QCD and dynamical phase transitions, II

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- QGP and Bjorken Hydrodynamics

  QGP: From Free-Streaming to Perfect-Fluid

- AdS/CFT and QCD Hydrodynamics

  $4 \to 5d$: *Holographic renormalization*

- AdS/CFT $\Rightarrow$ asymptotic perfect fluid dynamics

  Horizons vs. singularities

- Fast Thermalization to a perfect fluid

  AdS/CFT: Quasi-Normal Modes of a Moving Black Hole

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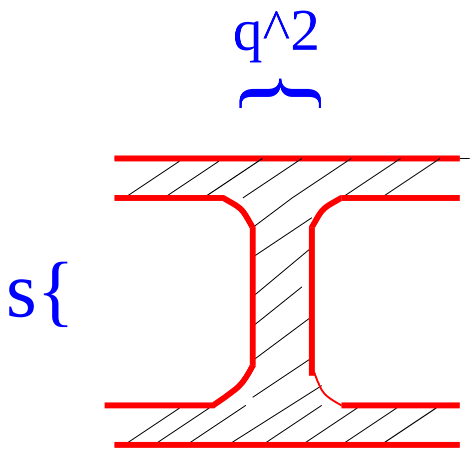
[a](with **Romuald Janik**, Cracow U.) hep-th/0512162, hep-th/06..... to appear

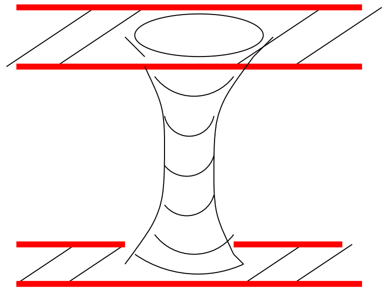
AdS/CFT Correspondence (1)

