QCD and dynamical phase transitions, I

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- **Introduction: QCD** *"in situ"*

  Phase transitions in collision

- *Saturation and Non-Linear QCD Evolution:*

  The Gluon Phase-space at saturation

- *Mapping to random polymers:*

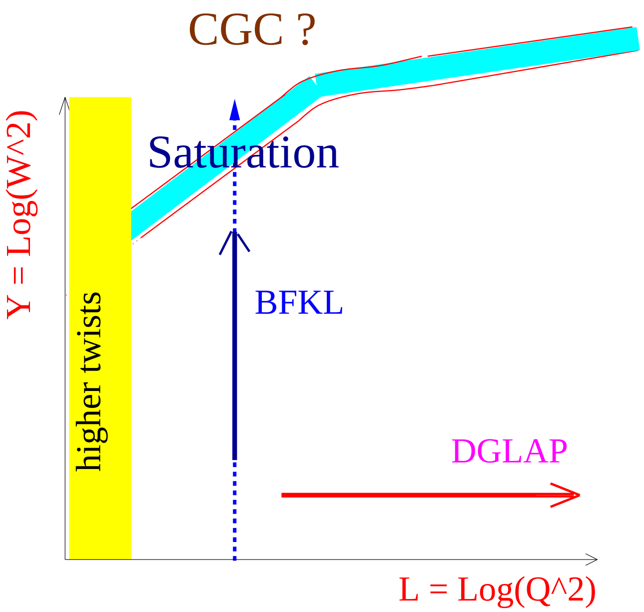
  A Spin-Glass Phase transition

- *Gluons near saturation:*

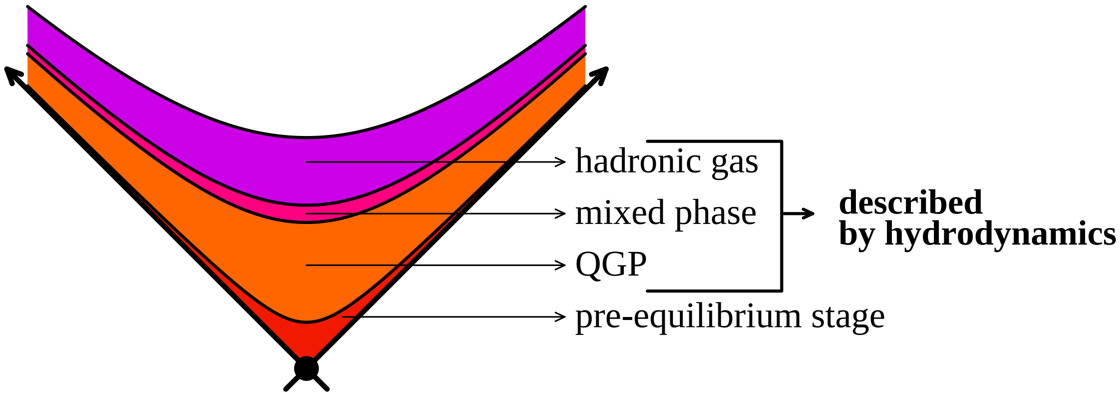
  Clustering Phase and "Hot Spots"

