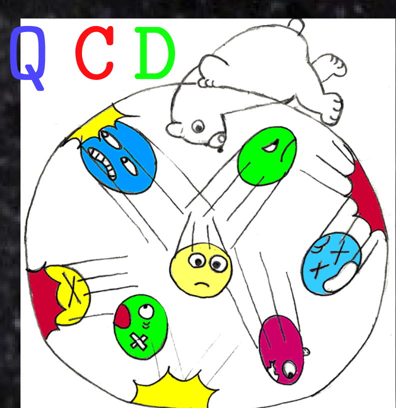
Lattice Study of Gluon Viscosities

-- A Step towards RHIC Physics –

Cracow School of Theoretical Physics

May 27 – June 5, 2006, Zakopane, Poland

Atsushi NAKAMURA
RIISE, Hiroshima University



Linear Response Theory

Many Thanks to the Organizers !

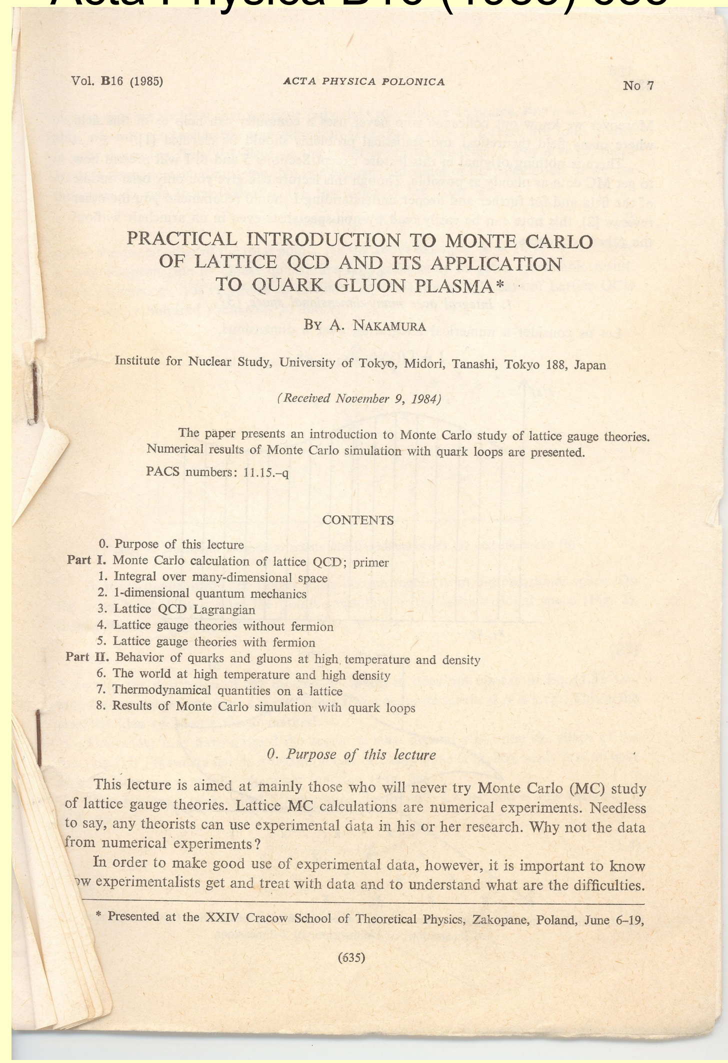
- I am very happy to be able to come here again after 20 years !

Part I. Monte Carlo calculation of lattice QCD : primer

Part II. Behavior of quarks and gluons at high temperature and density

* Presented at XCIV Cracow School of Theoretica Physics. Zakopane, Poland, June 6-19, 1984.

Acta Physica B16 (1985) 635



Vol. B16 (1985) ACTA PHYSICA POLONICA No 7

PRACTICAL INTRODUCTION TO MONTE CARLO OF LATTICE QCD AND ITS APPLICATION TO QUARK GLUON PLASMA*

By A. Nakamura

Institute for Nuclear Study, University of Tokyo, Midori, Tanashi, Tokyo 188, Japan

(Received November 9, 1984)

The paper presents an introduction to Monte Carlo study of lattice gauge theories. Numerical results of Monte Carlo simulation with quark loops are presented.

PACS numbers: 11.15.–q

CONTENTS

0. Purpose of this lecture

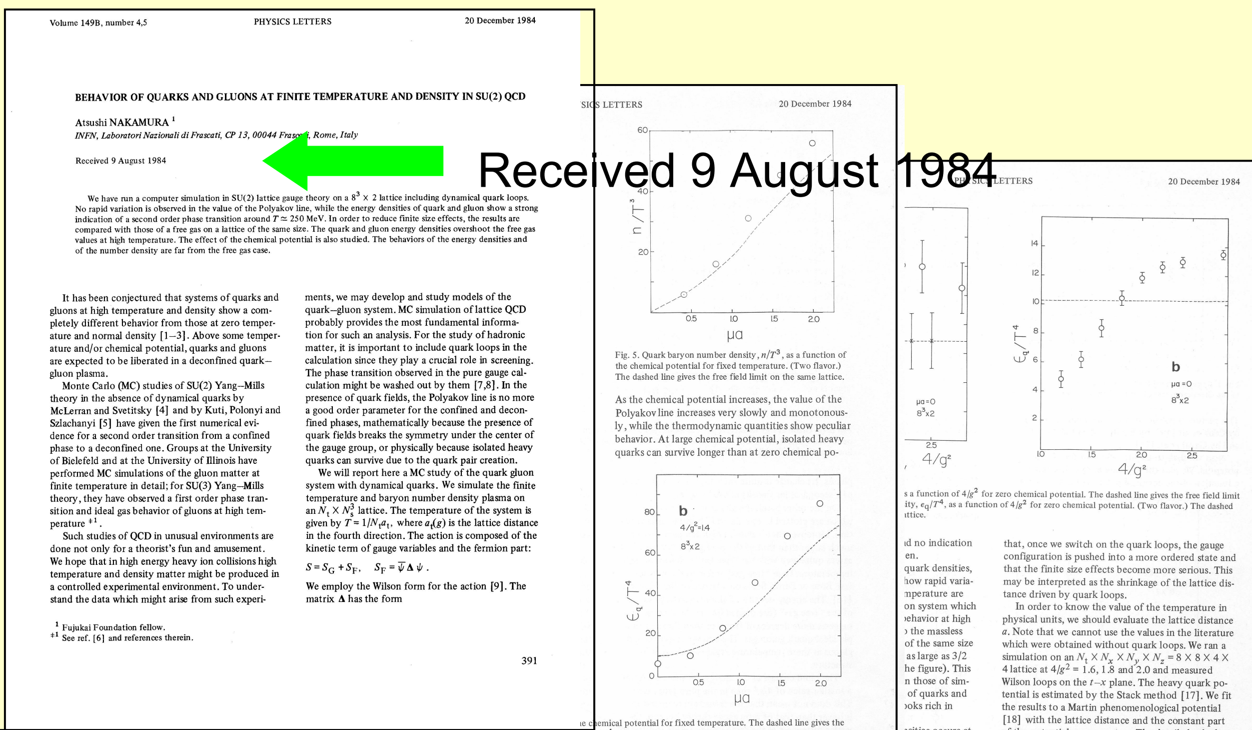
This lecture is aimed at mainly those who will never try Monte Carlo (MC) study of lattice gauge theories. Lattice MC calculations are numerical experiments. Needless to say, any theorists can use experimental data in his or her research. Why not the data from numerical experiments?

In order to make good use of experimental data, however, it is important to know ›w experimentalists get and treat with data and to understand what are the difficulties.

* Presented at the XXIV Cracow School of Theoretical Physics, Zakopane, Poland, June 6–19,

And Thanks again (after 20 years) to Andrzej and Larry !

- Based on the 2nd part, I worte a paper "Behavior of Quarks and Gluons at Finite Temperature and Density in SU(2) QCD" during my stay in Crakow.


Volume 149B, number 4,5 PHYSICS LETTERS 20 December 1984

BEHAVIOR OF QUARKS AND GLUONS AT FINITE TEMPERATURE AND DENSITY IN SU(2) QCD

Atsushi NAKAMURA [1]
INFN, Laboratori Nazionali di Frascati, CP 13, 00044 Frascati, Rome, Italy

Received 9 August 1984

We have run a computer simulation in SU(2) lattice gauge theory on a $8^3 \times 2$ lattice including dynamical quark loops. No rapid variation is observed in the value of the Polyakov line, while the energy densities of quark and gluon show a strong indication of a second order phase transition around $T \simeq 250$ MeV. In order to reduce finite size effects, the results are compared with those of a free gas on a lattice of the same size. The quark and gluon energy densities overshoot the free gas values at high temperature. The effect of the chemical potential is also studied. The behaviors of the energy densities and of the number density are far from the free gas case.

It has been conjectured that systems of quarks and gluons at high temperature and density show a completely different behavior from those at zero temperature and normal density [1–3]. Above some temperature and/or chemical potential, quarks and gluons are expected to be liberated in a deconfined quark–gluon plasma.

Monte Carlo (MC) studies of SU(2) Yang–Mills theory in the absence of dynamical quarks by McLerran and Svetitsky [4] and by Kuti, Polonyi and Szlachanyi [5] have given the first numerical evidence for a second order transition from a confined phase to a deconfined one. Groups at the University of Bielefeld and at the University of Illinois have performed MC simulations of the gluon matter at finite temperature in detail; for SU(3) Yang–Mills theory, they have observed a first order phase transition and ideal gas behavior of gluons at high temperature [‡1].

Such studies of QCD in unusual environments are done not only for a theorist's fun and amusement. We hope that in high energy heavy ion collisions high temperature and density matter might be produced in a controlled experimental environment. To understand the data which might arise from such experiments, we may develop and study models of the quark–gluon system. MC simulation of lattice QCD probably provides the most fundamental information for such an analysis. For the study of hadronic matter, it is important to include quark loops in the calculation since they play a crucial role in screening. The phase transition observed in the pure gauge calculation might be washed out by them [7,8]. In the presence of quark fields, the Polyakov line is no more a good order parameter for the confined and deconfined phases, mathematically because the presence of quark fields breaks the symmetry under the center of the gauge group, or physically because isolated heavy quarks can survive due to the quark pair creation.

We will report here a MC study of the quark gluon system with dynamical quarks. We simulate the finite temperature and baryon number density plasma on an $N_t \times N_s^3$ lattice. The temperature of the system is given by $T = 1/N_t a_t$, where $a_t(g)$ is the lattice distance in the fourth direction. The action is composed of the kinetic term of gauge variables and the fermion part:

$$S = S_G + S_F, \quad S_F = \bar{\psi} \Delta \psi .$$

We employ the Wilson form for the action [9]. The matrix Δ has the form

[1] Fujukai Foundation fellow.
[‡1] See ref. [6] and references therein.

Fig. 5. Quark baryon number density, n/T^3, as a function of the chemical potential for fixed temperature. (Two flavor.) The dashed line gives the free field limit on the same lattice.

As the chemical potential increases, the value of the Polyakov line increases very slowly and monotonously, while the thermodynamic quantities show peculiar behavior. At large chemical potential, isolated heavy quarks can survive longer than at zero chemical po-

...ie chemical potential for fixed temperature. The dashed line gives the

...s a function of $4/g^2$ for zero chemical potential. The dashed line gives the free field limit
...ity, ϵ_q/T^4, as a function of $4/g^2$ for zero chemical potential. (Two flavor.) The dashed
...attice.

...d no indication ...en. ...quark densities, ...how rapid varia- ...mperature are ...on system which ...ehavior at high ... the massless ...of the same size ...as large as 3/2 ...he figure). This ...n those of sim- ...of quarks and ...oks rich in

that, once we switch on the quark loops, the gauge configuration is pushed into a more ordered state and that the finite size effects become more serious. This may be interpreted as the shrinkage of the lattice distance driven by quark loops.

In order to know the value of the temperature in physical units, we should evaluate the lattice distance a. Note that we cannot use the values in the literature which were obtained without quark loops. We ran a simulation on an $N_t \times N_x \times N_y \times N_z = 8 \times 8 \times 4 \times 4$ lattice at $4/g^2 = 1.6$, 1.8 and 2.0 and measured Wilson loops on the t–x plane. The heavy quark potential is estimated by the Stack method [17]. We fit the results to a Martin phenomenological potential [18] with the lattice distance and the constant part

Received 9 August 1984

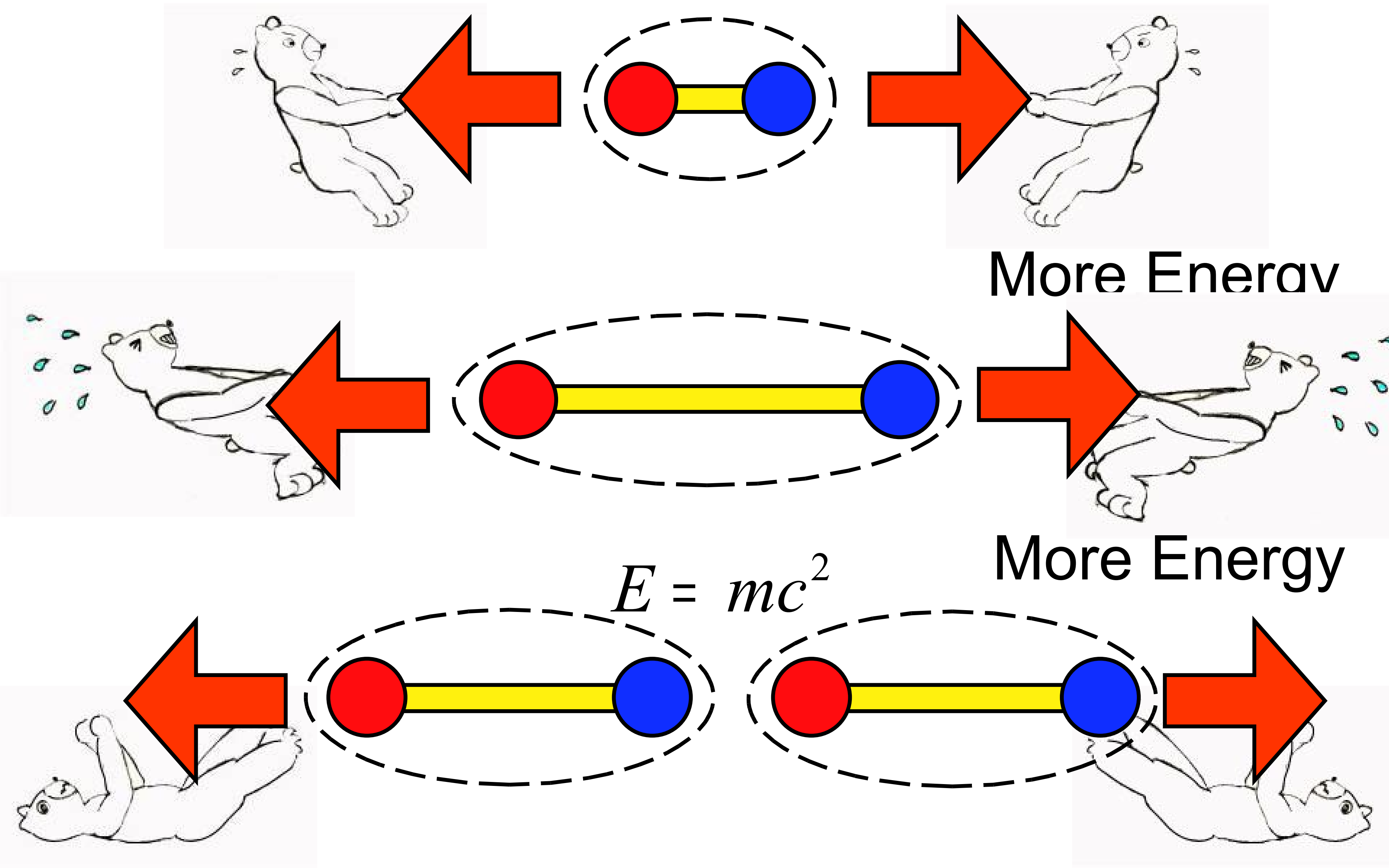
Acknowledgement

I am grateful to … participants of Zakopane school … , and especially A. Bialas and L.McLerran for valuable discussions and critical reading of the manuscript.

