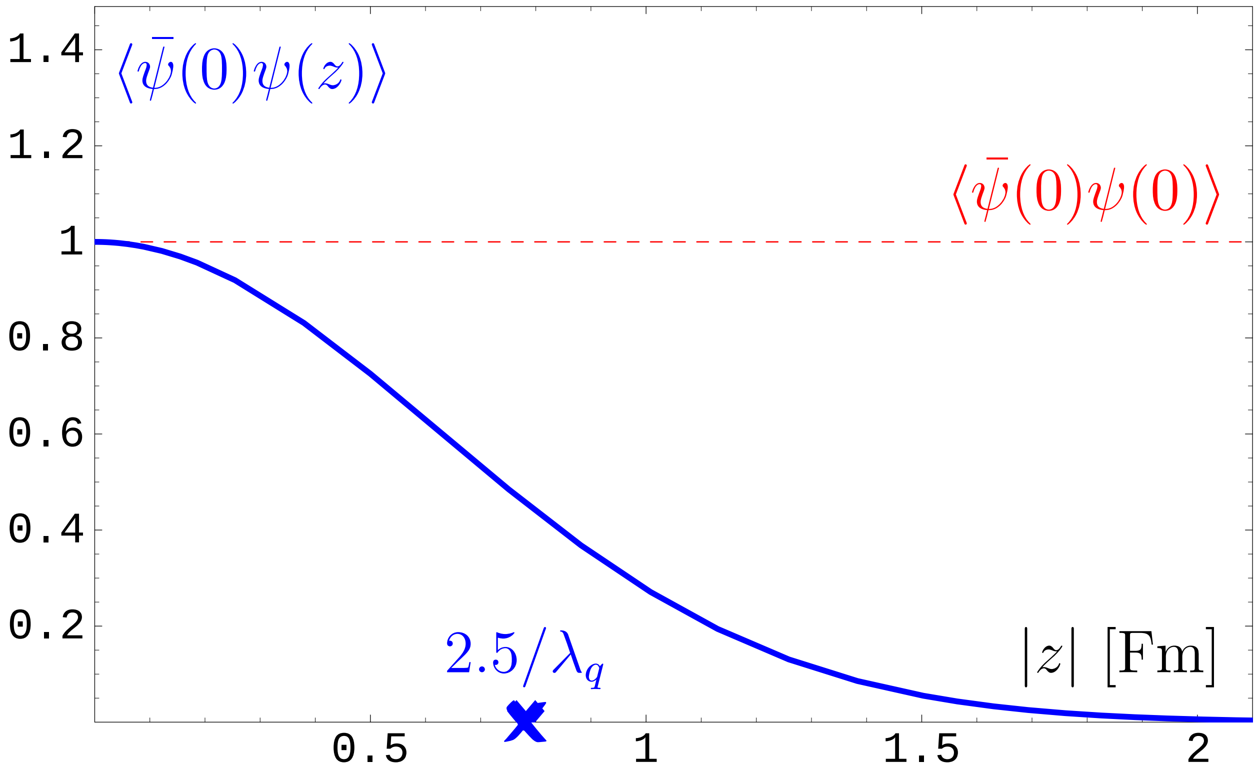
→ **Gaussian Model**

Lattice data of Pisa group



Nonlocality of quark condensates from lattice data of Pisa group in comparison with local limit.

Even at $|z| \simeq 0.5$ Fm nonlocality is quite important!

Bilocal quark condensates

Parameterization for scalar and vector condensates:

$$\langle\bar{\psi}(0)\psi(x)\rangle = \langle\bar{\psi}\psi\rangle \int_0^\infty \boxed{f_S(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$

$$\langle\bar{\psi}(0)\gamma_\mu\psi(x)\rangle = -ix_\mu A_0 \int_0^\infty \boxed{f_V(\alpha)}\, e^{\alpha x^2/4}\, d\alpha\,,$$

where $A_0 = 2\alpha_s\pi\langle\bar{\psi}\psi\rangle^2/81$.