QCD Dynamical Phase transitions

- The transition to Saturation at HERA

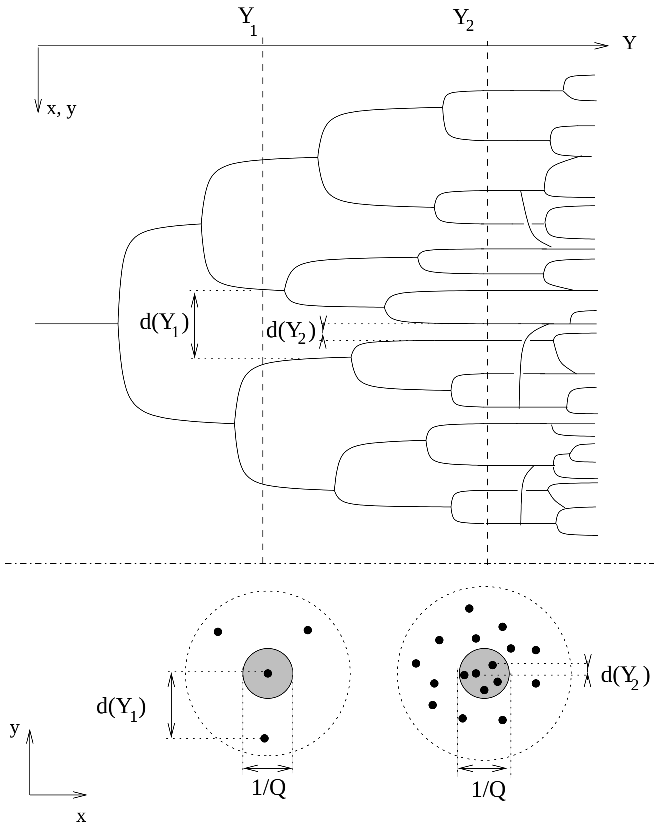


- The transition to QGP at RHIC



The QCD transition to Saturation

The "Tree" of Gluons / Dipoles



$d(Y) \to 0 \Rightarrow$ what happens?

$Y \sim Y_1$: Exponential growth: BFKL

$Y \sim Y_2$: **Transition to Saturation**

BFKL $\to$ BK, JIMWLK, Fluctuations

$Y > Y_2$: Beyond: the CGC?

The QCD rapidity evolution

- **The BFKL Evolution Operator**

Diffusive Approx. (1d momenta $l \equiv \log k^2$):

$$\chi(-\partial_l) \equiv 2\psi(1) - \psi(-\partial_l) - \psi(1+\partial_l) \sim A_0 + A_1\partial_l + A_2\partial_l^2 + \mathcal{O}(\partial_l^3)$$

A_0 : Exponential term

A_1 : "Drift" term

A_2 : Diffusion term

- **"Mean-Field" Amplitude :** $\mathcal{T} \sim cst.\mathbb{I}$

$$\partial_Y \mathcal{T} = \bar{\alpha}\chi\left(-\partial_l\right)\mathcal{T} - \bar{\alpha}\,\mathcal{T}^2$$

$$\mathcal{T}(l,Y) \sim \log\left[\frac{k^2}{Q_s^2(Y)}\right]\left(\frac{k^2}{Q_s^2(Y)}\right)^{-\sqrt{A_0/A_2}} \exp\left\{-\frac{1}{A_2\,Y}\log^2\left[\frac{k^2}{Q_s^2(Y)}\right]\right\}$$

- **Saturation Scale:**

$$\log Q_s^2(Y) \propto \left[2\sqrt{A_0 A_2} - A_1\right] Y - \frac{3}{2\sqrt{A_0/A_2}}\log Y + \mathcal{O}(1)$$

In red: geometric scaling

The Balitskiĭ-Kovchegov Equation

- **The Non-Linear BK Equation for $\mathcal{T}$:**

$$\boxed{\partial_Y \mathcal{T} = \bar{\alpha}\chi\left(-\partial_L\right)\mathcal{T} - \bar{\alpha}\,\mathcal{T}^2}$$

- **Equation BK $\Rightarrow$ F-KPP**

S.Munier, R.P., 2003,2004

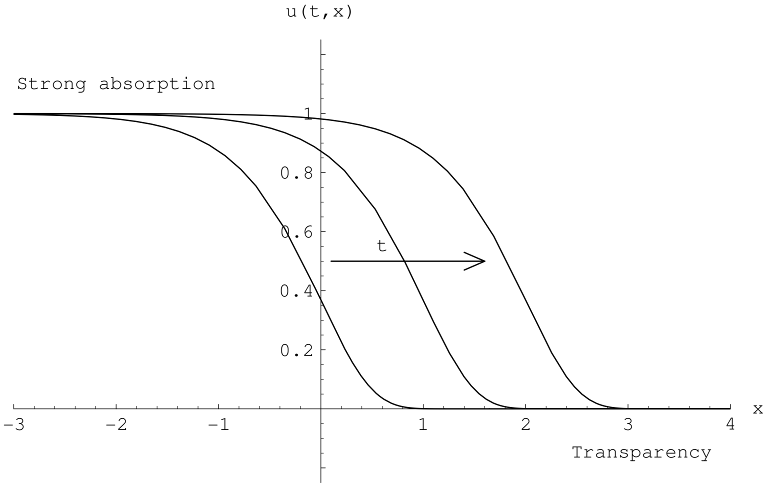
$$\boxed{\partial_t u(t,x) = \partial_x^2 u(t,x) + u(t,x)(1-u(t,x))}$$