The Origin of String Theory



Veneziano Amplitude

$A_R(s,q\hat{}2)$

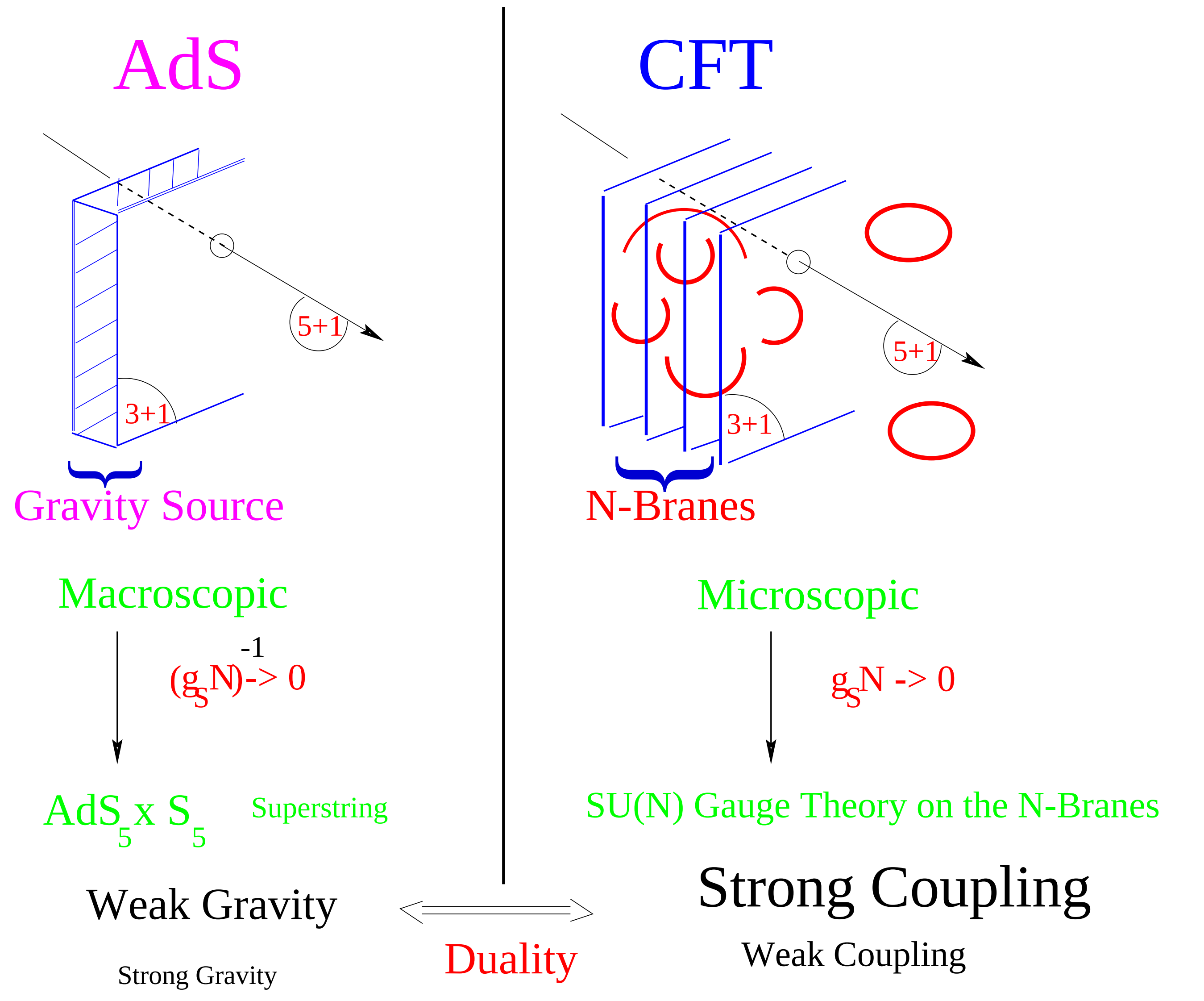


Shapiro-Virasoro Amplitude

$A_P(s,q\hat{}2)$

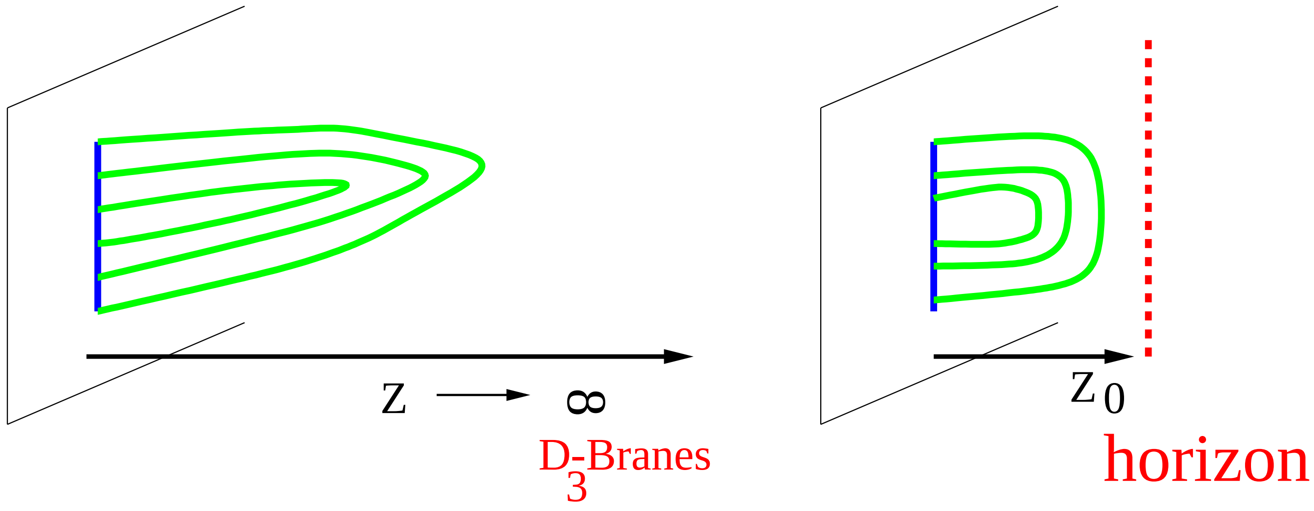
AdS/CFT Correspondence (2)