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Very easy to obtain local limits:

$$f_S(\alpha) = \delta\,(\alpha)\,, \qquad f_V(\alpha) = \delta'(\alpha)\,.$$

Quark-Gluon-Quark condensate

Convenient to term the 3-local condensate in fixed-point gauge by introducing three scalar functions:

$$\langle\bar{\psi}(0)\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle = (x_\mu y_\nu - g_{\mu\nu}(xy))\overline{M}_1(y^2, x^2, (x-y)^2) + (x_\mu x_\nu - g_{\mu\nu}x^2)\overline{M}_2(y^2, x^2, (x-y)^2)\,,$$

$$\langle\bar{\psi}(0)\gamma_5\gamma_\mu(-g\widehat{A}_\nu(x))\psi(y)\rangle = i\varepsilon_{\mu\nu xy}\overline{M}_3(y^2, x^2, (x-y)^2)\,,$$

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$$\overline{M}_i(y^2, x^2, (x-y)^2) = A_i \iiint_0^\infty d\alpha_1\, d\alpha_2\, d\alpha_3 \boxed{f_i(\alpha_1, \alpha_2, \alpha_3)}\, e^{\left(\alpha_1 y^2 + \alpha_2 x^2 + \alpha_3 (x-y)^2\right)/4}\,.$$

Coefficients are $A_i = \{-\frac{3}{2}, 2, \frac{3}{2}\}A_0$, (Mikhailov, Radyushkin).

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Easy transition to local case:

$$f_i\,(\alpha_1, \alpha_2, \alpha_3) = \delta\,(\alpha_1)\;\delta\,(\alpha_2)\;\delta\,(\alpha_3)\;.$$

Minimal Gaussian Model

Bakulev, Mikhailov, Radyushkin, and Stefanis use the minimal Gaussian ansatz:

$$f_S(\alpha) = \delta\left(\alpha - \Lambda_{\rm S}\right)\ , \quad f_V(\alpha) = \delta'(\alpha - \Lambda_{\rm V})\ , \quad \Lambda_{\rm V} = \Lambda_{\rm S} = \lambda_q^2/2\,,$$

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There is one parameter which was estimated in sum rules and has value $\lambda_q^2 = 0.4 - 0.55\,\mathrm{GeV}^2$.

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Problems:

- **QCD equations of motion are violated**

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Problems:

- **QCD equations of motion are violated**
- **Vector current correlator is not transverse $\Rightarrow$ gauge invariance is broken**

The aim of our investigations to cure these deficiencies.

QCD equation of motion for condensates

QCD equation of motion for splitted vector quark current

$$j_\mu(x) = \bar{\psi}(0)\gamma_\mu\psi(x)$$

in massless QCD in fix-point gauge:

$$(\partial_\mu - ig\hat{A}_\mu(x))j_\mu(x) \;\; = \;\; 0\,.$$

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If we average it over physical QCD vacuum, then we obtain the remarkable equation for condensates:

$$\partial_\mu\langle\bar{\psi}(0)\gamma^\mu\psi(x)\rangle = i\langle\bar{\psi}(0)g\hat{A}_\mu(x)\gamma^\mu\psi(x)\rangle\,.$$

Minimal Gaussian ansatz does not satisfy this equation.

Improved Gaussian model

We modify functions f_i by introducing new parameters:

$$
\begin{aligned}
f_i^{\mathrm{imp}}(\alpha_1, \alpha_2, \alpha_3) = \quad & (1 + X_i \partial_x + Y_i \partial_y + Z_i \partial_z) \\
& \delta(\alpha_1 - x\Lambda_{\mathrm{V}})\, \delta(\alpha_2 - y\Lambda_{\mathrm{V}})\, \delta(\alpha_3 - z\Lambda_{\mathrm{V}}) \ .
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What does it give?:

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What does it give?:

- **If these conditions are fulfilled**

$$12\left(X_2 + Y_2\right) - 9\left(X_1 + Y_1\right) = 1\,, \quad x + y = 1\,,$$

than QCD equations of motion are satisfied;

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- **We minimize nontransversity of polarization operator by special choice of model parameters.**

Transversity of VV correlator

We study correlator

$$\Pi^N_{\mu\nu} = i \int d^4x\, e^{iqx} \langle 0 | T \left[J^N_\mu(0) J^+_\nu(x) \right] | 0 \rangle \,,$$

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of two vector currents

$$J^+_\nu(x) = \bar{u}(x)\gamma_\nu d(x) \,; \qquad J_\mu(0) = \bar{d}(0)\gamma_\mu u(0) \,,$$

which correspond to charged ρ-meson.