- **"Dictionnary"**

$$\begin{aligned} Time &= t \propto Y \\ Space &= x \propto L + \frac{\bar{\alpha}D}{2}Y \\ Traveling\ Wave &= u(t,x) \sim u(t-vx)\ \propto\ \mathcal{T}(Y,k) \end{aligned}$$

Traveling wave Solutions

Bramson (1983)



- Traveling wave $\longrightarrow$ Geometric Scaling

$$u(t,x) \xrightarrow[t \to \infty]{} u(x - v(\gamma_c)t) \Rightarrow \boxed{\mathcal{T}(Y,k) = \mathcal{T}\left(\frac{k}{Q_s(Y)}\right)}$$

- Saturation Scale: $\longrightarrow$ "Universal terms"

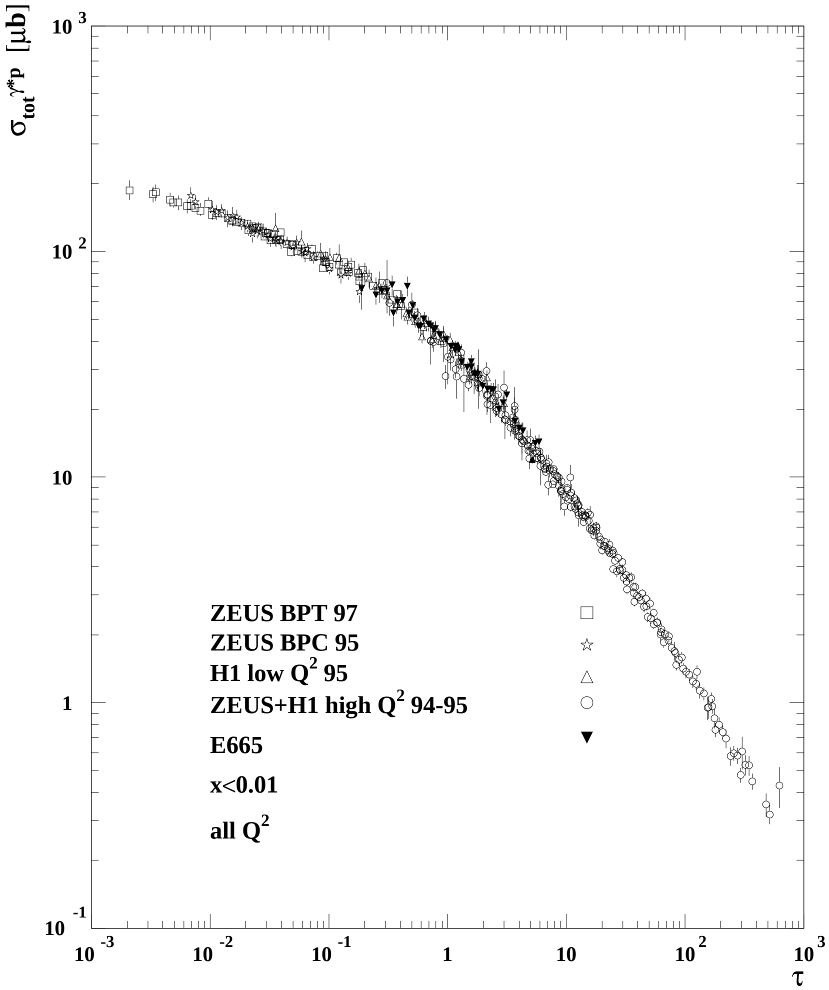
$$\boxed{\log Q_s^2(Y) = \bar{\alpha}\frac{\chi(\gamma_c)}{\gamma_c}Y - \frac{3}{2\gamma_c}\log Y - \frac{3}{(\gamma_c)^2}\sqrt{\frac{2\pi}{\bar{\alpha}\chi''(\gamma_c)}}\frac{1}{\sqrt{Y}} + \mathcal{O}(1/Y)}$$

Geometric Scaling

K.Golec-Biernat, J.Kwiecinski, A.Stasto (2000)