J.Maldacena (1998)



AdS/CFT Correspondence (3)

Holography



More on AdS$_5$

- **D_3-brane Solution of Super Gravity:**
  $\equiv$ low energy IIB SuperString

$$ds^2 = f^{-1/2}(-dt^2 + \sum_{1-3} dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)$$

"On-Branes×Out-Branes"

$$f = 1 + \frac{R^4}{r^4} \; ; \; R = 4\pi g_{YM}^2 \alpha'^2 N$$

- **"Maldacena limit":**

$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \; , \; R \; fixed \; \Rightarrow g_{YM}^2 N \to \infty$$

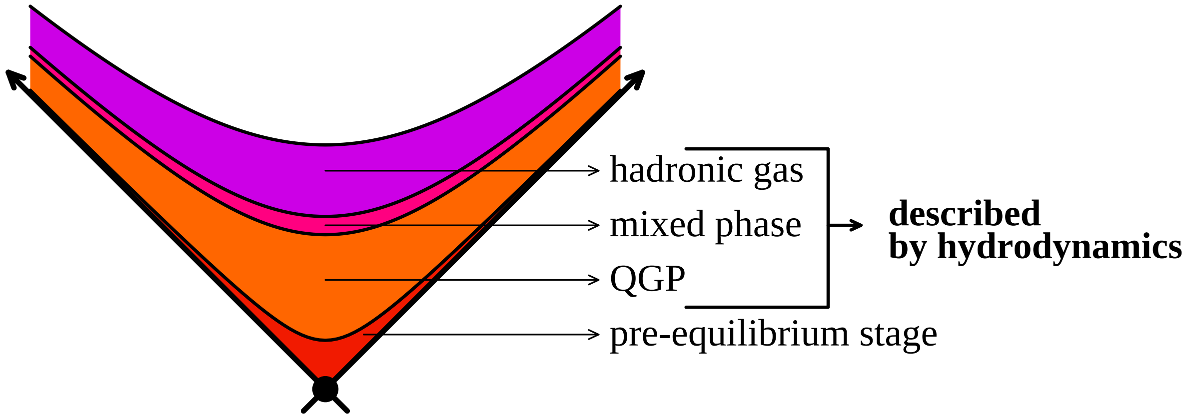
Strong coupling limit

$$ds^2 = \frac{1}{z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5$$

Background Structure: AdS$_5$ $\times$ S_5

QGP formation and Relativistic Hydrodynamics

J.D.Bjorken (1982)



- Boost Invariance

$$\tau = \sqrt{x_0^2 + x_1^2} \; ; \; y = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} \; ; \; x_T \to x_1, x_2$$

- QGP: Perfect fluid behaviour

- Pre-equilibrium stage: Fast

The 4d Energy-Momentum Tensor

- Constraint Equations

$$\mathrm{T}^{\mu}{}_{\mu} \equiv -T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} + 2T_{xx} = 0$$
$$\mathrm{D}_{\nu}T^{\mu\nu} \equiv \tau\frac{d}{d\tau}T_{\tau\tau} + T_{\tau\tau} + \frac{1}{\tau^2}T_{yy} = 0$$