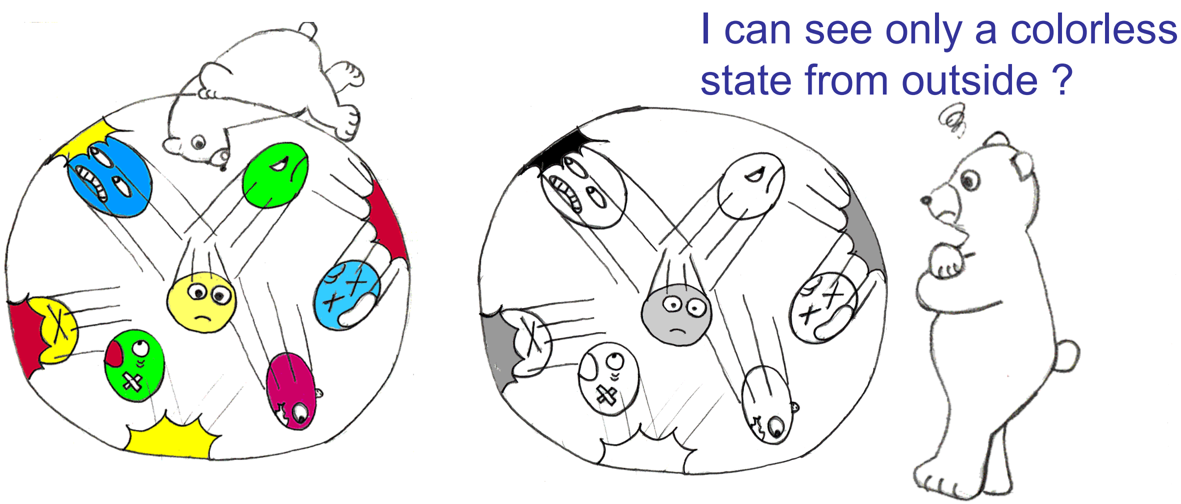
□ Contents

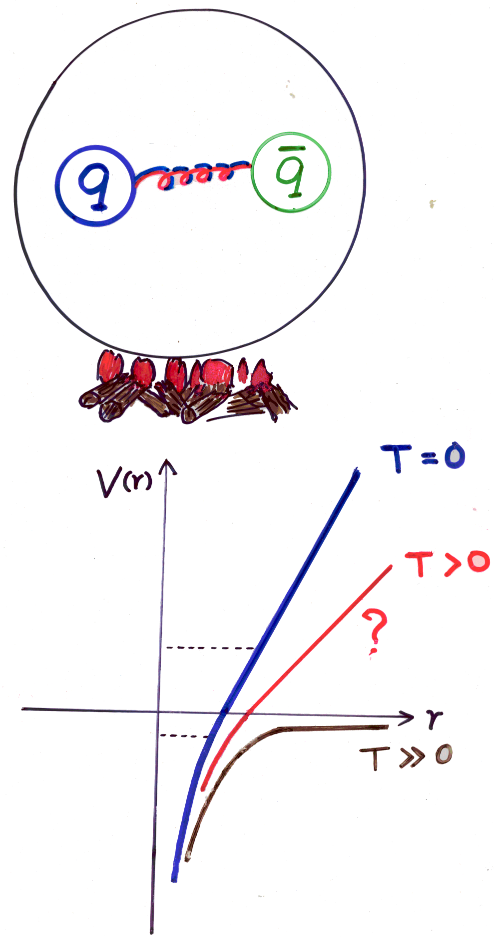
- Introduction
- Brief (and biased) Overview of Lattice QCD Study at Finite Temperature and Density
- Viscosity by Lattice QCD
- Summary

Confinement



Confinement (2)



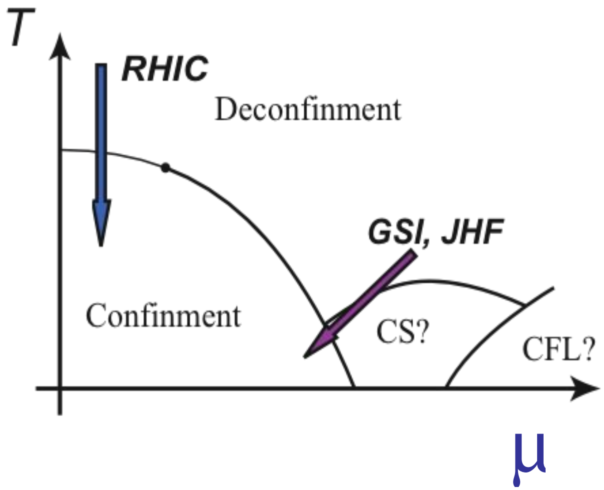


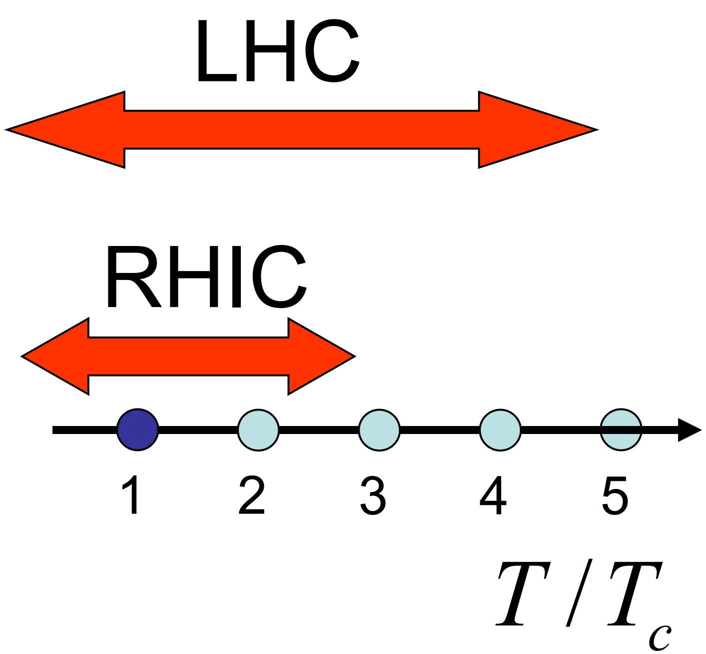
Confinement Potential is "screened" at finite temperature.

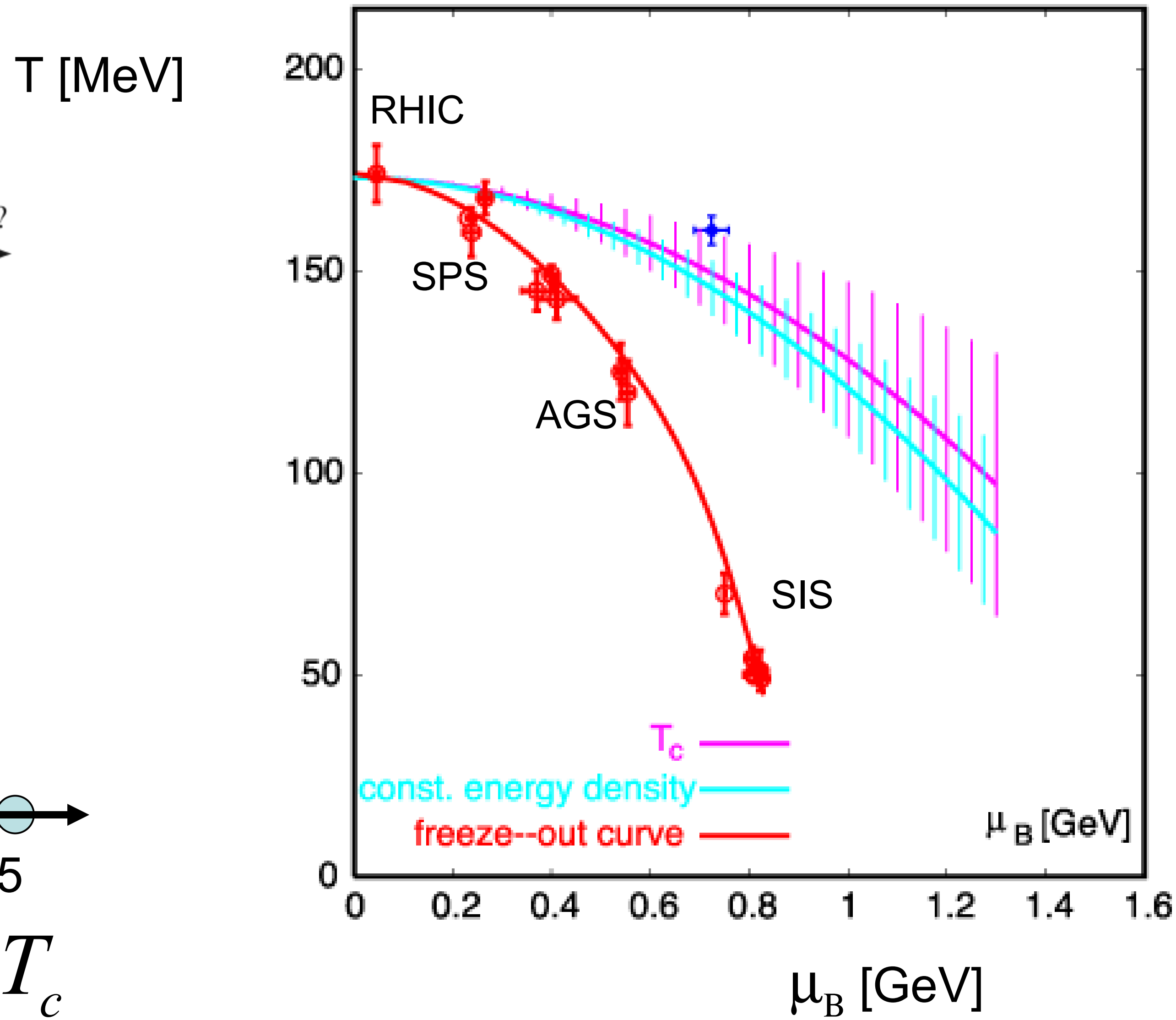
Deconfinement

A Comparison with Lattice Results

P. Braun-Munzinger, K. Redlich and J. Stachel







Observation of a Phase Transition at Finite Temperature on the Lattice

1981, McLerran and Svetitsky, Kuti, Polonyi and Szlachanyi, Engels et al.

$$Z = e^{-\beta F} = \mathrm{Tr}\, e^{-\beta (H-\mu N)} = \int_{\phi} \left\langle \phi \left| e^{-\beta (H-\mu N)} \right| \phi \right\rangle$$

$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+A Static Quark})}{Z(\text{Gluons})} = \left\langle L(\vec{x}) \right\rangle$$

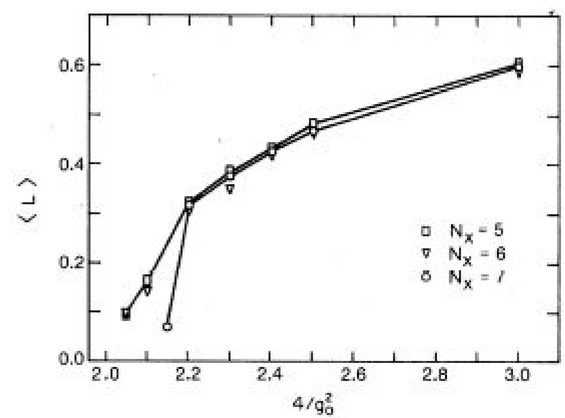
Excess Energy when a quark exists.

$$e^{-\beta \Delta F} = \frac{Z(\text{Gluons+Static Quark+Anti-Quark})}{Z(\text{Gluons})}$$

$$= \left\langle L(\vec{x}) L^{\dagger}(\vec{y}) \right\rangle$$

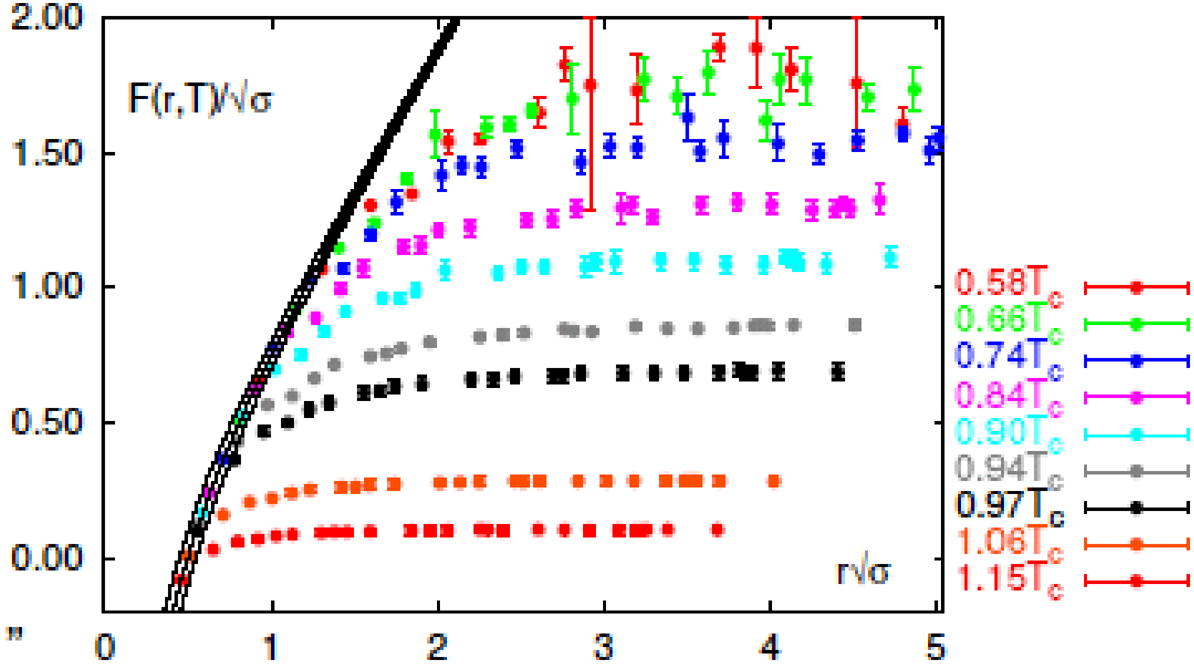
Excess Energy when a quark and an anti-quark exist.

⇨ Heavy Quark Potential



McLerran and Svetitsky, PRD24, □□□□□□

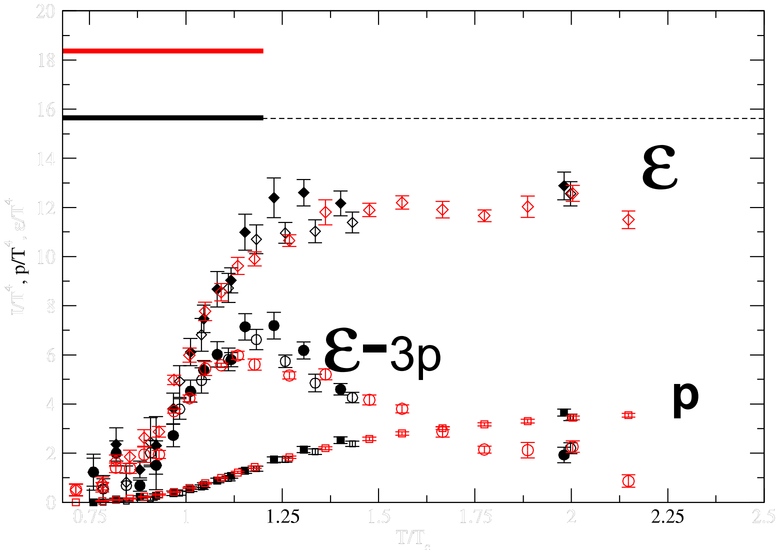
Heavy Quark Potential with Dynamical Quarks



Bielefeld

$$T = 1/N_t a_t$$

$$a_t \to 0 \text{ (continuum limit)} \quad N_t \to \infty$$



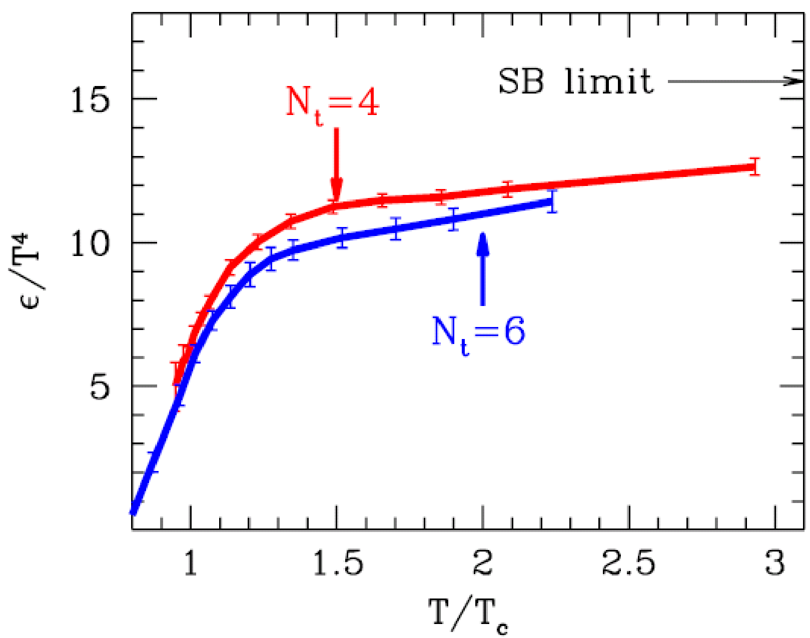
MILC Collaboration, Nf=2+1

hep-lat/0509053

Red Nt=4 **Black Nt=6**

Y.Aoki et al.,

hep-lat/0510084



Progress of Lattice Technology (1)
- Gauge Fixing and Calculation of Color Dependent Objects -

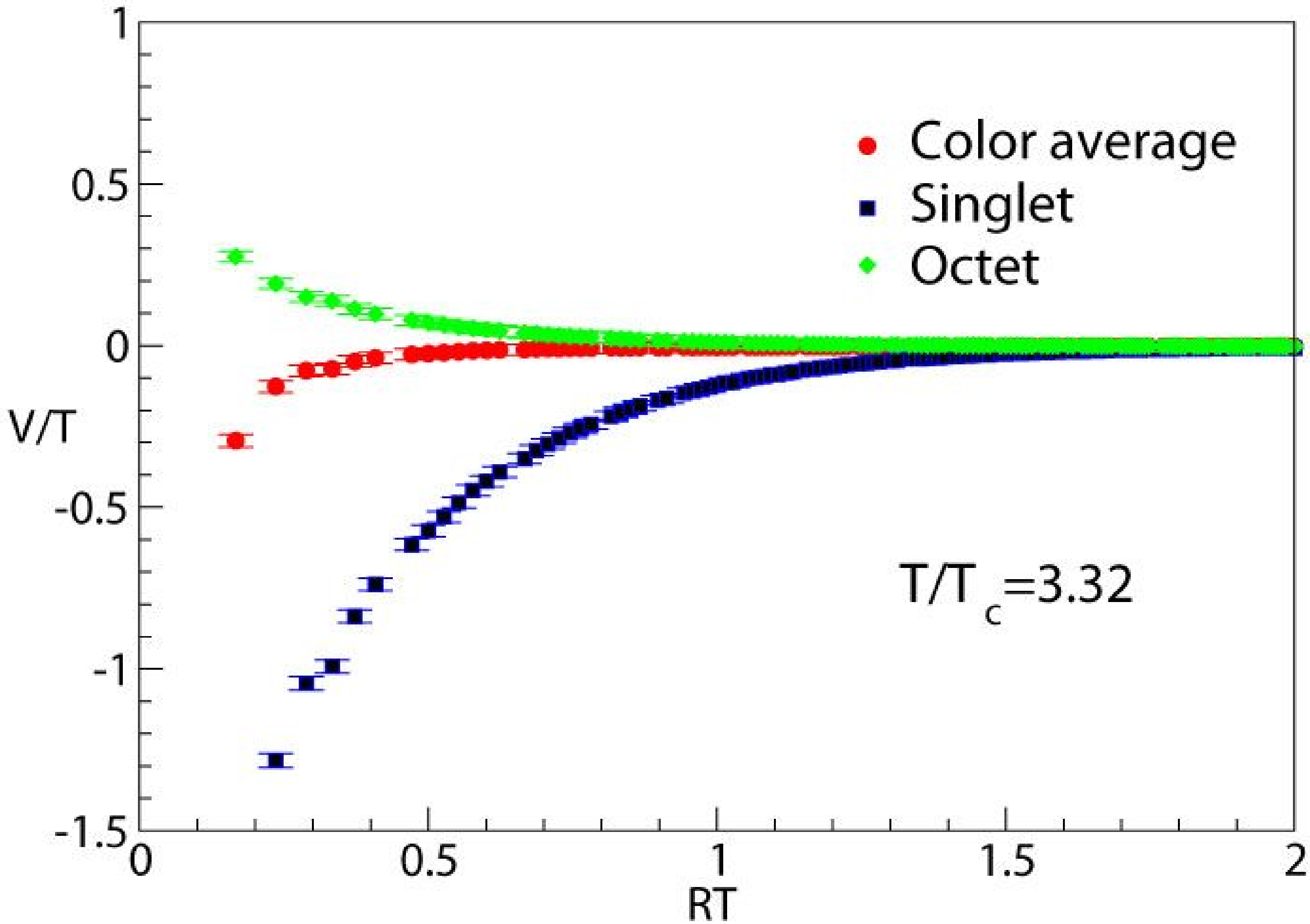
Color Dependent Potentials

$$3 \times 3^* = 1 + 8$$

In early days, we measured the "Color-Averaged" Potential, although the color-singlet formulation was given by McLerran and Svetitsky

Now we can measure "Color-Singlet" Potential.