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of two vector currents, with generalized first current:

$J^N_\mu(0) = \bar{d}(0)\gamma_\mu \left(-in\nabla_y\right)^N u(y)\Big|_{y=0}$,where $n^2 = 0,\ nq \neq 0.$

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$$\Pi^{\text{nonpert}}_{\mu\nu} \frac{n_\mu q_\nu}{nq}$$

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$$\widehat{B}_{-q^2 \to M^2} \left[\Pi^{\text{nonpert}}_{\mu\nu} \frac{n_\mu q_\nu}{nq} \right]$$

where $\widehat{B}$ is Borel operator.

Transversity of VV correlator

We study correlator

$$\Pi_{\mu\nu}^{N} = i \int d^4x \, e^{iqx} \langle 0 | T \left[J_\mu^N(0) J_\nu^+(x) \right] | 0 \rangle \, ,$$

of two vector currents, with generalized first current:

$J_\mu^N(0) = \bar{d}(0)\gamma_\mu \left(-in\nabla_y\right)^N u(y)\Big|_{y=0}$,where $n^2 = 0$, $nq \neq 0$.

Our correlator consists of perturbative and nonperturbative parts: $\Pi_{\mu\nu}^N = \Pi_{\mu\nu}^{\text{pert}} + \Pi_{\mu\nu}^{\text{nonpert}}$.

Condition of the correlator transversity is

$$\Delta\Pi_L^N \equiv \frac{M^4}{2A_0} \widehat{B}_{-q^2\to M^2} \left[\Pi_{\mu\nu}^{\text{nonpert}} \frac{n_\mu q_\nu}{nq} \right] = \int\limits_0^1 x^N \varphi(x) \, dx = 0 \, ,$$

where $\widehat{B}$ is Borel operator.

Realization of this requirement is laborious, because we choose the Gaussian behavior by hand.

Minimization of nontransversity terms

Our prime interest are the linear combinations of $\Delta\Pi_L^N$ for $N = 0, 1, 2, \ldots$, which correspond to conformal moments

$$\left\langle \xi^{2N} \right\rangle_L \equiv \int\limits_0^1 (2x-1)^{2N} \varphi(x)\, dx = \sum_{k=0}^{2N} (-2)^{2N-k} \binom{2N}{k} \Delta\Pi_L^{2N-k} .$$

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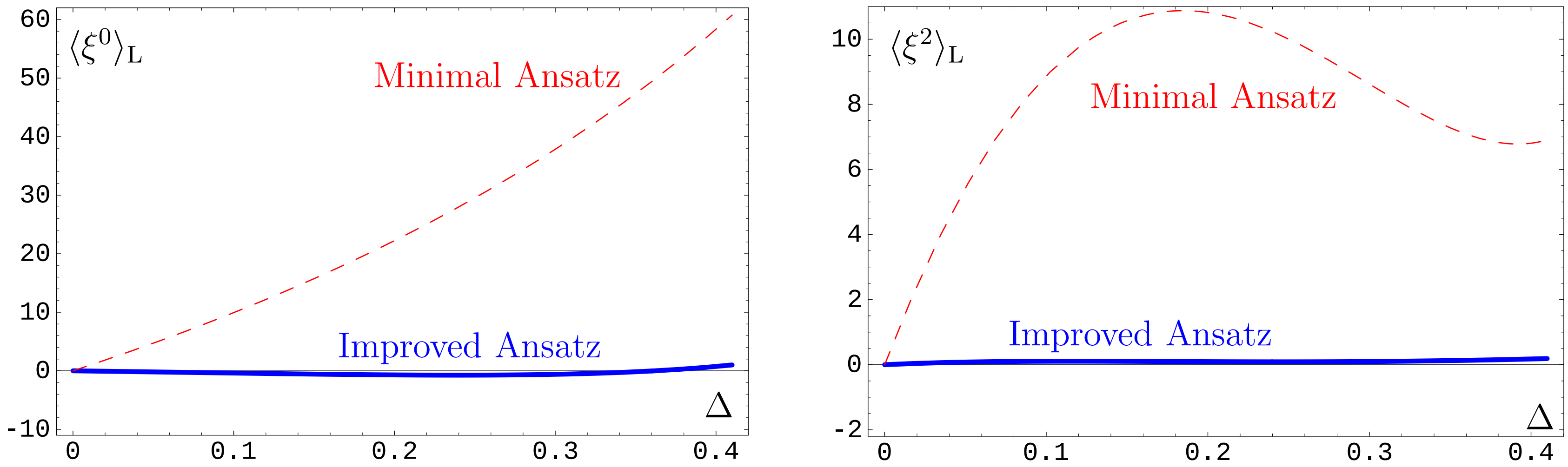
Minimization of $\left|\langle \xi^{2N} \rangle_L\right|$ for $N = 0, 1, ..., 5$ gives us the set of parameters:

$$\begin{aligned}
X_1 &= -0.082\,;\;\; Y_1 = Z_1 = -2.243\,;\;\; x = 0.788\,;\\
X_2 &= -1.298\,;\;\; Y_2 = Z_2 = -0.239\,;\;\; y = z = 1 - x = 0.212\,;\\
X_3 &= +1.775\,;\;\; Y_3 = Z_3 = -3.166\,;\;\; X_v = \Lambda_{\rm V}/\Lambda_{\rm S} = 1.00\,.
\end{aligned}$$

Comparison between models



Confrontation between improved model (solid line) and minimal model (dotted line) $|\langle\xi^{2N}\rangle_L|$ functions.

The improved Gaussian model makes violation of polarization operator transversity 100 times smaller as compared with the minimal model.

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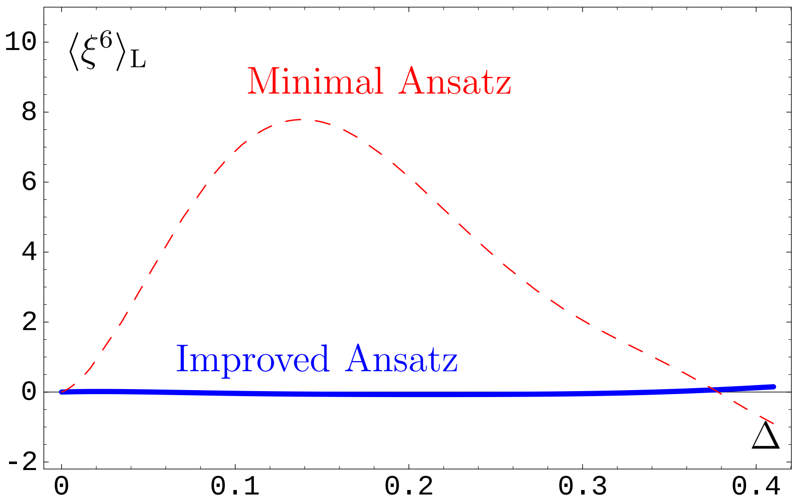

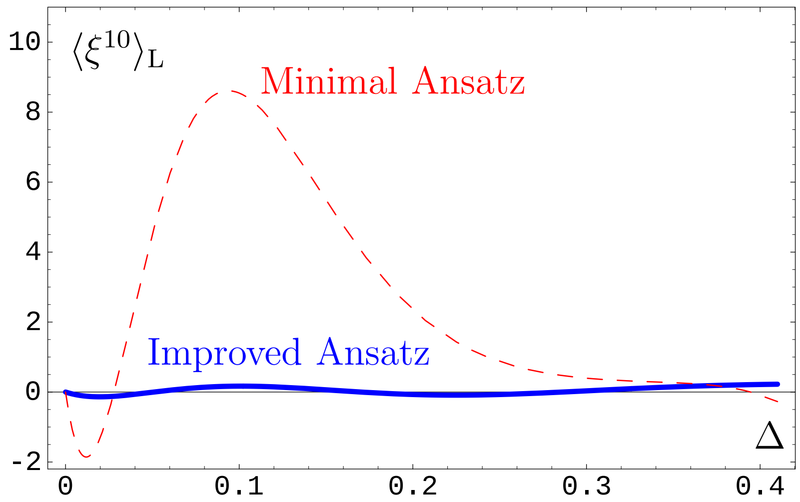

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Pion distribution amplitude $\varphi_\pi(x,\mu^2)$

The pion DA parameterizes this matrix element:

$$\langle 0|\,\bar{d}(z)\gamma_\nu\gamma_5 \boldsymbol{E}(\boldsymbol{z},\boldsymbol{0}) u(0)\,|\,\pi(P)\rangle\Big|_{z^2=0} = i f_\pi P_\nu \int_0^1 dx\; e^{ix(zP)}\;\varphi_\pi(x,\mu^2)$$