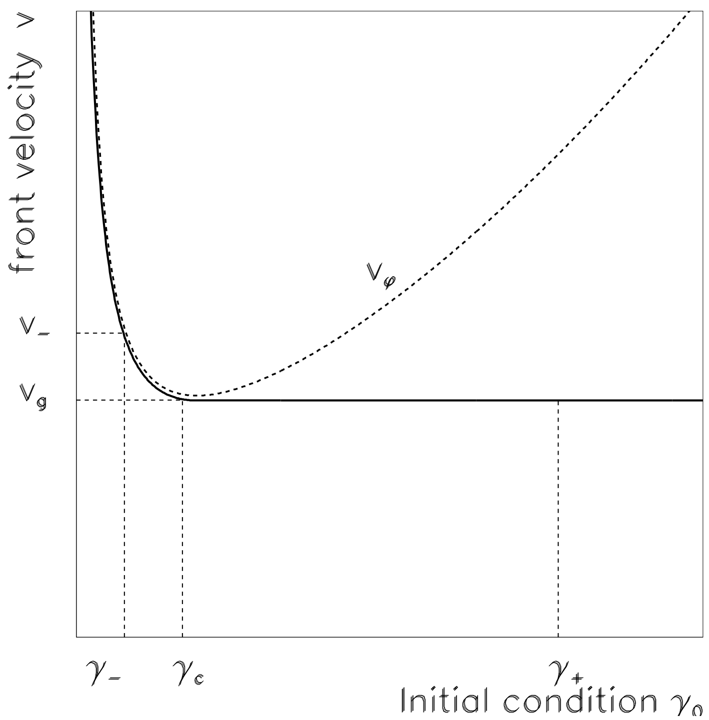
The Dipole Tree Observed in DIS:

$$\sigma^{\gamma^*}(Y,Q) = \int_0^\infty x_{01}^3\, dx_{01}\, |\psi(x_{01}Q)|^2 \int kdkJ_0(kx_{01})\ \mathcal{T}(Y,k)$$



More on the (Super)Critical Regime

Nonlinear Dynamics



$\chi(\gamma)/\gamma$

- Sub-critical regime: phase velocity

$$\gamma_0 = \gamma_- \Rightarrow v \equiv v_\varphi(\gamma) = \bar{\alpha}\chi(\gamma)/\gamma$$

- Critical regime: phase $\equiv$ group velocity

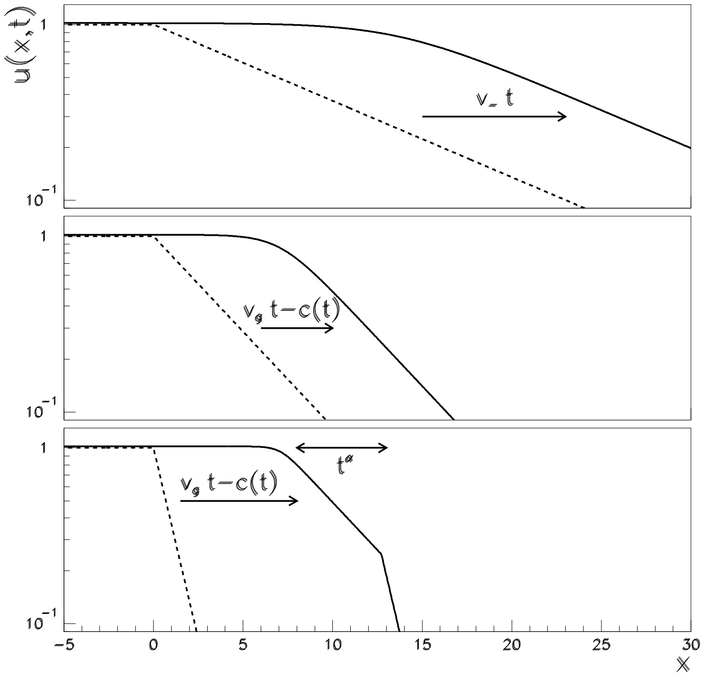
$$\gamma_0 = \gamma_c = .6275... \Rightarrow v_\varphi(\gamma_c) \equiv v_g(\gamma_c) = \bar{\alpha}\chi'(\gamma_c)$$

- Super-critical regime (cf. QCD at $\gamma_+ = 1$)

$$\gamma_0 = \gamma_+ \Rightarrow \bar{v} \equiv v_g(\gamma_c) = \min[v_\varphi(\gamma)]$$

The (Super)Critical Wave Front

Derrida, Van Saarlos: "Pulled vs. Pushed fronts"