Color-dependent Potentials (Landau Gauge)



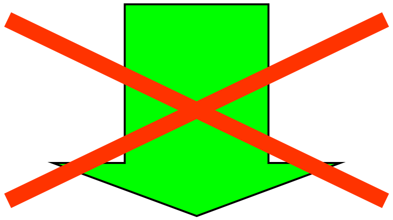
$24^3 \times 6$

Quench

T.Saito and A.Nakamura.

Deconfinement

(Disappearing of the confinement potential)

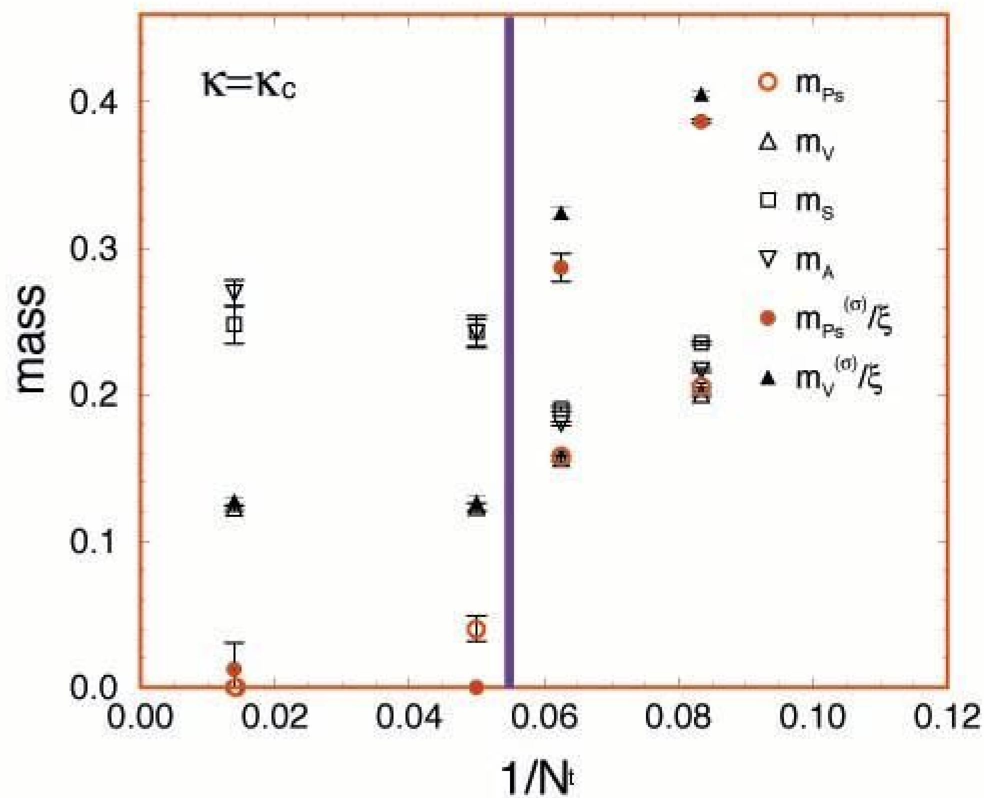


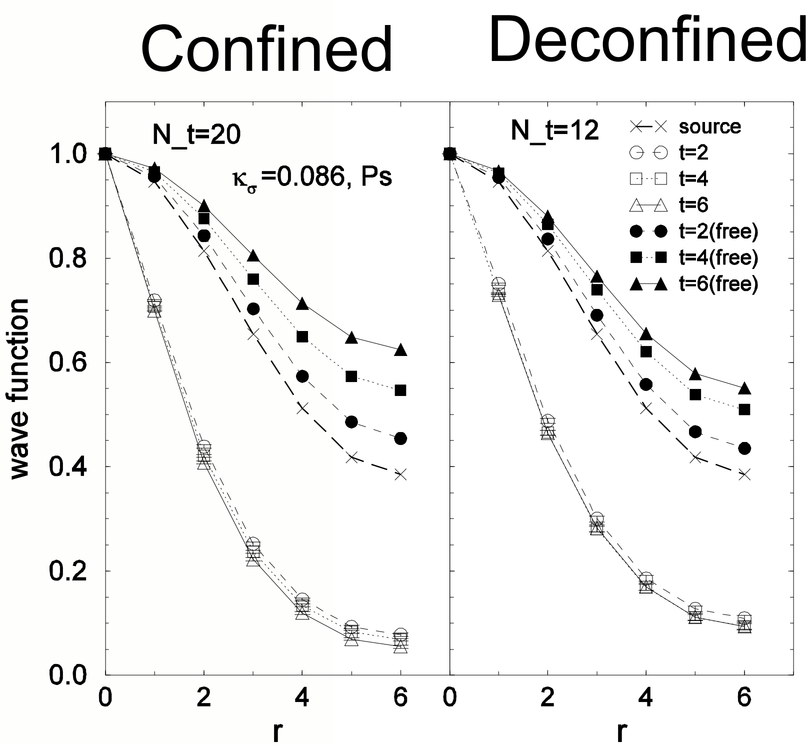
No Bound State

- QED is a Deconfinement theory, but there are Positroniums.
- Mass and Width may change.

Progress of Lattice Technology (2)
- Hadrons at finite Temperature -

QCD-Taro Collaboration, Phys.Rev. D63 (2001) 054501,hep-lat/0008005

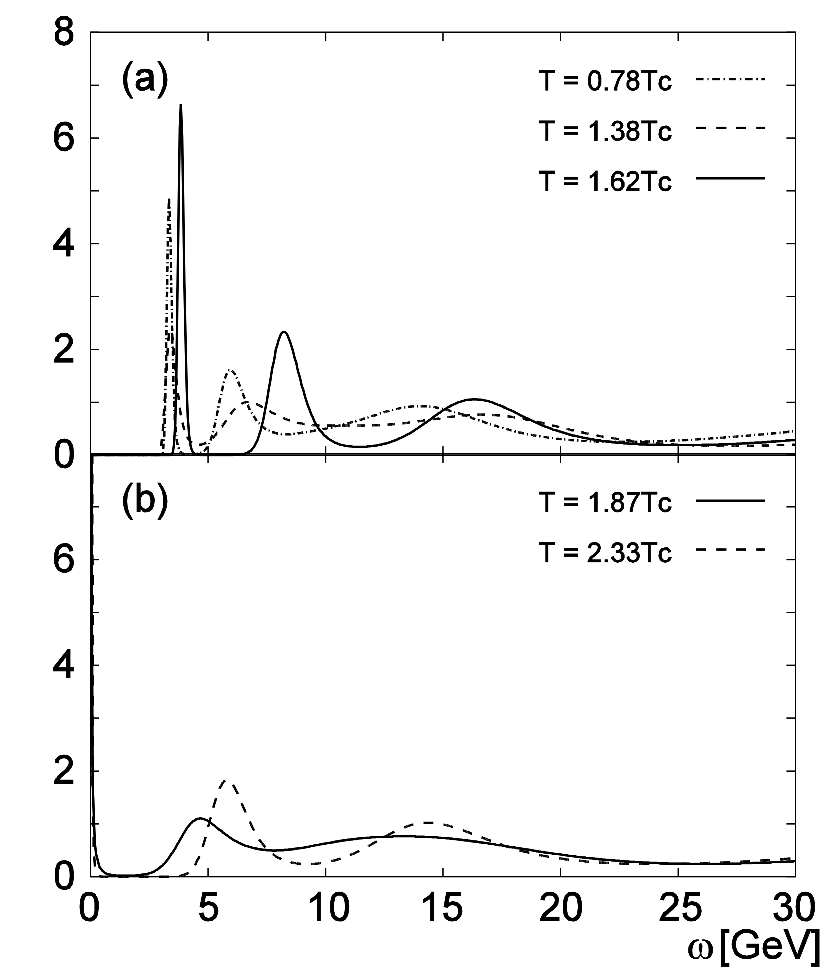


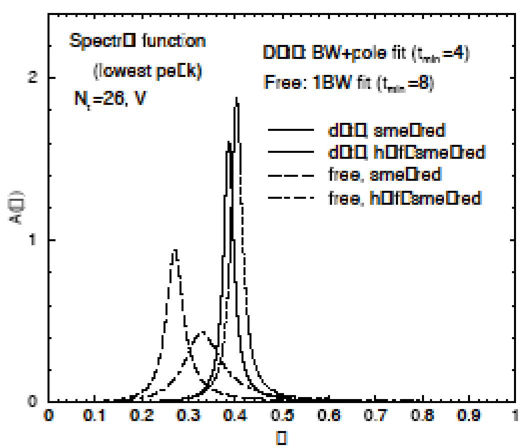


Spectral Functions at finite *T*

- Asakawa-Hatsuda
  - Phys.Rev.Lett. 92 (2004) 012001
- Umeda et al.
  - Nucl.Phys. A721 (2003) 922
- Datta et al. □
  - Phys.Rev. D69 (2004) 094507

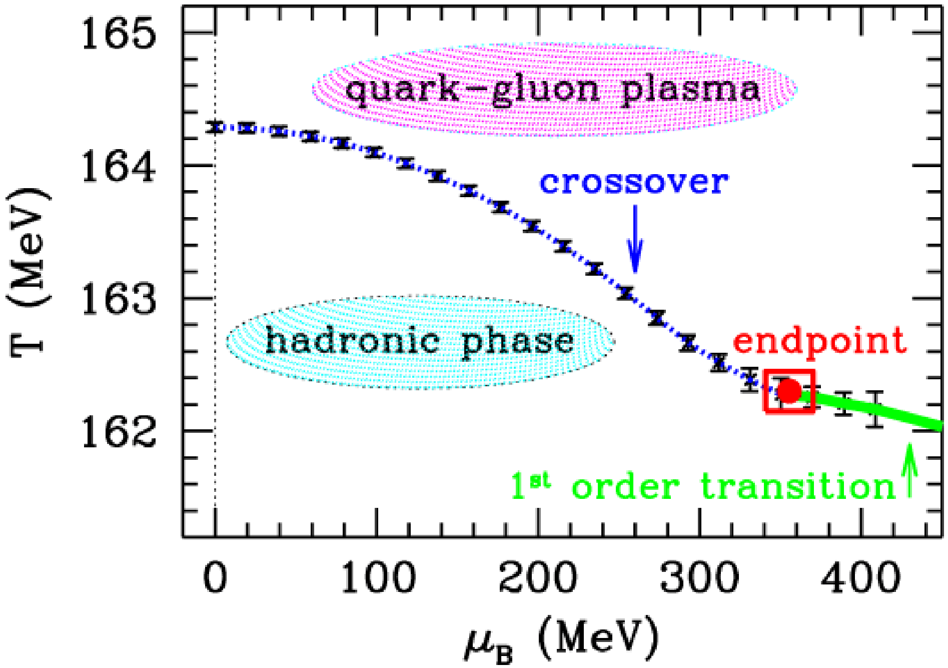
Asakawa-Hatsuda

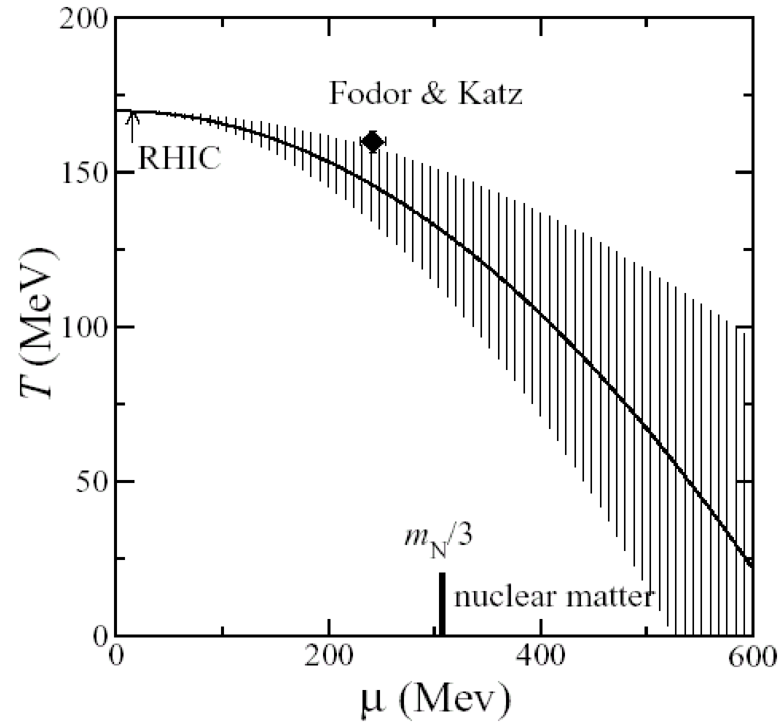


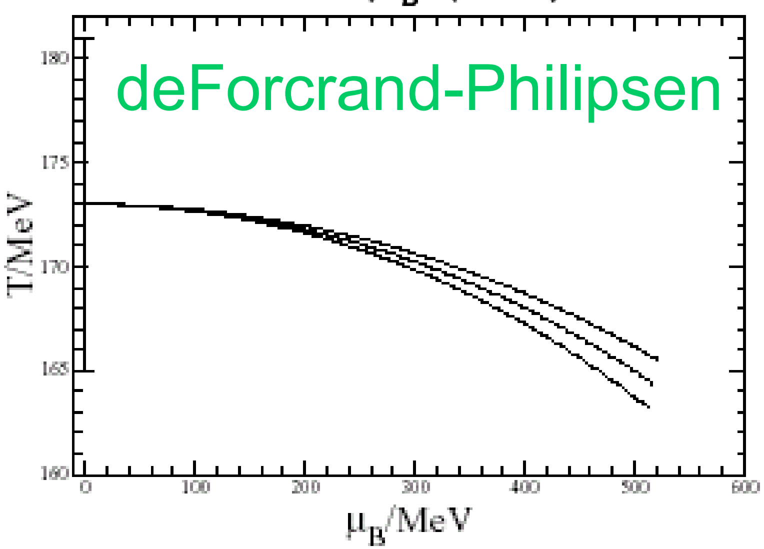


Umeda et al.