- Boost-invariant Tensor

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & \ldots \end{pmatrix}$$

- 1-parameter family

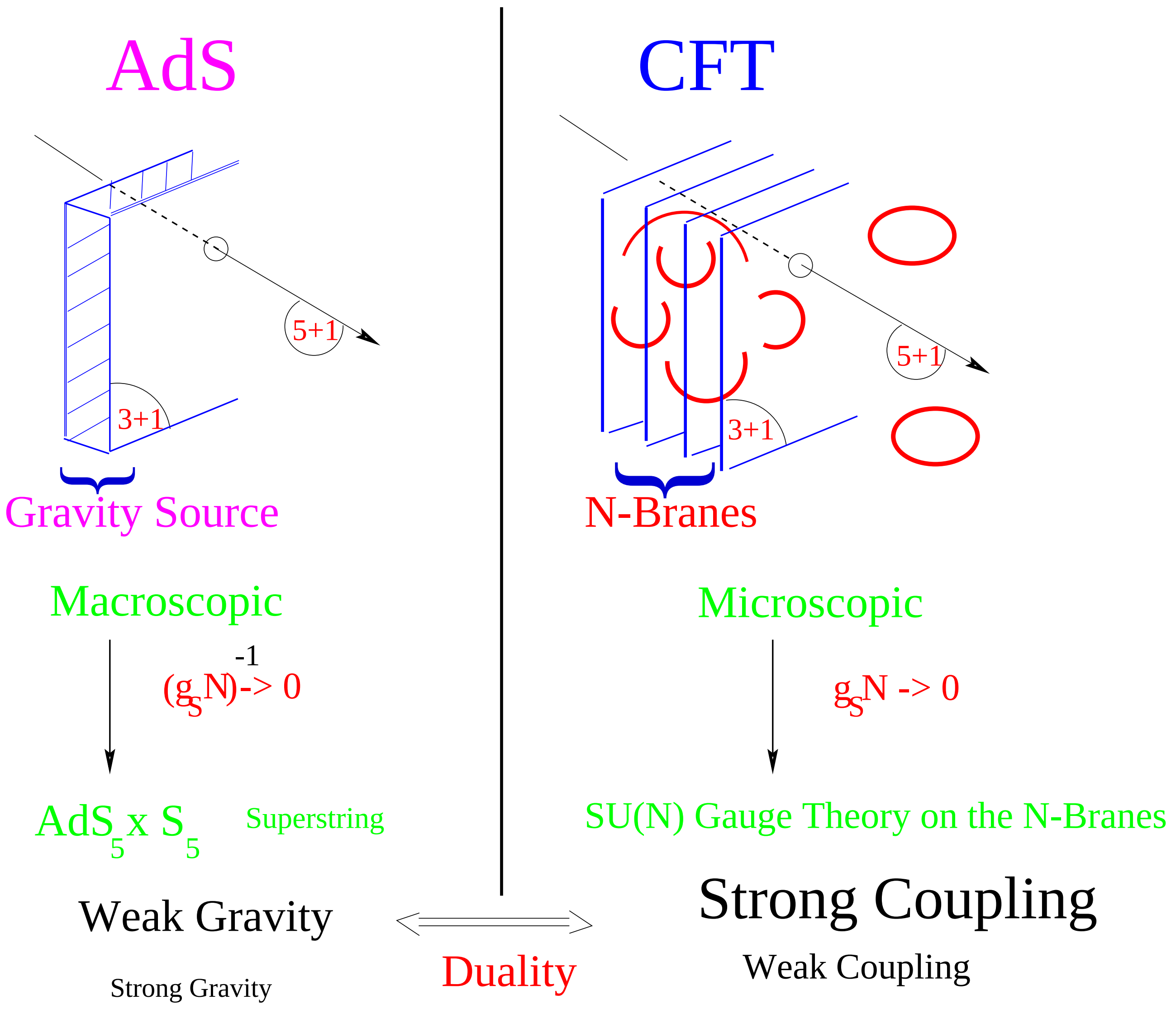
$f(\tau) \propto \tau^{-s}$: $(0 < s < 4 \; ; \; T_{\mu\nu}t^{\mu}t^{\nu} \geq 0)$

$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid

$f(\tau) \propto \tau^{-1}$: Free streaming

AdS/CFT Correspondence

J.Maldacena (1998)



$4 \longrightarrow 5d$:
Holographic Renormalization

K.Skenderis (2002)

- **Fefferman-Graham Coordinates:**

$$ds^2 = \frac{g_{\mu\nu} dx^\mu dx^\nu + dz^2}{z^2}$$

- $4d \Leftrightarrow 5d$ **metric:**

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} (= \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu} (= 0) + z^4 g^{(4)}_{\mu\nu} (\propto \langle T_{\mu\nu} \rangle) + \ldots$$

$+ \ldots$: from Einstein Eqs.