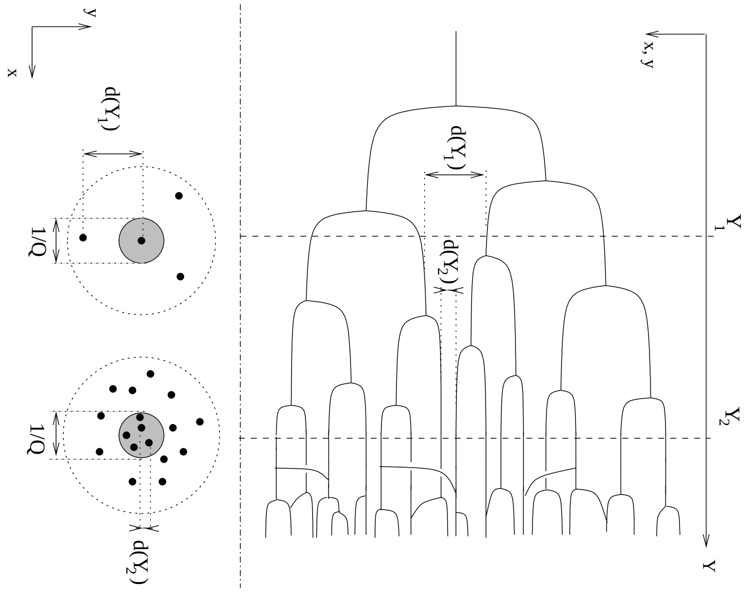


$-1-\quad -2- -3- -4-$

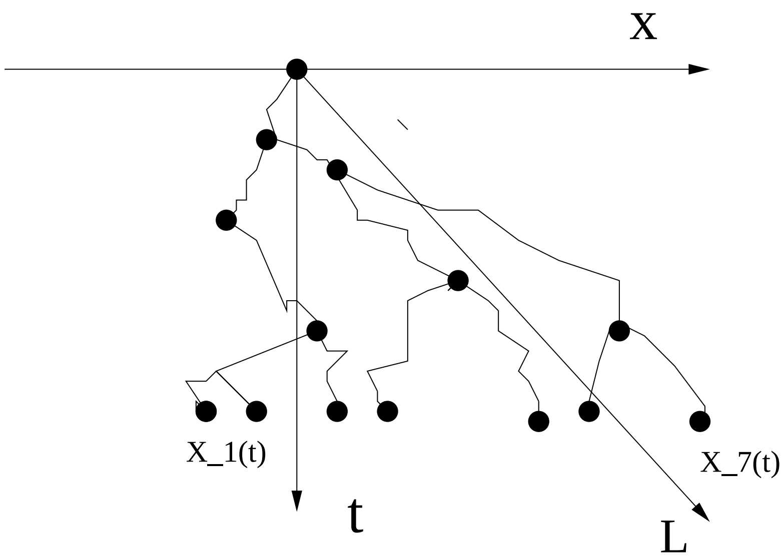
(Super)critical "Pulled" Fronts

Question: Is it the sign of a QCD dynamical phase transition?

Mapping to random polymers



$$y = \frac{t}{A_0}\ ; \qquad L_i \equiv \log k_i^2(y) \Leftrightarrow -\beta\ (x_i(t) - x(0)) + (A_0 - A_1)y$$



$$x_i(t) :\ Branching + Diffusion + Shift$$

Thermodynamics of random polymers

- **The Partition Function:**

$$Z(t)_{for\ one\ event} \equiv \sum_{i=1}^{n} e^{-\beta x_i(t)} \Leftrightarrow \frac{1}{n}\sum_{i=1}^{n} k_i^2(y) \equiv \bar{k}^2(y)$$

- **Generating functionale**

$$G(x,t) \equiv \sum_p \frac{1}{p!}\, \langle\, \left[-e^{\,\beta x}\ Z(t)\right]^p \rangle_{all\ events} \Leftrightarrow 1 - u(x,t)$$