Progress of Lattice Technology (3)
- QCD Simulations at Finite Density -

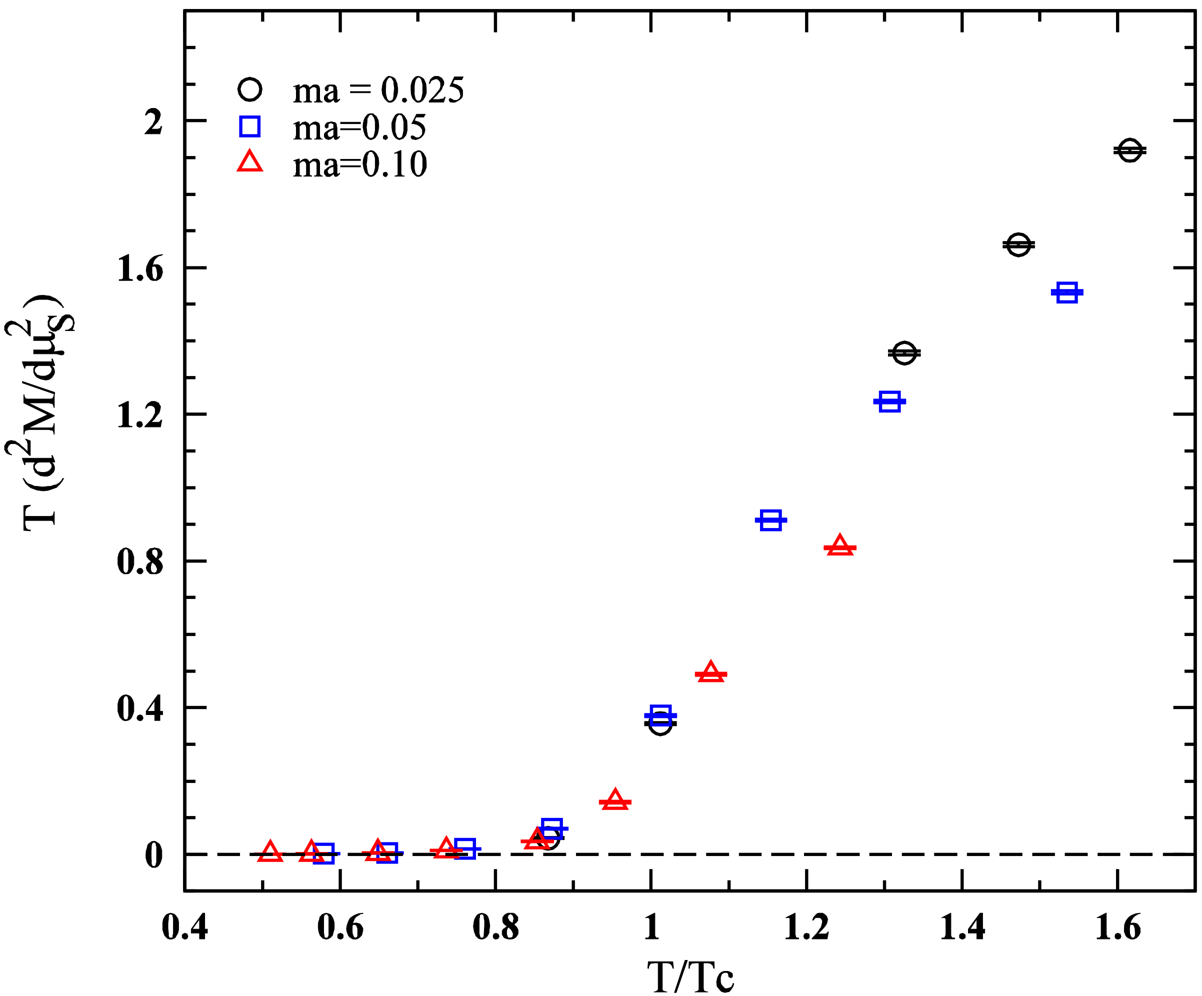






Screening Mass and Density

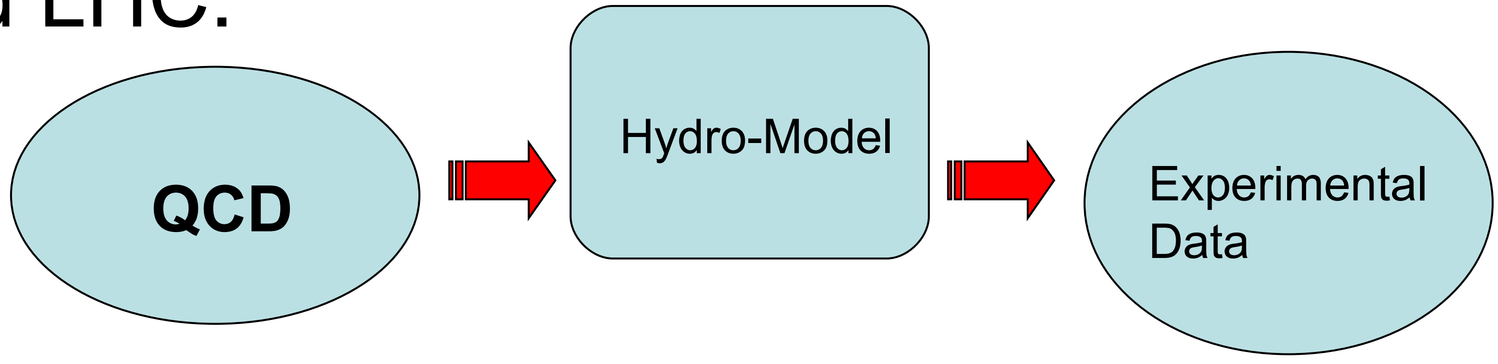
$$M(\mu) = M(0) + \mu \left.\frac{dM}{d\mu}\right|_0 + \frac{1}{2}\mu^2 \left.\frac{d^2M}{d\mu^2}\right|_0 + \cdots$$



Transport Coefficients

A. Nakamura S.Sakai R.Gupta

- A Step towards Gluon Dynamical Behavior.
- They can be (in principle) calculated by a well established formula (Linear Response Theory).
- They are important to understand QGP which is realized in RHIC (and CERN-SPS) and LHC.



Another Personal Motivation

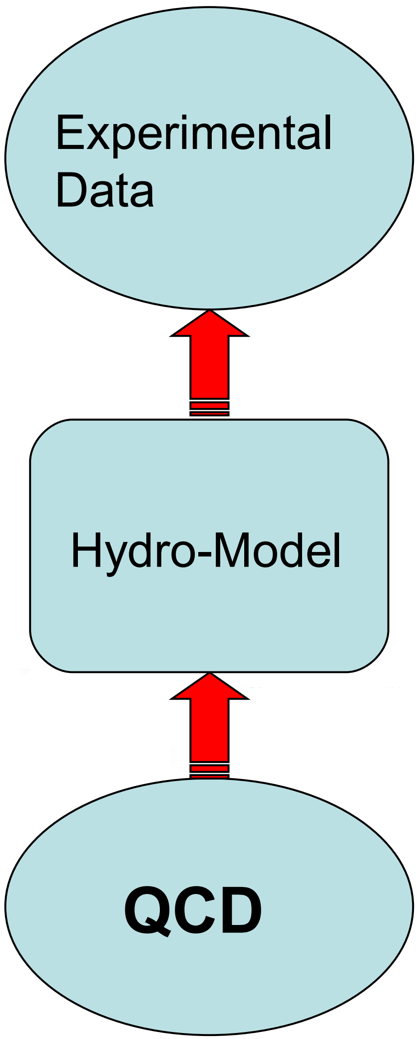
Long time ago, when I was young, I was studying in a Lab as a graduate student of Profs. Namiko and Ohba. (Prof. Bialas once kindly visited and stayed with us.)

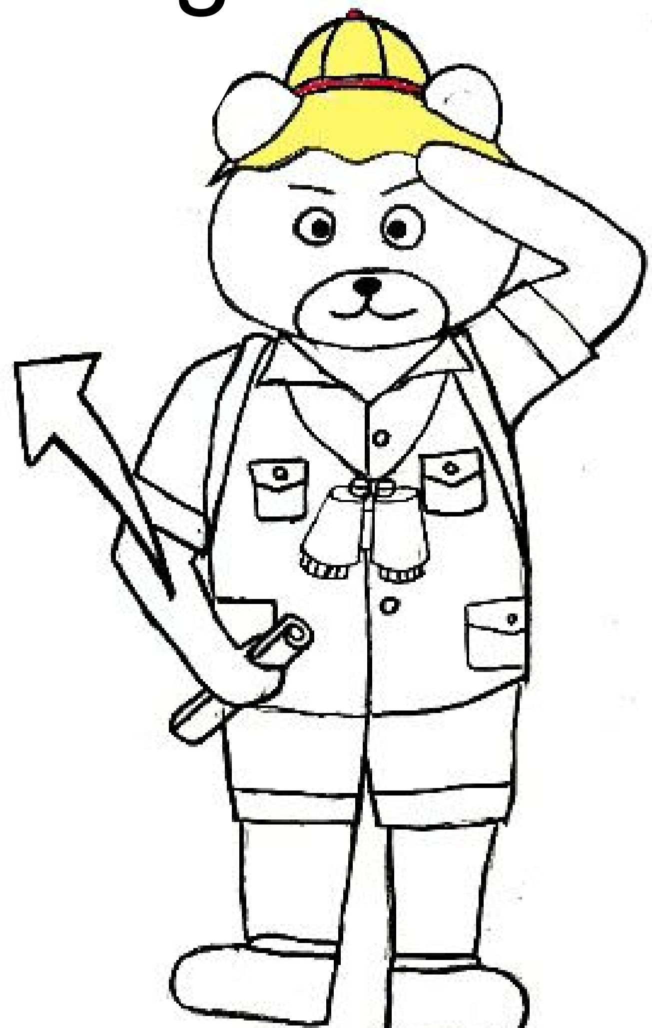
My Supervisor, Prof. Namiki, had studied Landau Hydro-dynamical Model from Field Theory point of view.

It was the only place at that time in Japan, where the hydro was daily discussed.

From the Lab came Muroya, Nonaka, Hirano, Morita … who now actively study the hydro-dynamical model.

Yes, I will also study the hydro for supporting young friends.





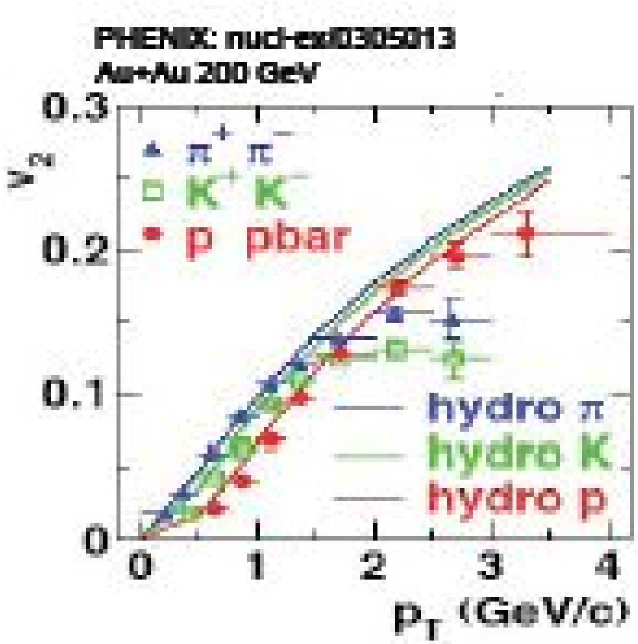
RHIC-data ⇨ Big Surprise !

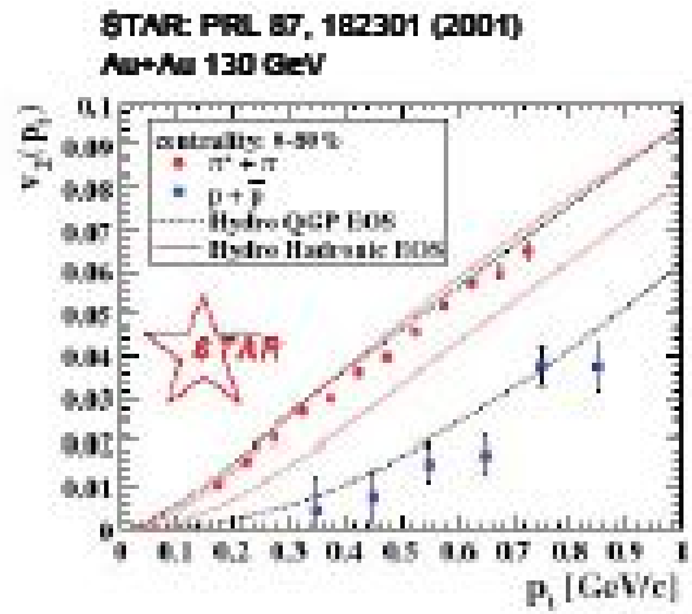
Hydro-dynamical Model describes RHIC data well !

At SPS, the Hydro describes well one-particle distributions,

HBT etc., but fails for the elliptic flow.

Hydro describes well v2





Hydrodynamical calculations are based on Ideal Fluid, i.e., zero shear viscosity.

Or not so surprise …

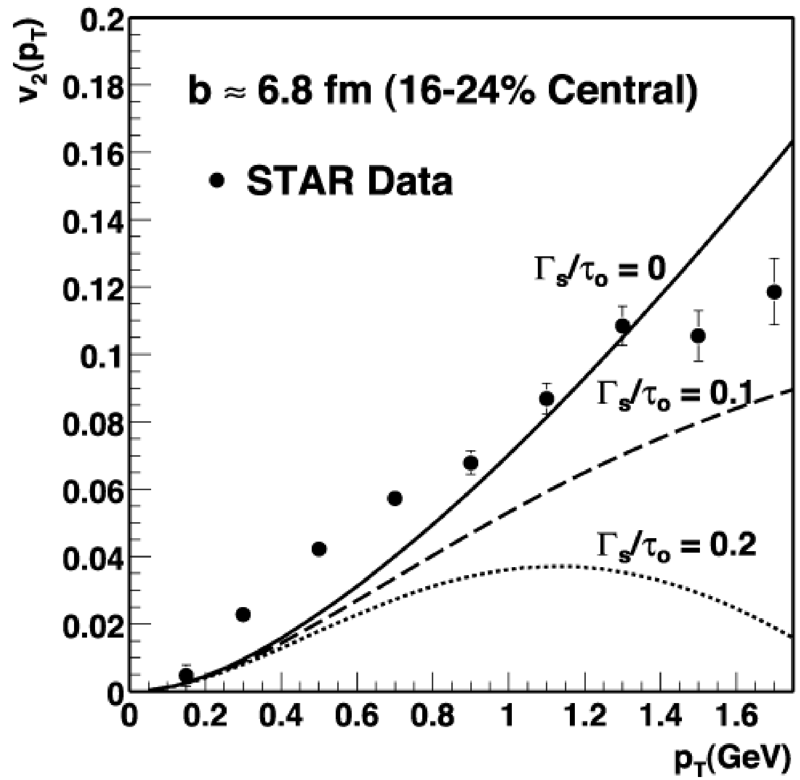
- E. Fermi, Prog. Theor. Phys. 5 (1950) 570
  - Statistical Model
- S.Z.Belen'skji and L.D.Landau, Nuovo.Cimento Suppl. 3 (1956) 15
  - Criticism of Fermi Model
    "Owing to high density of the particles and to strong interaction between them, one cannot really speak of their number."

Hagedorn, Suppl. Nuovo Cim. 3 (1956) 147. Limiting Temperature

Teaney, Phys.Rev. C68 (2003) 034913
(nucl-th/0301099)

$$\Gamma_s \; \S \; \frac{\frac{4}{3}\eta}{sT}$$

η : shear viscosity

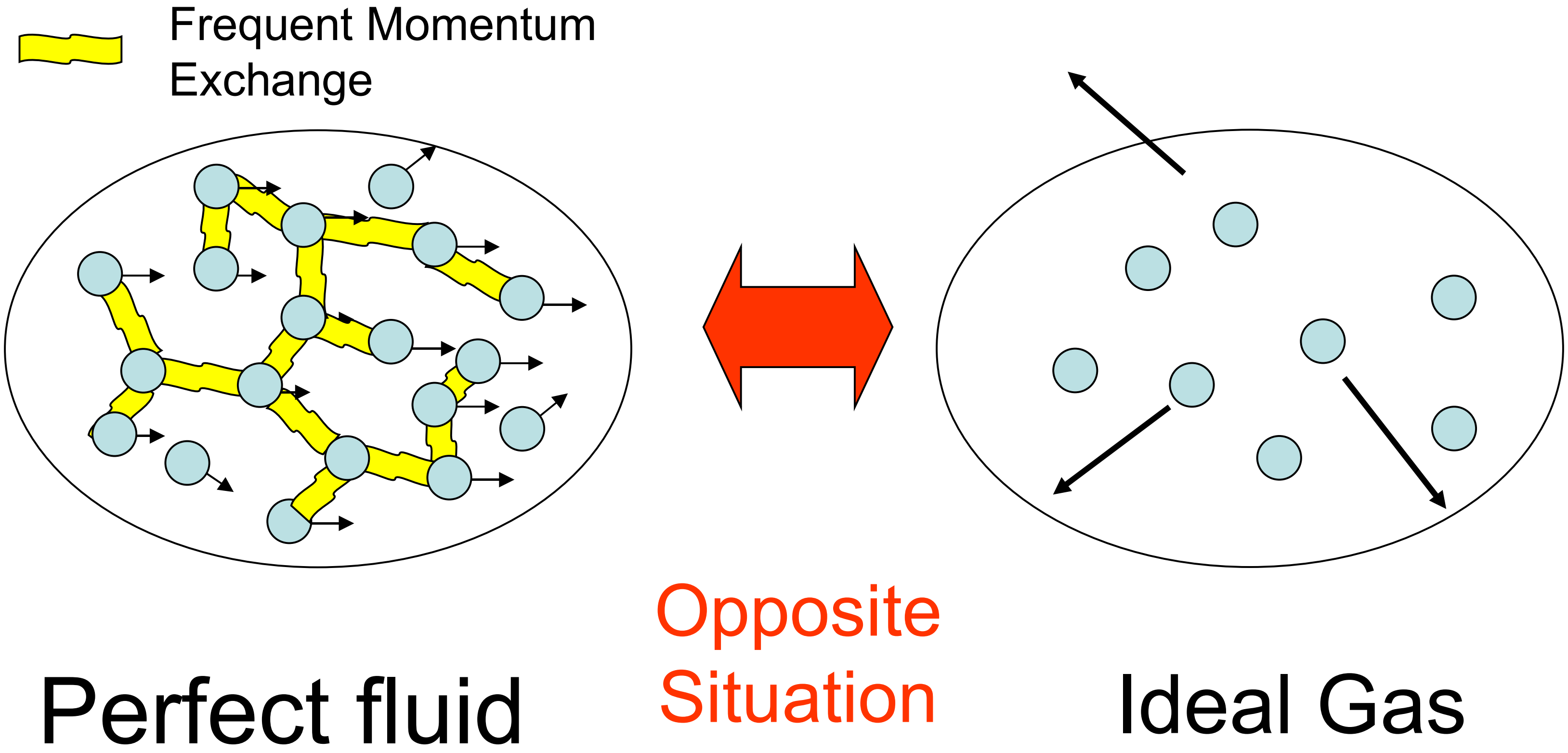


$\tau = \sqrt{t^2 - z^2}$: Time scale of the expansion

Another Big Surprise !