Test on Statics:
4d Perfect Fluid $\Leftrightarrow$ 5d Black Hole

- 4d Perfect Fluid

$$\langle T_{\mu\nu} \rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 & 0 & 0 & 0 \\ 0 & 1/z_0^4 & 0 & 0 \\ 0 & 0 & 1/z_0^4 & 0 \\ 0 & 0 & 0 & 1/z_0^4 \end{pmatrix}$$

- Derivation of the Fefferman-Graham metrics

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4) z^2} dt^2 + (1 + z^4/z_0^4) \frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

- It is indeed the Static Black Hole

$$\tilde{z} = \frac{z}{\sqrt{1 + \frac{z^4}{z_0^4}}}$$

$$ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4} \frac{d\tilde{z}^2}{\tilde{z}^2}$$

Boost-Invariant Geometries

- **Boost-Invariant 5-d F-G metric:**

$$ds^2 = \frac{-e^{a(\tau,z)} d\tau^2 + \tau^2 e^{b(\tau,z)} dy^2 + e^{c(\tau,z)} dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- **Einstein Equation(s):**

$$[a(\tau,z), b(\tau,z), c(\tau,z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

$$v = \frac{z}{\tau^{s/4}}$$

$$v(2a'(v)c'(v) + a'(v)b'(v) + 2b'(v)c'(v)) - 6a'(v) - 6b'(v) - 12c'(v) + vc'(v)^2 = 0$$
$$3vc'(v)^2 + vb'(v)^2 + 2vb''(v) + 4vc''(v) - 6b'(v) - 12c'(v) + 2vb'(v)c'(v) = 0$$
$$2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 +$$
$$4vsc''(v) + 4sc'(v) - 2vsa'(v)c'(v) + 2vsc'(v)^2 = 0\ .$$

- **Asymptotic Solution**

$$\begin{aligned} a(v) &= A(v) - 2m(v) \\ b(v) &= A(v) + (2s-2)m(v) \\ c(v) &= A(v) + (2-s)m(v) \end{aligned} \quad (1)$$

Dual of a Perfect Relativistic fluid

$$\boxed{v = \frac{z}{\tau^{1/3}}}$$

- **Asymptotic metric**

$$\boxed{ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\ \frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)\left(\tau^2 dy^2+dx_\perp^2\right)\right]+\frac{dz^2}{z^2}}$$

- **Black Hole off in the 5th dimension**

$$Horizon:\ \ z_0 = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}}\ .$$

$$Temperature:\ \ T(\tau) \sim \frac{1}{z_0} \sim \tau^{-\frac{1}{3}}$$

$$Entropy:\ \ S(\tau) \sim Area \sim \tau \cdot \frac{1}{z_0^3} \sim const$$

Free Streaming and general cases

$$\boxed{v = \frac{z}{\tau^{1/4}}}$$

- **Asymptotic metric**

$$ds^2 = \frac{\left(-(1+\frac{v^4}{\sqrt{8}})^{\frac{1-2\sqrt{2}}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1+2\sqrt{2}}{2}} dt^2 + (1+\frac{v^4}{\sqrt{8}})^{\frac{1}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1}{2}} \tau^2 dy^2 + \right.}{z^2}$$

$$\frac{\left. + (1+\frac{v^4}{\sqrt{8}})^{\frac{1+\sqrt{2}}{2}}(1-\frac{v^4}{\sqrt{8}})^{\frac{1-\sqrt{2}}{2}} dx_\perp^2 \right)}{z^2} + \frac{dz^2}{z^2}$$