Fock–Schwinger string to ensure the gauge-independence:

$$\boldsymbol{E}(\boldsymbol{z},\boldsymbol{0}) = \mathcal{P}\,exp\left[ig\int\limits_0^z A_\mu(\tau)d\tau^\mu\right]$$

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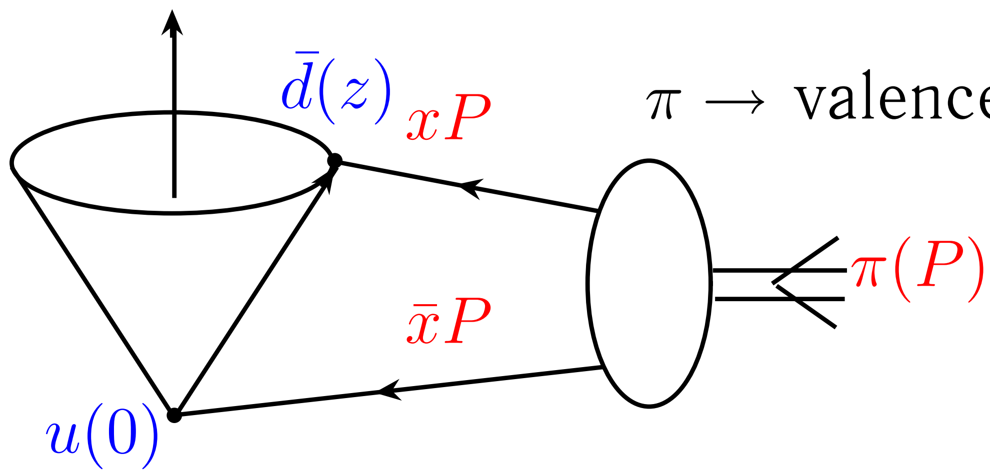
Pion DA describes the transition of a physical pion into two valence quarks, separated at light cone.



Representation for DA

Pion DA in a form of Gegenbauer expansion:

$$\varphi_\pi(x;\mu^2) = 6x\bar{x}\Big[1 + a_2 C_2^{3/2}(2x-1) + a_4 C_4^{3/2}(2x-1) + \ldots\Big]$$

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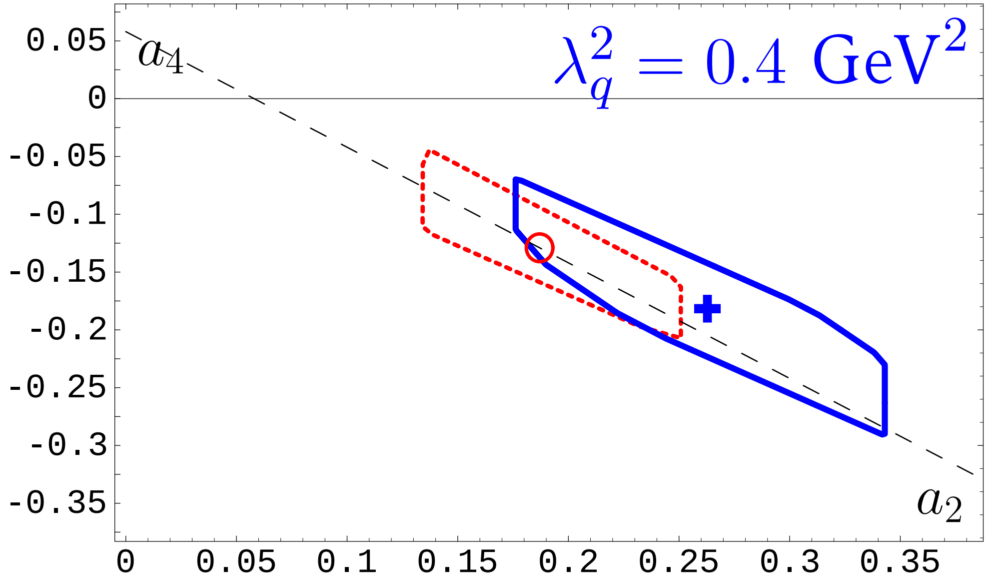

Region for $(a_2,\ a_4)$ Gegenbauer coefficients of the pion DA for improved model (solid line) in comparison with minimal result: BMS model (○) and bunch (dashed line).

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- The pion DA has changed, but in a weak way.
- The BMS model, obtained in the minimal Gaussian approach, is a very good model for pion DA.
- The region of allowed by QCD SR pion DAs shifted to larger values for a_2 and a_4.