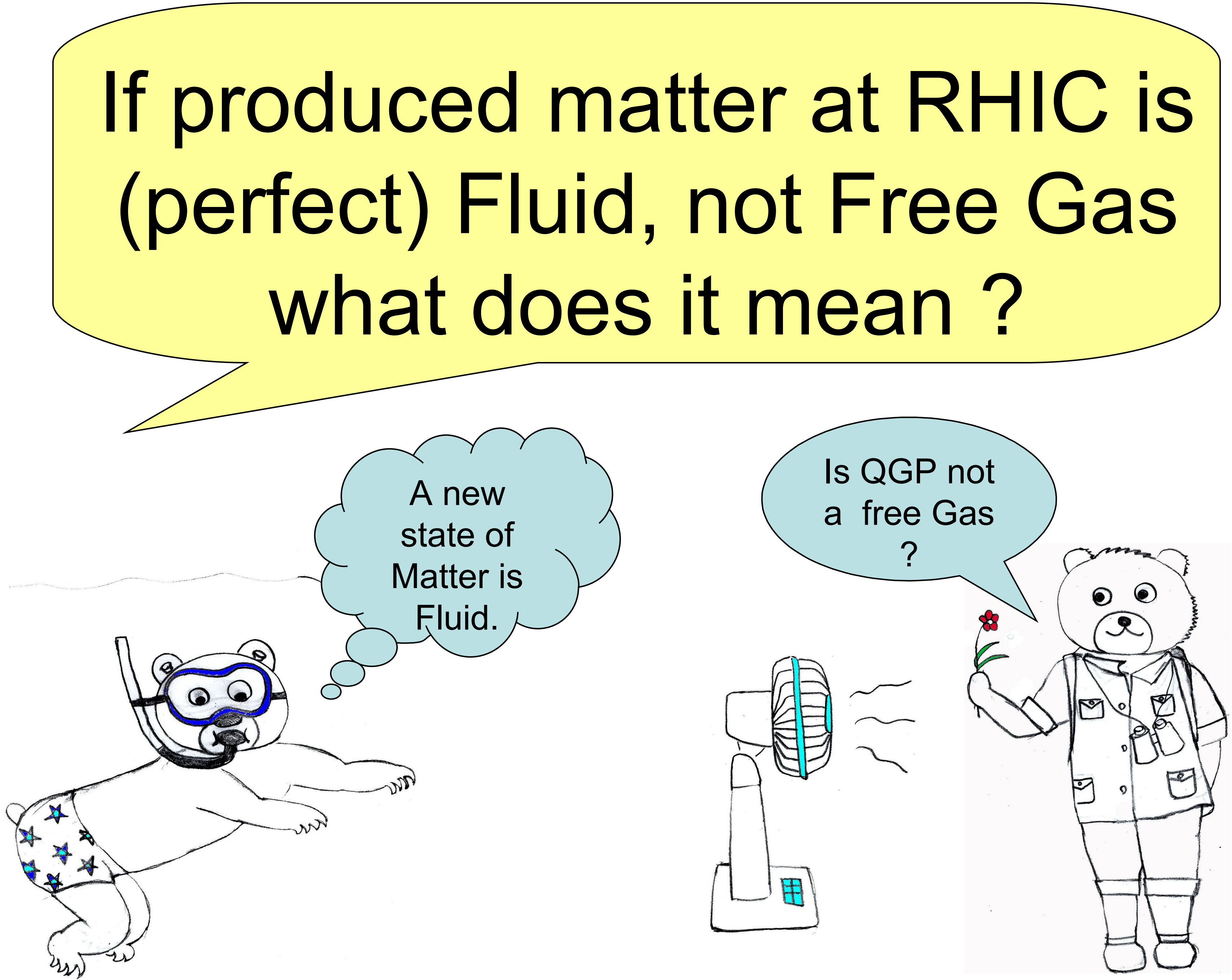
- The Hydrodynamical model assumes zero viscosity,
  i.e., Perfect Fluid.
- Phenomenological Analyses suggest also small viscosity.

Liquid or Gas ?



Literature (1)

- Iso, Mori and Namiki, Prog. Theor. Phys. 22 (1959) pp.403-429
  - The first paper to analyze the Hydrodyanamical Model from Field Theory.
  - Applicability Conditions were derived:
    - Correlation Length << System Size
    - Relaxation time << Macroscopic Characteristic Time
    - Transport Coefficients must be small


If produced matter at RHIC is (perfect) Fluid, not Free Gas what does it mean ?
A new state of Matter is Fluid.
Is QGP not a free Gas ?

Lowest Perturbation
(Illustration purpose only)

Pressure

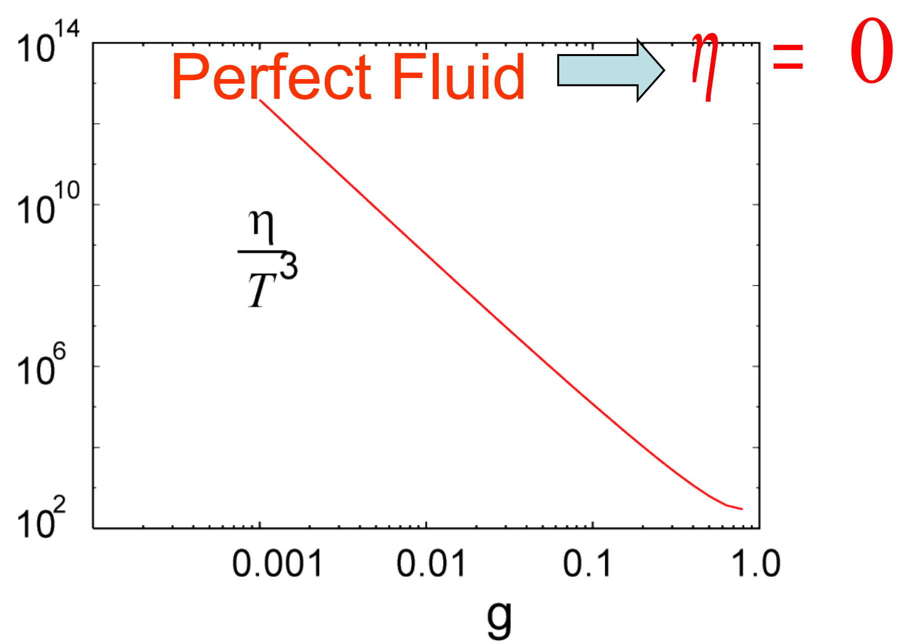
$$P = \frac{\pi^2}{90} T^4 \left(1 - \frac{15}{8} \left(\frac{g}{\pi}\right)^2 + \cdots \right)$$

Ideal Free Gas

Viscosity

$$\eta = \kappa \frac{T^3}{g^4 \ln g^{-1}}$$

$$\kappa = 27.126 (N_f = 0), \quad 86.473 (N_f = 2)$$



- At weak coupling,

Literature (2)

- G. Baym, H. Monien, C. J. Pethick and D. G. Ravenhall,
  - Phys. Rev. Lett. 16 (1990) 1867.
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0011 (2000) 001, (hep-ph/0010177).
  - Leading-log results"
- P. Arnold, G. D. Moore and L. G. Yaffe
  - JHEP 0305 (2003) 051, (hep-ph/0302165).
  - Beyond leading log"

Literature (3)

- Hosoya, Sakagami and Takao, Ann. Phys. 154 (1984) 228.
  - Transport Coefficients Formulation
- Hosoya and Kayantie, Nucl. Phys. B250 (1985) 666.
- Horsley and Shoenmaker, Phys. Rev. Lett. 57 (1986) 2894; Nucl. Phys. B280 (1987) 716.
- Karsch and Wyld, Phys. Rev. D35 (1987) 2518.
  - The first Lattice QCD Calculation
- Aarts and Martinez-Resco, JHEP0204 (2002)053
  - Criticism against the Spectrum Function Ansatz.
- Petreczky and Teaney, hep-ph/0507318
  - Impossible to determine Heavy Quark Transport coefficient

Literature (4)

- Masuda, A.N.,Sakai and Shoji
  Nucl.Phys. B(Proc.Suppl.)42, (1995),526
- A.N., Sakai and Amemiya
  Nucl.Phys. B(Proc.Suppl.)53, (1997), 432
- A.N, Saito and Sakai
  Nucl.Phys. B(Proc.Suppl.)63, (1998), 424
- Sakai, A.N. and Saito
  Nucl.Phys. A638, (1998), 535c
- A.N, Sakai
  Phys.Rev.Lett. 94 (2005) 072305
  hep-lat/0406009

Linear Response Theory

- Zubarev
  "Non-Equilibrium Statistical Thermo-dynamics"
- Kubo, Toda and Saito
  "Statistical Mechanics"

$$\rho \sim e^{-A+B} : \text{ non-equilibrium statistical operator}$$

$$A = \int d^3x \beta(x,t) u^{\nu} T_{0\nu}(x,t)$$

$$B = \int d^3x \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} T_{\mu\nu}(x,t) \partial^{\mu}(\beta(x,t) u^{\nu})$$

Using: $e^{-A+B} = e^{-A} + \int_0^1 d\tau\, e^{A\tau} B e^{-A\tau} e^{-A} + \cdots$

$$\rho \approx \rho_{eq} + \int_0^1 d\tau \left(e^{A\tau} B e^{-A\tau} e^{-A} - \langle B \rangle_{eq}\right) \rho_{eq}$$

$$\rho_{eq} \equiv e^{-A} / \mathrm{Tr} e^{-A} \rightarrow \exp(-\beta H) / \mathrm{Tr} e^{-A}$$

in the co-moving frame, $u^{\mu} = (1 \quad 0 \quad 0 \quad 0)$

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu} \rangle_{eq} +$$

$$+ \int d^3x' \int_{-\infty}^{t} dt' e^{\varepsilon(t'-t)} (T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq} \partial^{\rho} (\beta u^{\sigma})$$

where $(T_{\mu\nu}(x,t), T_{\rho\sigma}(x',t'))_{eq}$

$$\equiv \int_0^1 d\tau \left\langle T_{\mu\nu}(x,t) \left(e^{-A\tau} T_{\rho\sigma}(x',t') e^{A\tau} - \langle T_{\rho\sigma}(x',t') \rangle_{eq} \right) \right\rangle_{eq}$$

$$\langle T^{ij} \rangle = \eta (\partial^i u^j + \partial^j u^i)/2$$

$$\langle T^{0i} \rangle = -\chi (\beta^{-1}(x,t) \partial^i \beta + \partial_{\alpha} u^{\alpha})$$

$$\langle p \rangle - \langle p \rangle_{eq} = -\varsigma \partial_{\alpha} u^{\alpha}$$

$$p \equiv -\frac{1}{3} T^i{}_i$$

- One can show

$$(T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'))_{eq} = -\beta^{-1}\int_{-\infty}^{t'} dt''\left\langle T_{\mu\nu}(x,t),T_{\rho\sigma}(x',t'')\right\rangle_{ret}$$

Transport Coefficients are expressed by Quantities at Equilibrium

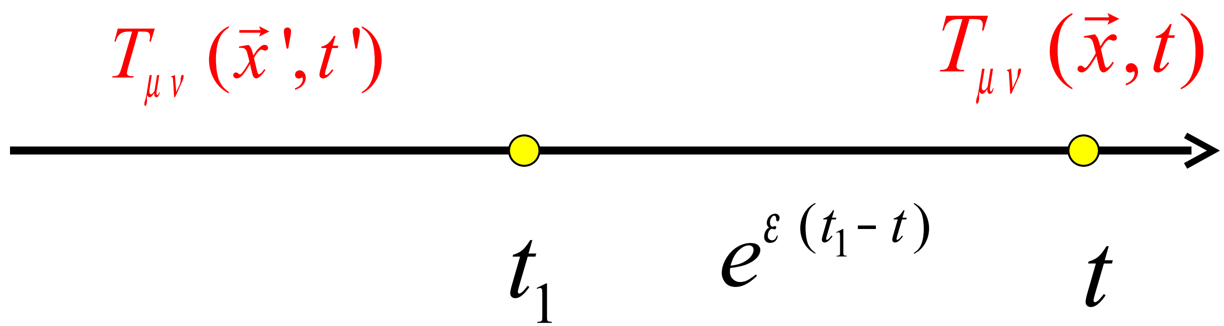
$$\eta = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{12}(\vec{x},t) T_{12}(\vec{x}',t') >_{ret}$$

$$\frac{4}{3}\eta + \varsigma = -\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{11}(\vec{x},t) T_{11}(\vec{x}',t') >$$

$$\chi = -\frac{1}{T}\int d^3x' \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)} \int_{-\infty}^{t_1} dt' < T_{01}(\vec{x},t) T_{01}(\vec{x}',t') >_{ret}$$

η **: Shear Viscosity** ς **: Bulk Viscosity**

χ **: Heat Conductivity** → we do not consider in Quench simulations.



$$-\infty < t' < t_1 < t$$

Energy Momentum Tensors

$$T_{\mu\nu} = 2Tr(F_{\mu\sigma}F_{\nu\sigma} - \frac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})$$

$$(T_{\mu\mu} = 0)$$

$$U_{\mu\nu}(x) = \exp(ia^2 gF_{\mu\nu}(x))$$

$$F_{\mu\nu} = \log U_{\mu\nu} / ia^2 g$$

or

$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}^{\dagger}\right) / 2ia^2 g$$

Real Time Green function vs. *Temperature* Green function

Hashimoto, A.N. and Stamatescu, Nucl.Phys.B400(1993)267

$$<<\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]>> \equiv \frac{1}{Z}\mathrm{Tr}\left(\frac{1}{i}[\phi(t,\vec{x}),\phi(t',\vec{x}')]e^{-\beta H}\right)$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\Lambda(\omega,\vec{p})$$

$$\phi(t,\vec{x}) = e^{itH}\phi(0,\vec{x})e^{-itH}$$

$$G_\beta^{ret/adv}(t,\vec{x};t',\vec{x}') = \pm\theta(t-t'/t'-t)<<\ldots>>$$

$$= F\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}K_\beta^{ret/adv}(\omega,\vec{p})$$

$$K_\beta^{ret/adv}(\omega,\vec{p}) = \int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\Lambda(\omega')}{\omega-\omega'\pm\varepsilon}$$

Temperature Green function

$$G_\beta(\tau,\vec{x};\tau',\vec{x}') = << T_\tau \phi(\tau,\vec{x})\phi(\tau',\vec{x}') >>$$

$$\phi(t,\vec{x}) = e^{\tau H}\phi(0,\vec{x})e^{-\tau H}$$

$$G_\beta(\tau,\vec{x};0,0) = G_\beta(\tau+\beta,\vec{x};0,0)$$

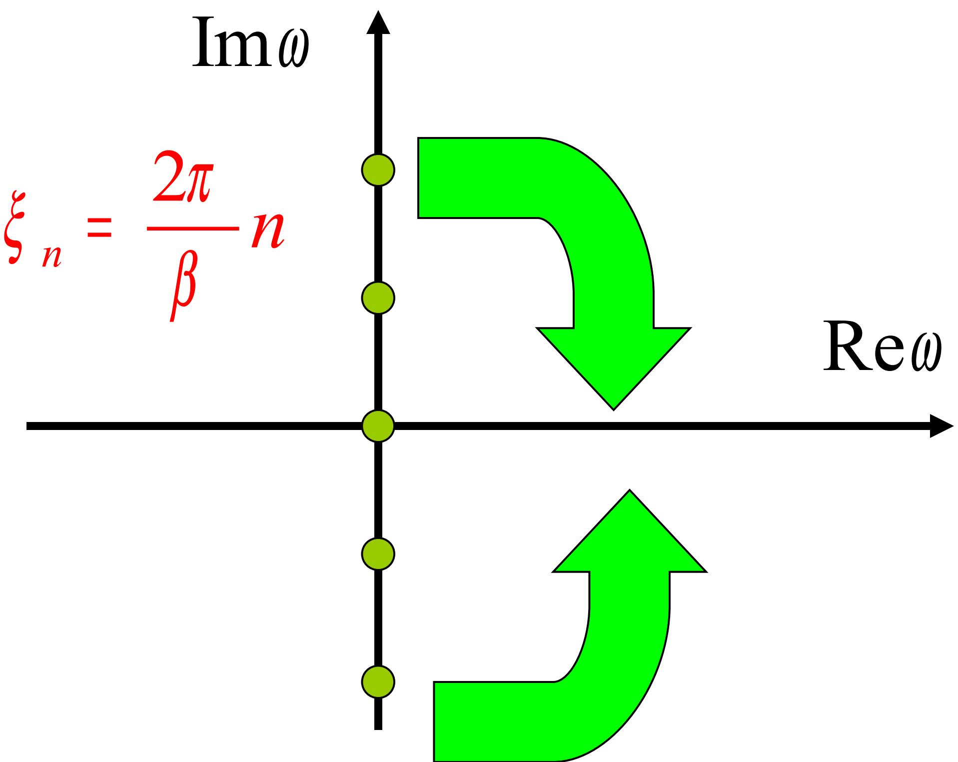
$$\hat{K}_\beta(\xi_n,\vec{p}) = F^{-1}\int_0^\beta d\tau\, e^{-i\xi_n(\tau-\tau')}G_\beta(\tau,\vec{x};\tau',\vec{x}')$$

$$\xi_n = \frac{2\pi}{\beta}n, n = 0,\pm 1,\pm 2,,,$$

Matsubara-frequencies

Abrikosov-Gorkov-Dzyalosinski-Fradkin Theorem

$$\hat{K}_{\beta}(\xi_n) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\Lambda(\omega)}{\omega - i\xi_n} = iK_{\beta}(i\xi_n)$$



$$\xi_n = \frac{2\pi}{\beta} n$$

On the lattice, we measure Temperature Green function at $\omega = \xi_n$

We must reconstruct Advance or Retarded Green function.