- **True Singularities ?**

Ricci scalar:

$$R = -20 + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right) \ .$$

Riemann tensor squared:

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$

AdS/CFT: Selection of the Perfect Fluid

- **Curvature invariant:** $\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$

$$\mathfrak{R}^2 = \frac{4}{\left(1-\Delta(s)^2 v^8\right)^4} \cdot \Big[10\,\Delta(s)^8 v^{32} - 88\,\Delta(s)^6 v^{24} + 42\,v^{24} s^2 \Delta(s)^4 + \\ +112\,v^{24}\Delta(s)^4 - 112\,v^{24}\Delta(s)^4 s + 36\,v^{20} s^3 \Delta(s)^2 - 72\,v^{20} s^2 \Delta(s)^2 + \\ +828\,\Delta(s)^4 v^{16} + 288\,v^{16}\Delta(s)^2 s - 288\,v^{16}\Delta(s)^2 - 108\,v^{16} s^2 \Delta(s)^2 + \\ -136\,v^{16} s^3 + 27\,v^{16} s^4 - 320\,v^{16} s + 160\,v^{16} + 296\,v^{16} s^2 + 36\,v^{12} s^3 + \\ -72\,v^{12} s^2 - 88\,\Delta(s)^2 v^8 + 42\,v^8 s^2 + 112\,v^8 - 112\,v^8 s + 10 \Big] + \mathcal{O}\left(\frac{1}{\tau^{\#}}\right)$$

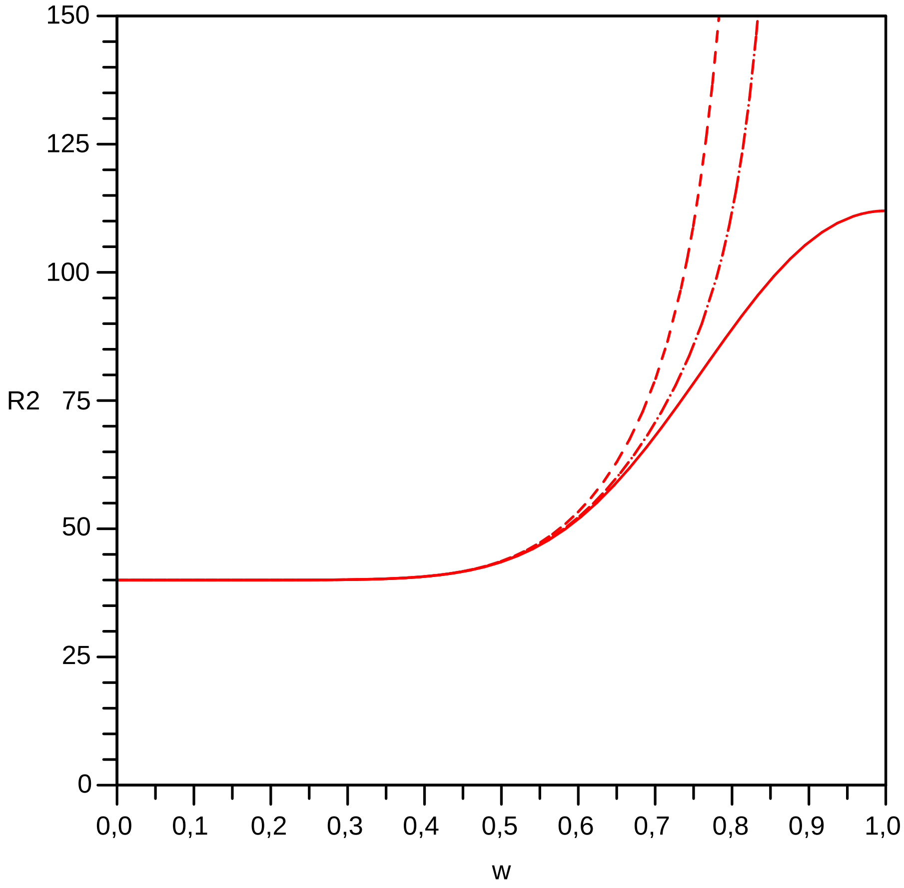
- $\mathfrak{R}^2$ for $s = \frac{4}{3}$**:**