- **"Dictionnary"**

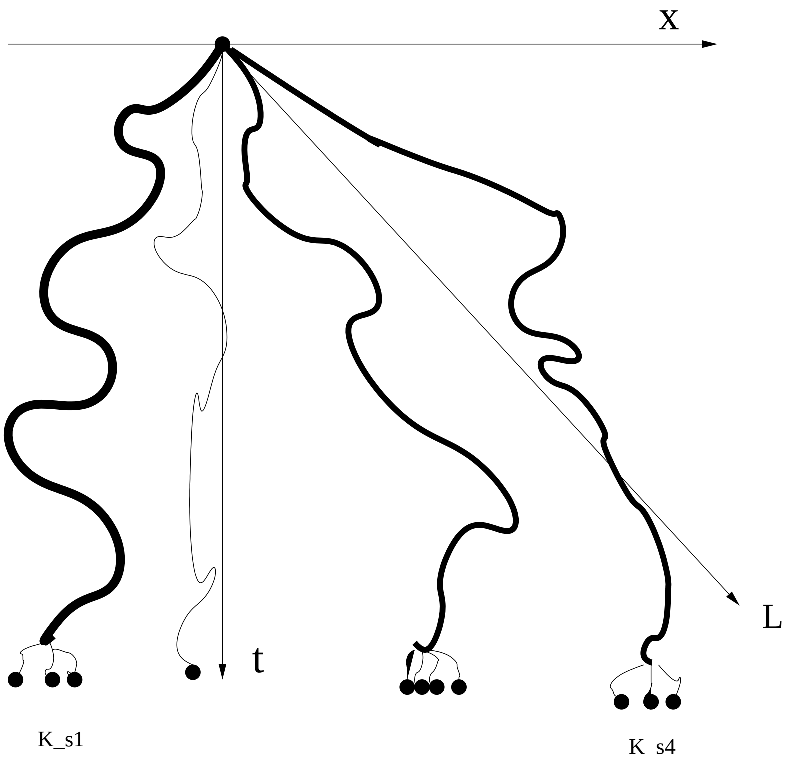
$$Free\ energy:\ \ F = -\frac{1}{\beta}\langle\ \log Z(t)\ \rangle \quad \Leftrightarrow \quad v \times t \sim \log(Q_s^2)$$

$$Spectrum:\ \ f = -\frac{1}{\beta}\log Z(t) + \frac{1}{\beta}\langle\ \log Z(t)\ \rangle \quad \Leftrightarrow \quad \mathcal{P}(f) \sim \mathcal{T}(Q^2/Q_s^2)$$

$$Spin-Glass\ Phase: \quad \Leftrightarrow \quad \beta > \beta_c:\ \ \gamma_0 > \gamma_c$$

The QCD spin-glass phase

Clustering structure



$s_1 \cdots s_4$ clusters near $k_{s1} \cdots k_{s4}$

- Equivalent temperature $T < T_c$

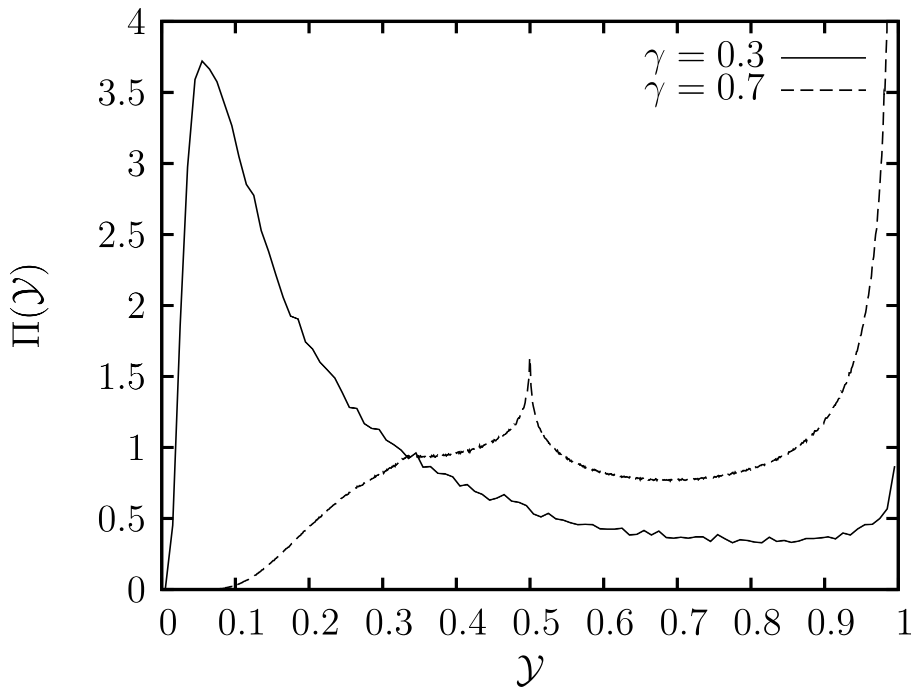
$$\frac{T_c - T}{T_c} \equiv 1 - \frac{\beta_c}{\beta} = 1 - \gamma_c$$