Transport Coefficients of QGP

We measure Correlations of Energy-Momentum tensors

$$\langle T_{\mu\nu}(0)T_{\mu\nu}(\tau)\rangle$$

Convert them (Matsubara Green Functions) to Retarded ones (real time).

Transport Coefficients (Shear Viscosity, Bulk Viscosity and Heat Conductivity)

Ansatz for the Spectral Functions

We measure Matsubara Green Function on Lattice (in coordinate space).

$$< T_{\mu\nu}(t,\vec{x})T_{\mu\nu}(0) > = G_\beta(t,\vec{x}) = F.T.G_\beta(\omega_n,\vec{p})$$

$$G_\beta(\vec{p},i\omega_n) = \int d\omega \frac{\rho(\vec{p},\omega)}{i\omega_n - \omega}$$

We assume (Karsch-Wyld)

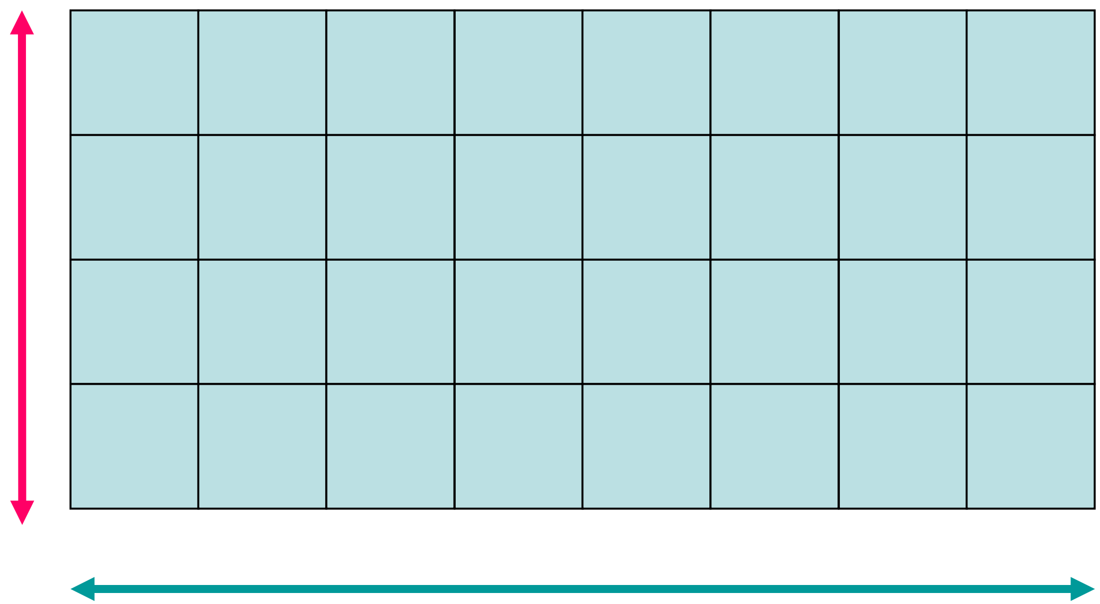
$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2} + \frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

and determine three parameters, A, m, γ.

We need large Nt !

Some Special Features of Lattice QCD at Finite Temperature

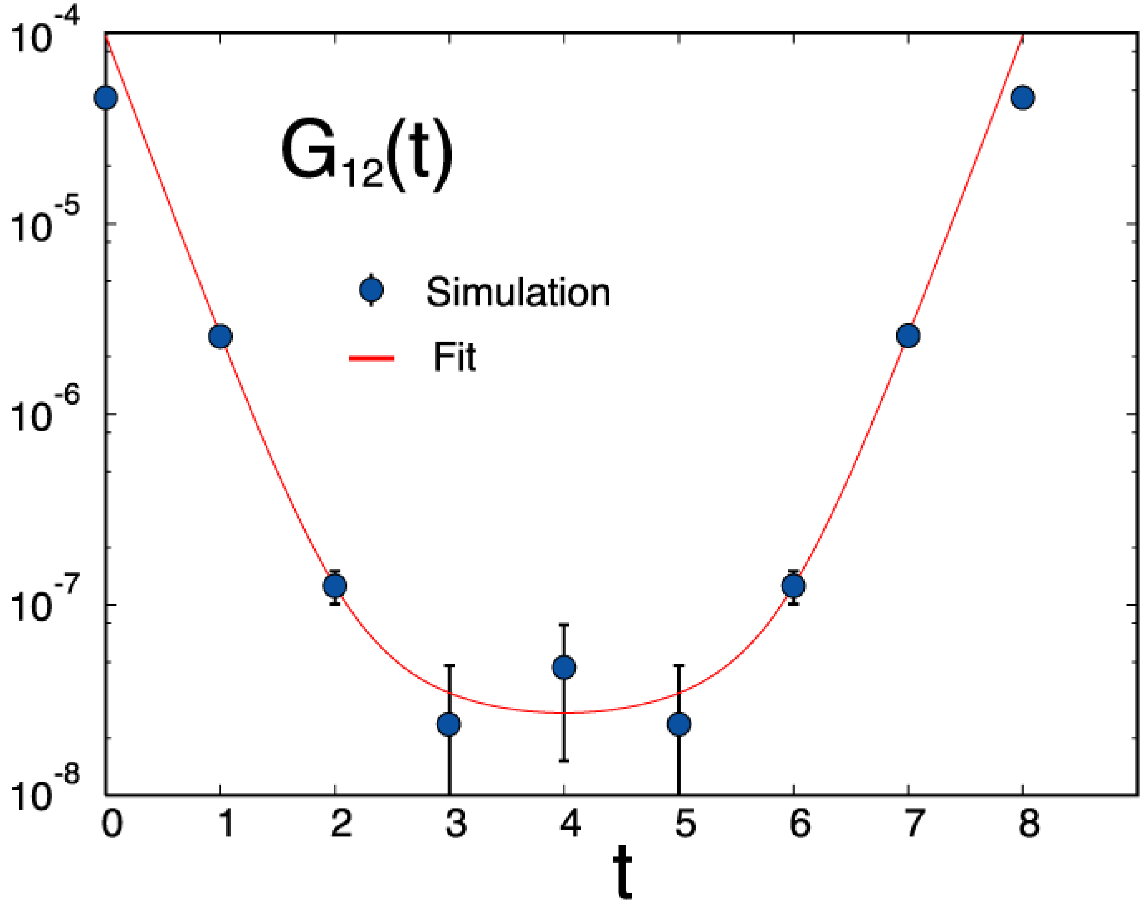
$$N_t a_t = \frac{1}{kT}$$



$$N_s a_s$$

High Temperature ⇒ $N_t a_t$: small

Nt=8



Lattice and Statistics

Iwasaki Improved Action

$16^3 \times 8$

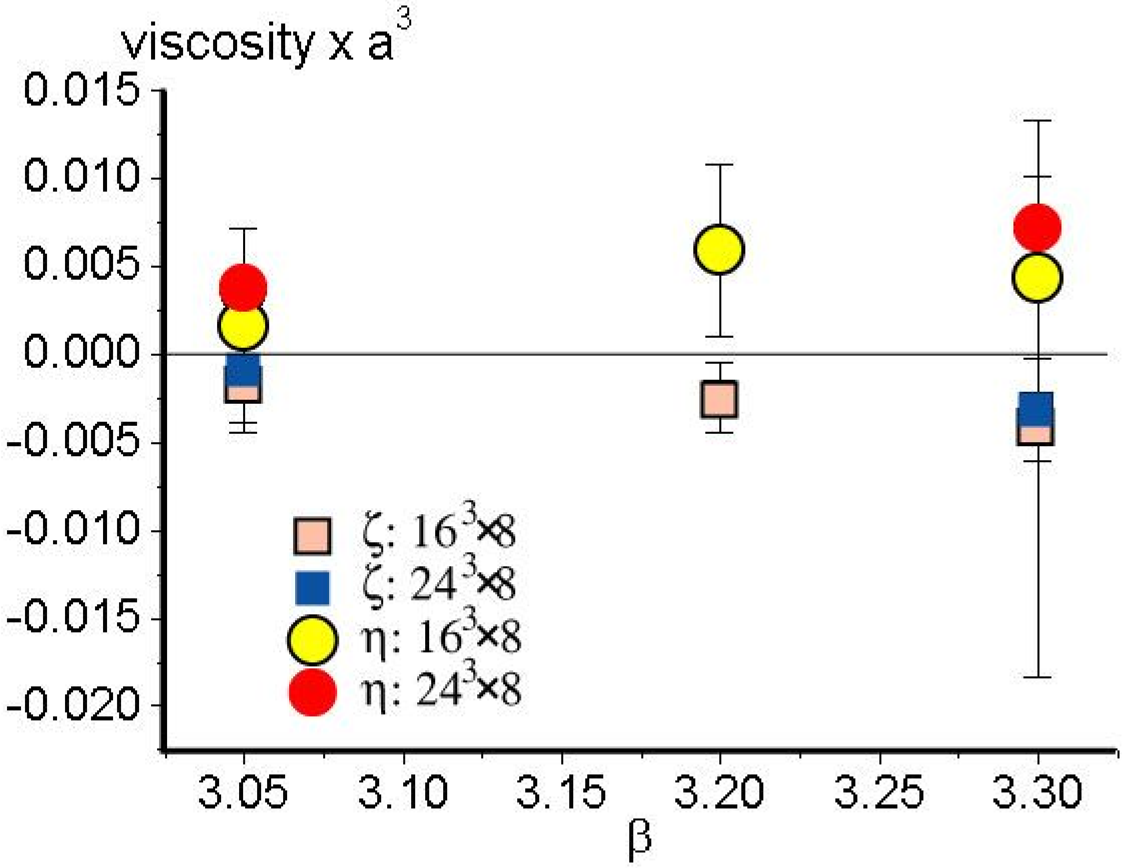
β=3.05 : 1333900 sweeps
β=3.20 : 1212400 sweeps
β=3.30 : 1265500 sweeps

$24^3 \times 8$

β=3.05 : 61000 sweeps
β=3.30 : 84000 sweeps

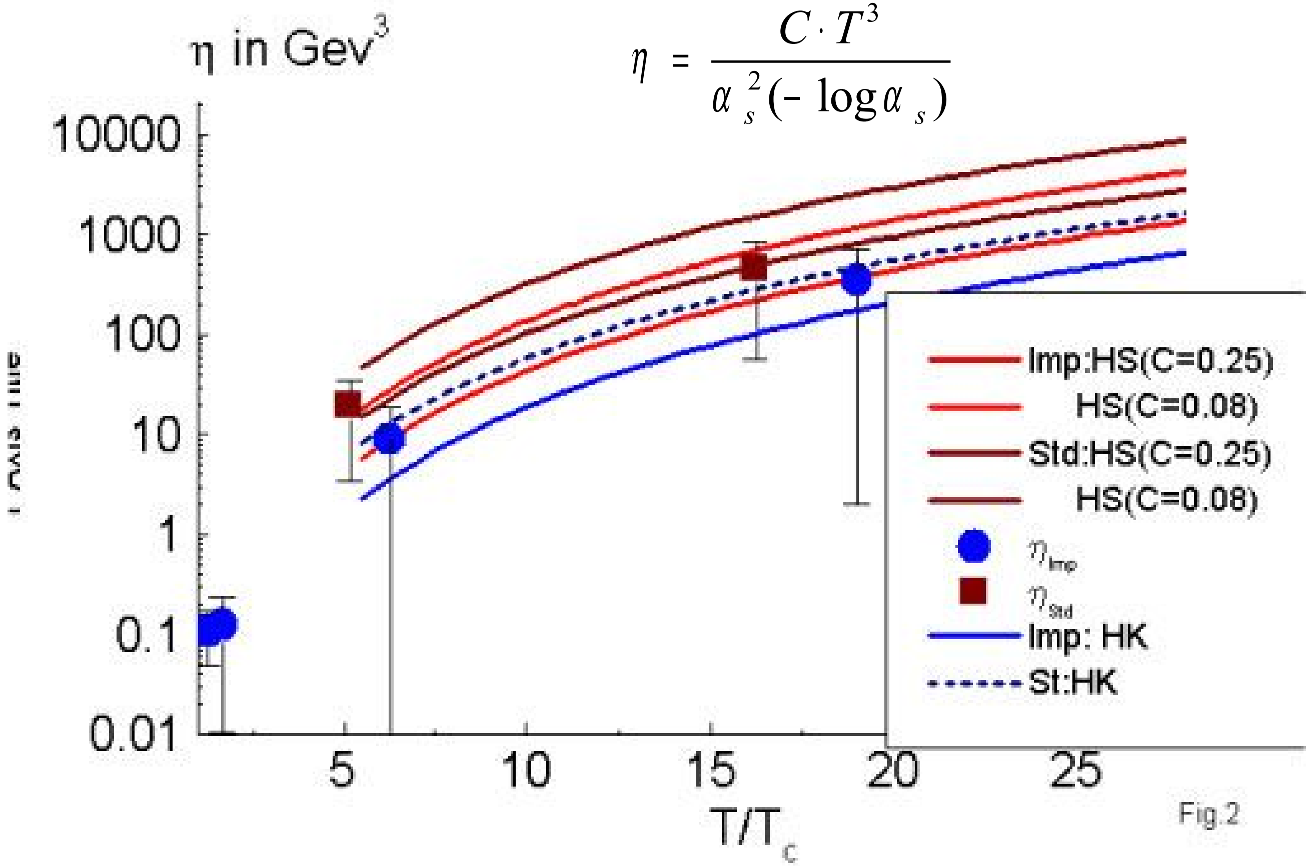
Quench

Results: Shear and Bulk Viscosities



Comparison with Pertubative Calculations

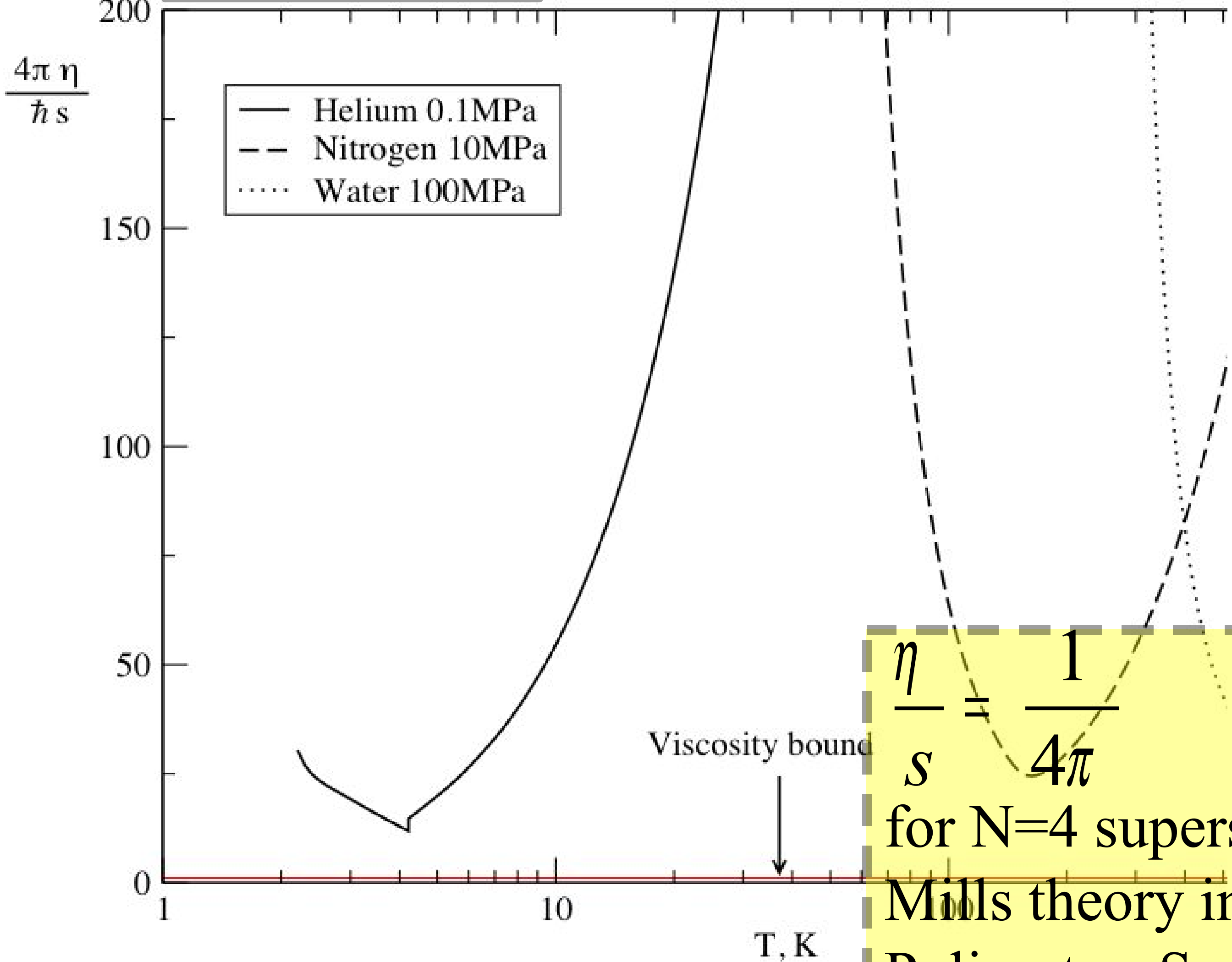
$$\eta = \frac{C \cdot T^3}{\alpha_s^2(-\log \alpha_s)}$$