$$\mathfrak{R}^2{}_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1+w^4)^4}$$

where $w = v/\Delta(\frac{4}{3})^{\frac{1}{4}} \equiv \sqrt[4]{3}\, v.$

AdS/CFT: Selection of the Perfect Fluid

$\mathfrak{R}^2$ for $s = \frac{4}{3} \pm .1$



Thermalization response-time of a perfect fluid

- **Quasi-normal scalar modes of a static Black Hole in Fefferman-Graham coordinates**

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

$$-\frac{1}{z^3}\frac{\left(1-\frac{z^4}{z_0^4}\right)^2}{\left(1+\frac{z^4}{z_0^4}\right)}\partial_t^2\phi(t,z)+\partial_z\left(\frac{1}{z^3}\left(1-\frac{z^8}{z_0^8}\right)\partial_z\phi(t,z)\right)=0\ .$$

- **Separation of variables** $\phi(t,z) = e^{i\omega t}\phi(z)$

$$\phi'' + \frac{1-\tilde{z}^2}{\tilde{z}(1-\tilde{z})(2-\tilde{z})}\ \phi' + \left(\frac{\omega}{\pi T}\right)^2\ \frac{1}{4\tilde{z}(1-\tilde{z})(2-\tilde{z})}\ \phi = 0$$

- **Dominant Decay Time**

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\ i$$

- **A note on Viscosity *vs.* Q-n modes**

Thermalization response-time of a perfect fluid

- **Quasinormal scalar modes for the boost-invariant Black Hole geometry**

$$\boxed{\Delta\phi \equiv \frac{1}{\sqrt{-g}}\partial_i\left(\sqrt{-g}g^{ij}\partial_j\phi\right) = 0}$$

$$\left[\partial_z \to \tau^{-\frac{1}{3}}\partial_v \;;\; \partial_\tau \to \partial_\tau - \frac{1}{3}\tau^{-\frac{4}{3}}\partial_v\right]$$

$$-\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\partial_\tau^2\phi(\tau,v) + \tau^{-\frac{2}{3}}\;\partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(\tau,v)\right) = 0$$

- **Separation of variables** $\phi(\tau,v) = f(\tau)\phi(v)$

$$\partial_\tau^2 f(\tau) = -\omega^2\tau^{-\frac{2}{3}}f(\tau) \;\Rightarrow\; f(\tau) = \sqrt{\tau}J_{\pm\frac{3}{4}}\left(\frac{3}{2}\omega\tau^{\frac{2}{3}}\right) \;\sim\; \tau^{\frac{1}{6}}\;e^{\frac{3}{2}i\omega\tau^{\frac{2}{3}}}$$

$$\boxed{\partial_v\left(\frac{1}{v^3}(1-v^8)\partial_v\phi(v)\right) + \omega^2\frac{1}{v^3}\frac{(1+v^4)^2}{1-v^4}\phi(v) = 0}$$

- **Dominant Decay Proper-Time**

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667\,i$$

Conclusions II

- AdS/CFT and Hydrodynamics
  Construction of the "Dual" of a relativistic fluid
- $4d \rightarrow 5d$: Holographic Renormalisation
  5-d Metric from the 4-d $T_{\mu\nu}$
- 5d Physical Criterium
  Non-singular 5d Horizon $\Rightarrow$ Asymptotic
  4d perfect fluid
- Quasi-Normal mode analysis:
  The perfect fluid is thermally "very stable"

Unexpected AdS/CFT consequences for QCD at strong coupling?