- Cluster weights

$$W_{si} = \frac{\sum_{i \in si} k_i^2}{\sum_i k_i^2}$$

The QCD spin-glass phase

Overlap Function



Overlap distribution $\Pi(\mathcal{Y})$

- Clustering Probability $\tilde{\mathcal{Y}}(q)$

$$\tilde{\mathcal{Y}}(q)\ dq = \frac{1}{\left\{\sum(k_i^2)\right\}^2} \sum_{i,j=1}^{n} k_i^2\ k_j^2\ \ \Theta\{k_i(y{=}qY) = k_j(y{=}qY) \text{ in } [q, q+dq]\}$$

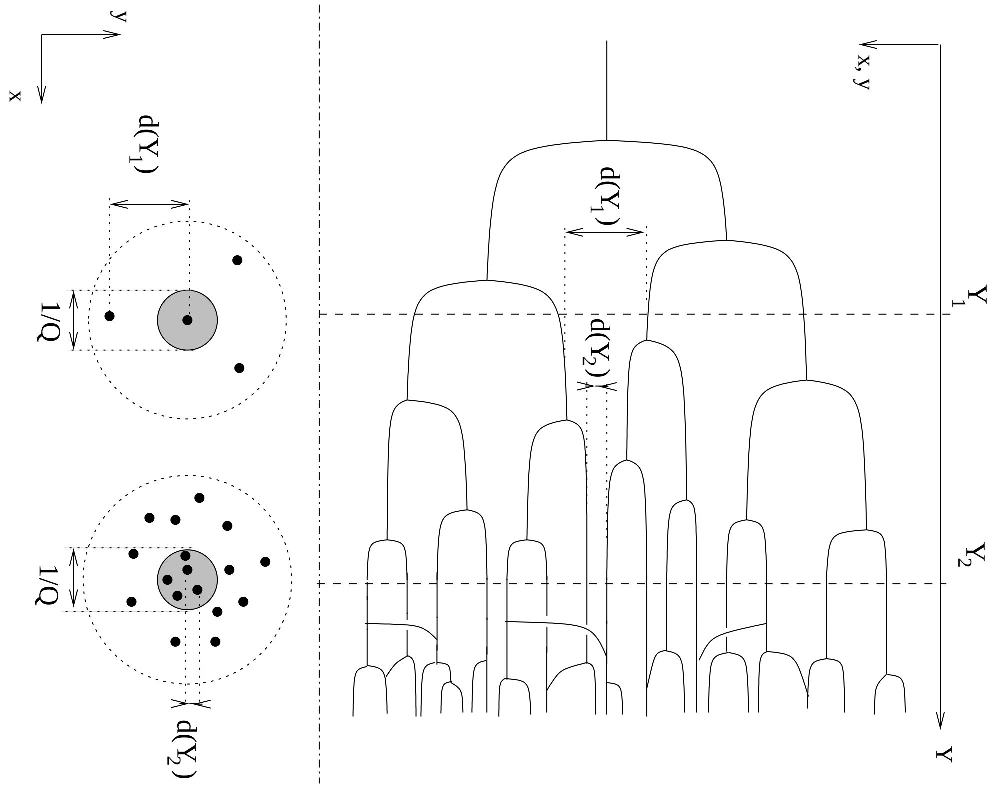
- Overlaps $\mathcal{Y} = \sum_{si} W_{si}^2$

$$\tilde{\mathcal{Y}}(q)\ dq\ = \delta(q-1)\ \mathcal{Y}\ + \delta(q)\ (1-\mathcal{Y})$$

- Cluster distribution $\Pi(\mathcal{Y})$

$$\langle\ \mathcal{Y}\ \rangle_\Pi = 1 - T/T_c\ ;\ \langle\ \mathcal{Y}^p\ \rangle_\Pi \Rightarrow \Pi(\mathcal{Y})\ \mathit{predicted}$$

Questions and Prospects



- Phenomenology

  Connect with Experiments: “Hot Spots”
- BFKL kernel, Fluctuations, Pomeron Loops

  $\sim$ Random polymers with constraints?
- The QCD phase beyond Saturation

  Relation with Advanced Statistical Physics

Conclusions (part I)

- The QCD transition to Saturation
  (Super)critical Dynamics
- Mapping to Random Polymers:
  QCD Spin-Glass phase
- Results:
  Clustering Structure: Overlaps
- Prospects
  New Relation with Mathematics (non-linear Eqs.) and Physics (Disordered systems, Polymer diffusion and Spin glass phase transitions)