Good for T/Tc>5

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \ !$$

Kovtun, Son and Starinets, hep-th/0405231

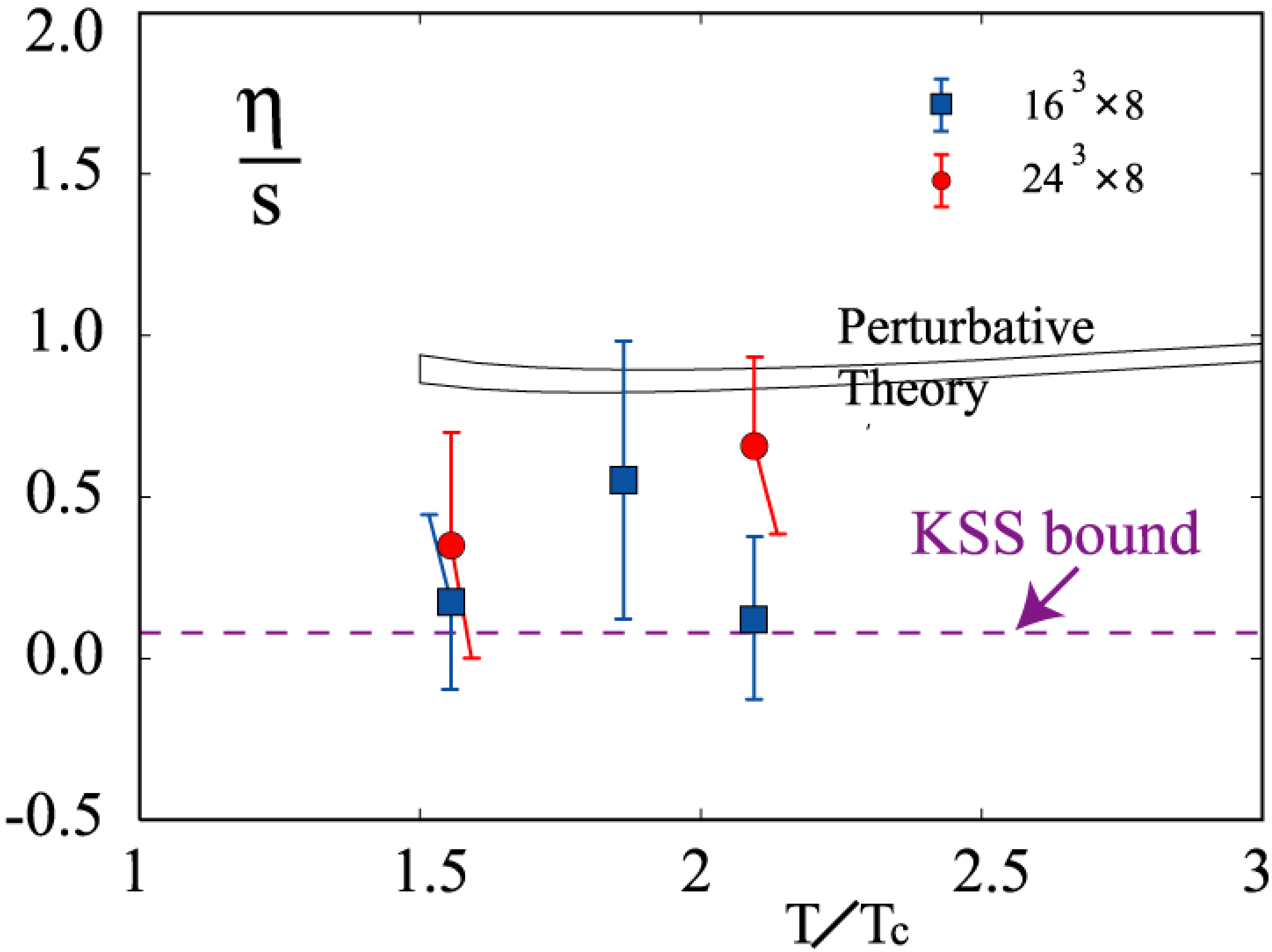


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

for N=4 supersymmetric Yang-Mills theory in the large N.

Policastro, Son and Starinets, Phys

Rev. Lett. 87 (2001) 081601



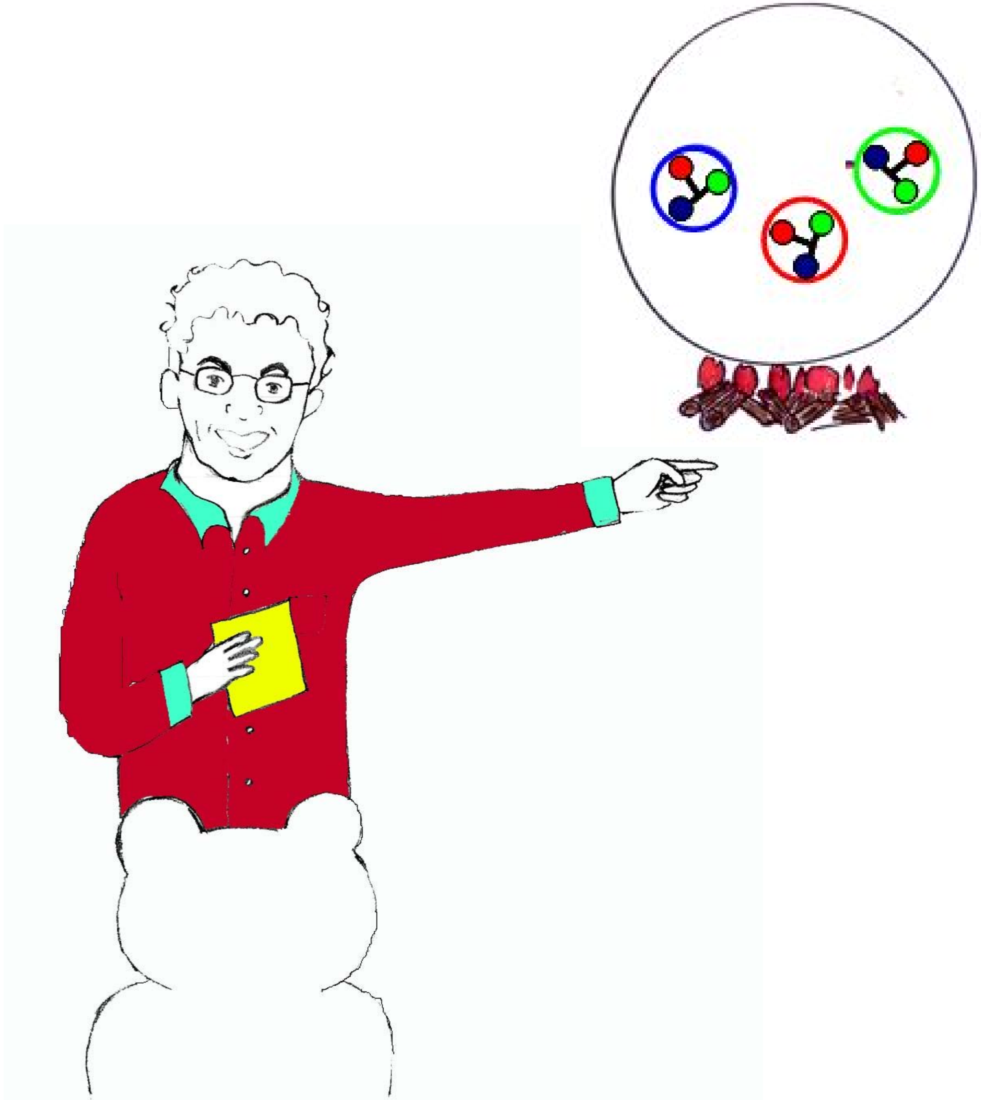
2.0
1.5
1.0
0.5
0.0
-0.5
η/s
16^3×8
24^3×8
Perturbative
Theory
KSS bound
1
1.5
2
2.5
3
T/Tc

$\frac{\eta}{s}$ can have the lower limit ?

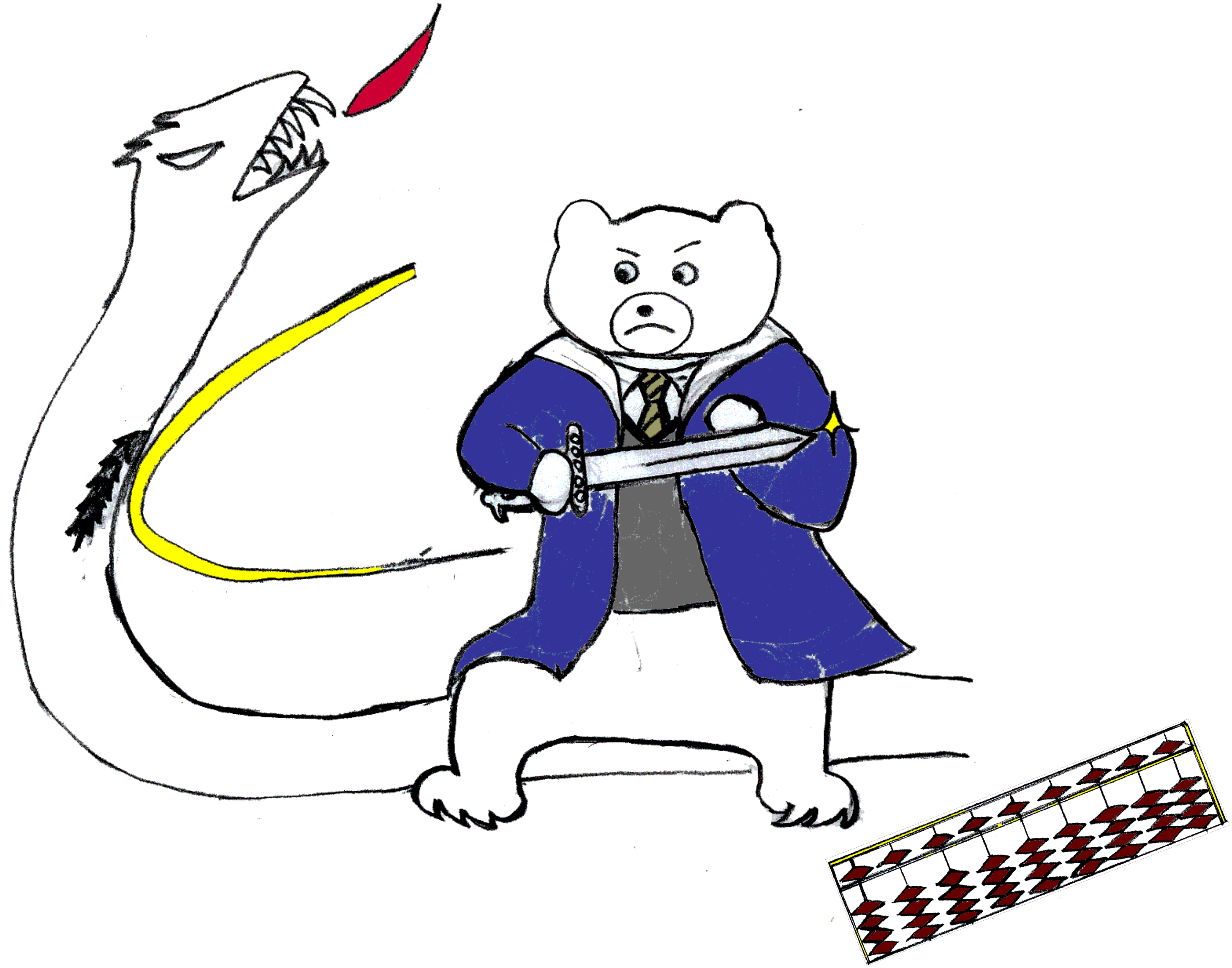
- Counter Example by Prof. Baym
  - We heat up Billiard Balls which have inter-structure. Then Entropy increases. If the surface of the balls does not change, the Viscosity should be the same.

$$\Rightarrow \quad \frac{\eta}{s} \to 0$$

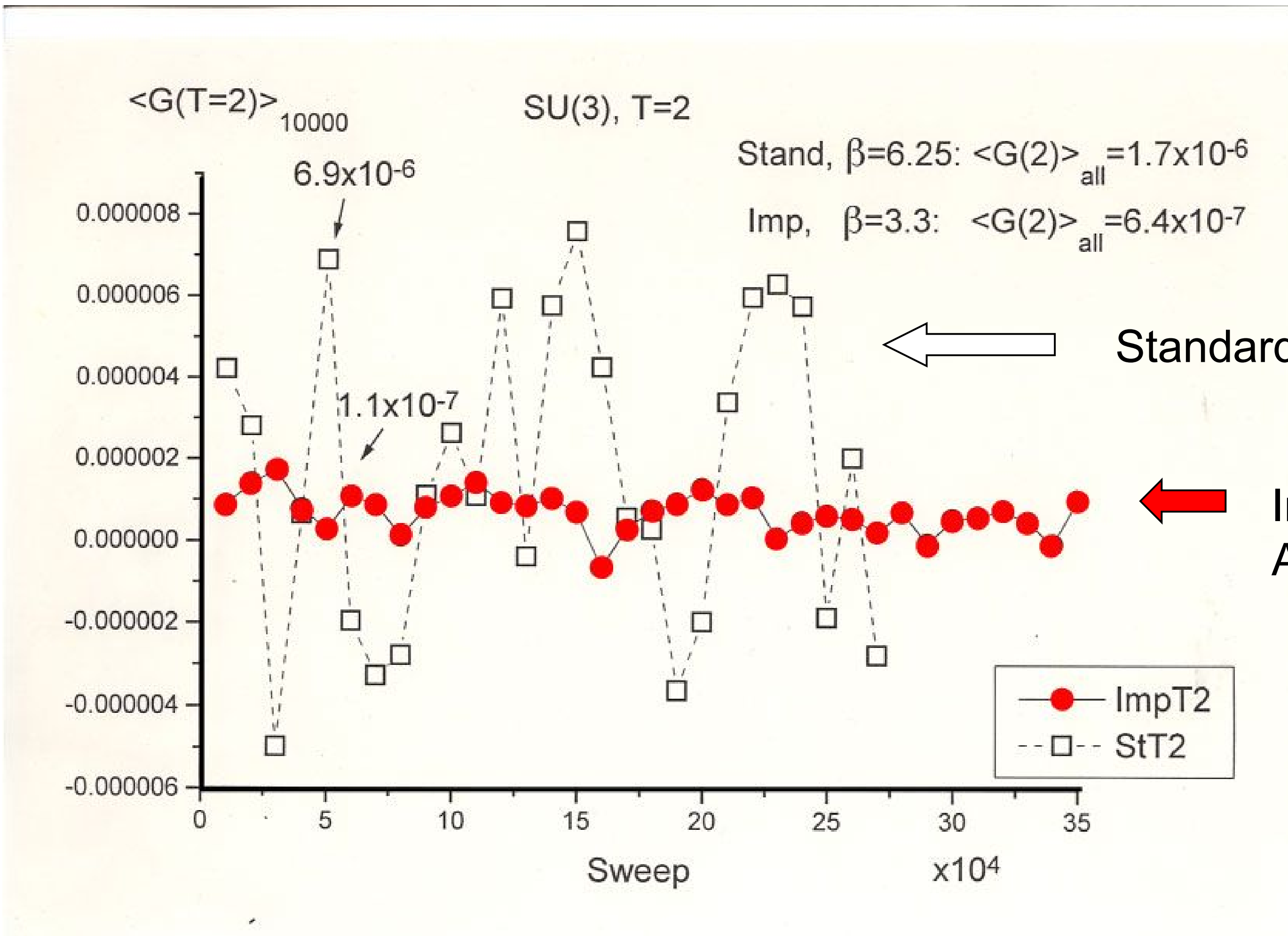
- We may give Counter-Argument ?



Fighting against Noise



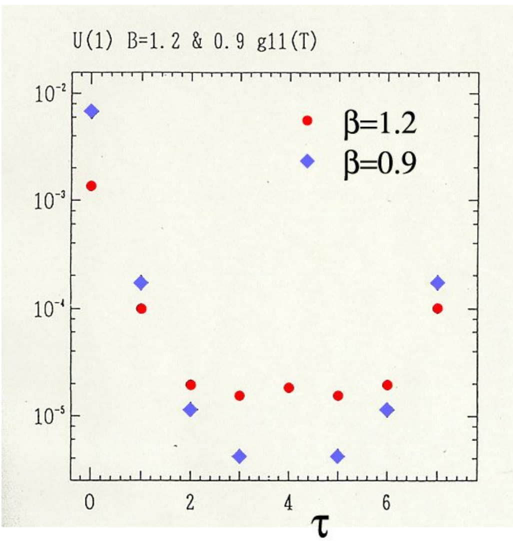
Fluctuations in MC sweeps



Standard Action

Improved Action

Correlators



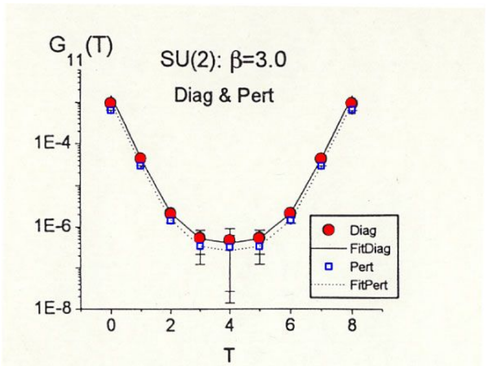
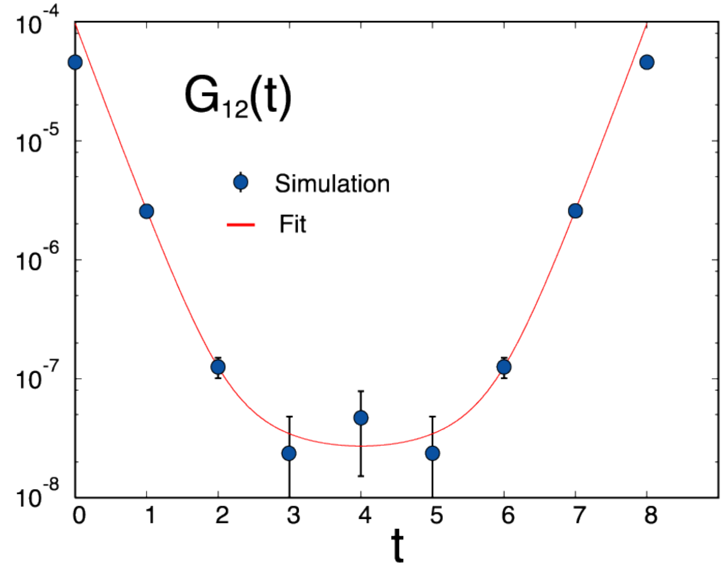


Figure 3. Fit of $G_{11}(T)$ by the parameters of spectral function for SU(2) at $\beta = 3.0$



U(1)
Coulomb and Confinement Phases

SU(2)

Two Definitions:

F=log U

F=U-1

SU(3)

Improved Action

Errors in U(1), SU(2), SU(3) standard and SU(3) improved

1995 U(1)

1997 SU(2)

1998 SU(3) preliminary

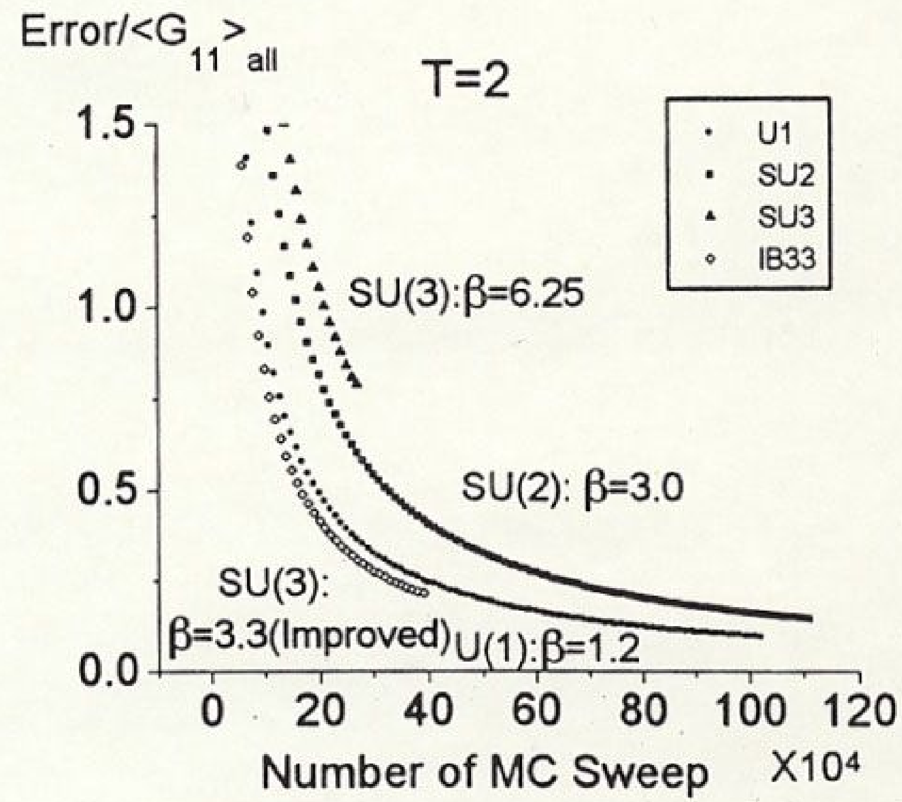


Figure 2. Error as a function of number of MC sweeps at $T = 2$ for U(1) $\beta = 1.2$, SU(2) $\beta = 3.0$, SU(3) $\beta = 6.25$ and improved action for SU(3) $\beta = 3.3$.

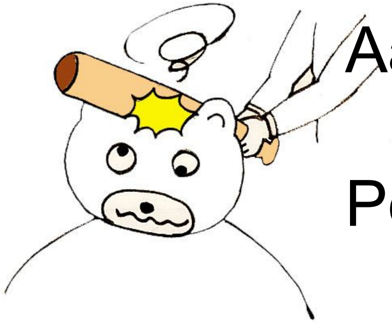
Low Frequency Region in Spectral Function $\rho(\omega)$ is Important

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$ Horsley and Shoenmaker

$(\varepsilon \longrightarrow 0)$ after the Thermo-Dynamics Limit

Long Range in τ of Thermal Green Function $\langle T_{\mu\nu}(0)T_{\mu\nu}(\tau)\rangle$ on the Lattice should be precisely determined.

⇨ The finite volume scaling will be required.



Aarts and Martinez-Resco, JHEP0204 (2002)053

Criticism against the Spectrum Function Ansatz.

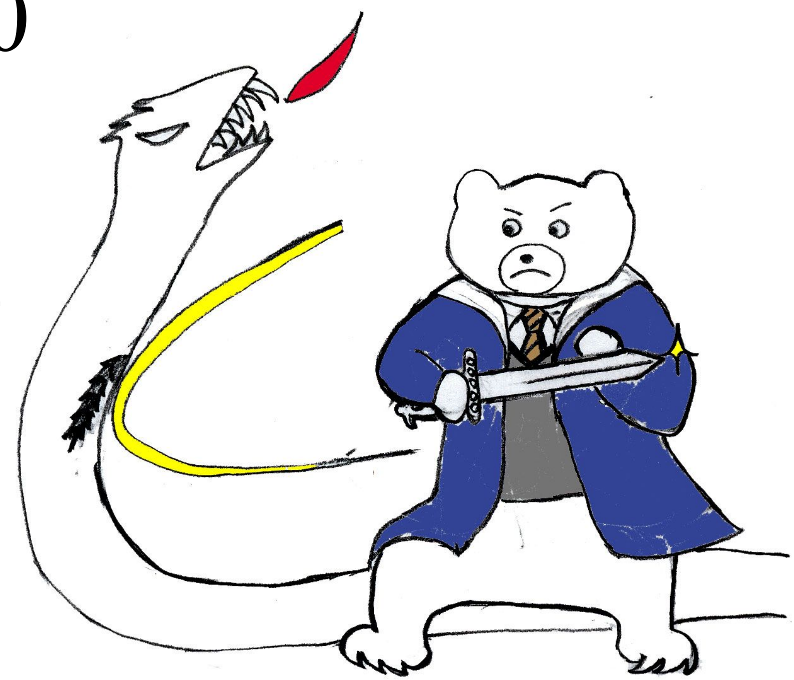
Petreczky and Teaney, hep-ph/0507318

Impossible to determine Heavy Quark Transport

Note that coefficient

Non-Equilibrium Calculations are in general subtle.

- Important Regions : $\omega : 0$
  - Physics is in Infra-Red i.e., Themodynamical Limit
- But this is Challenge of Lattice Simulation !



Summary

- We have calculated Transport Coefficients on Nt=8 Lattice. The limitations are
  - Quench Approximation
  - In order to convert Matsubara Green Function to Retarded one, we use Ansatz for Spectral Function with fitting parameters:

$$\rho = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2+\gamma^2}+\frac{\gamma}{(m+\omega)^2+\gamma^2}\right)$$

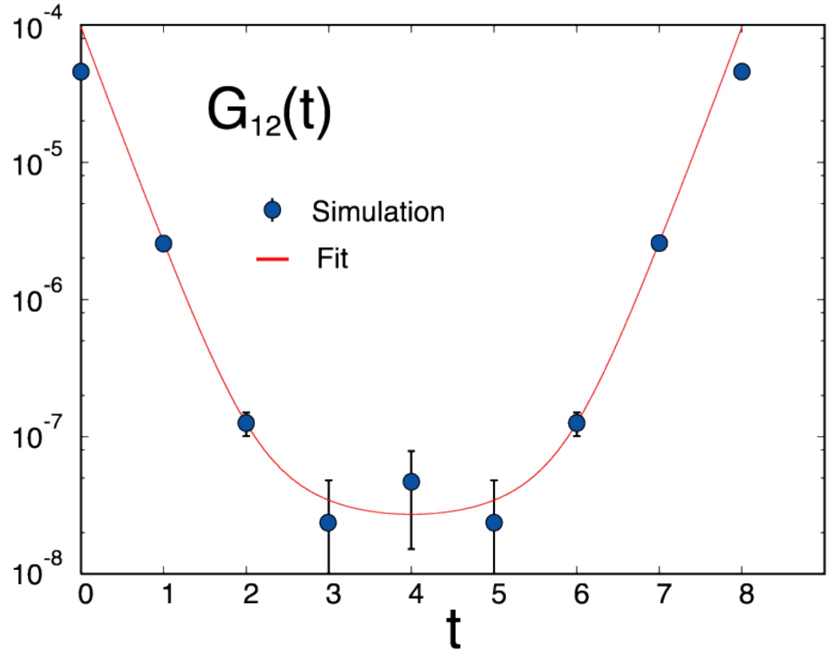
- Shear Viscosity
  - Positive $\eta/s \sim 0.1$
- Bulk Viscosity ~ 0
- Improved Action helps us a lot to get good Signal/Noise ratio.

Future direction ?

- If we can extract the Spectral Density $\rho(\omega)$ we can get the Transport Coefficients.
  - Maximum Entropy Method by Asakawa, Nakahara and Hatsuda
- We need (probably)
  - Anisotropic Lattice
  - Finite size scaling analysis
- Full QCD ?
  or
  with Quark Sector even in quench ?

We need data at large τ (small ω) with $O\left(\frac{1}{10}\right)$ Errors

- Brute Force ?
  - Not so crazy because the next Super-Computer is Peta-Flops Order.
- Good Operator
  - Extended
  - Renormalized



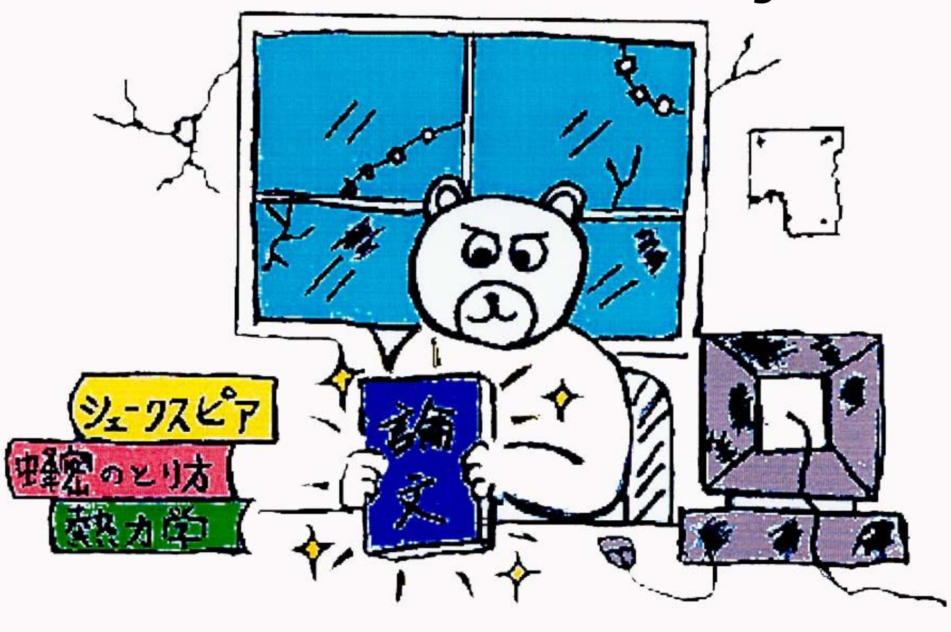
Limitations of the Current Lattice QCD Simulations for RHIC Physics

- Spectral Functions
  - Quench Approximation
- Transport Coefficients
  - Quench Approximation
- Finite Density Simulations
  - Still Quantitative, not yet Qualitative
- Dynamical QCD Simulations
  - Not yet with Chiral Fermions

We are the poorest group among Lattice Society

But Interesting QGP Physics motivates us go further as possible as we can !

Anyone is welcome to join !



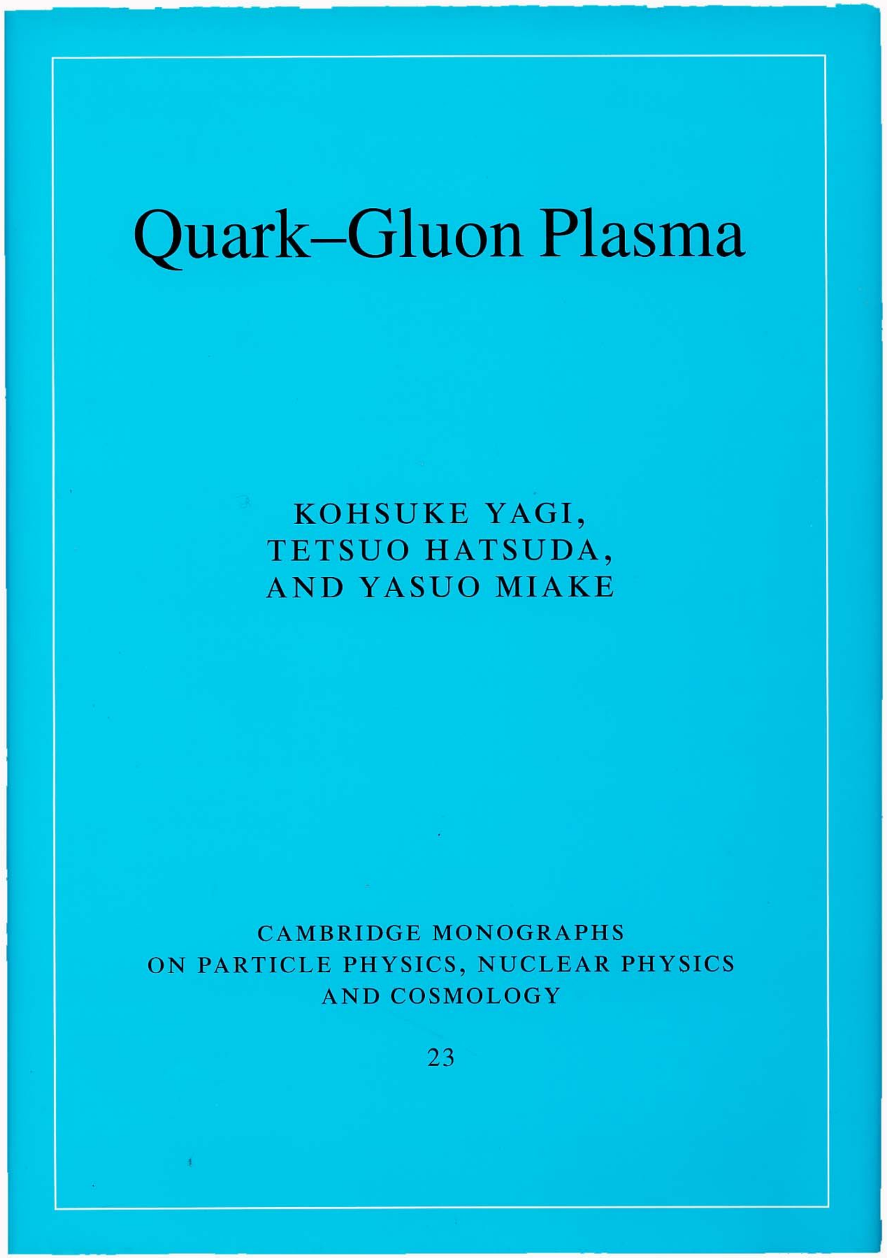
A Report to Andrzej

- Mr. and Mrs. Bialas visited Japan when I was a student. I learned lots from conversations with them.
- One day, Mrs. Bialas told me why you are not married, young gentleman !
- Andrzej was joking, “He is watching us, and has decided not to marry !”

- Andrzej continued, “You may doubt if a married man is happy or not, by watching me and others. But I storongly recommend you to marry some day !”
- Then Mrs. Bials gave me a Polish amber necklace, “This is for your future wife.”

Now this necklace is take by my wife.

Backup Slides



- What is the quark-gluon plasma ?

  Part I Basic Concept of Quark-Gluon Plasma
- Introduction to QCD
- Physics of the quark-hadron phase transition
- Field theory at finite temperature
- Lattice gauge approach to QCD phase transition

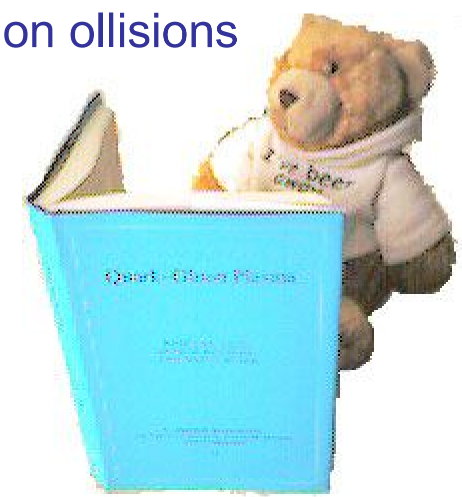
  ...

  Part II Quark-Gluon Plasma in Astrophysics

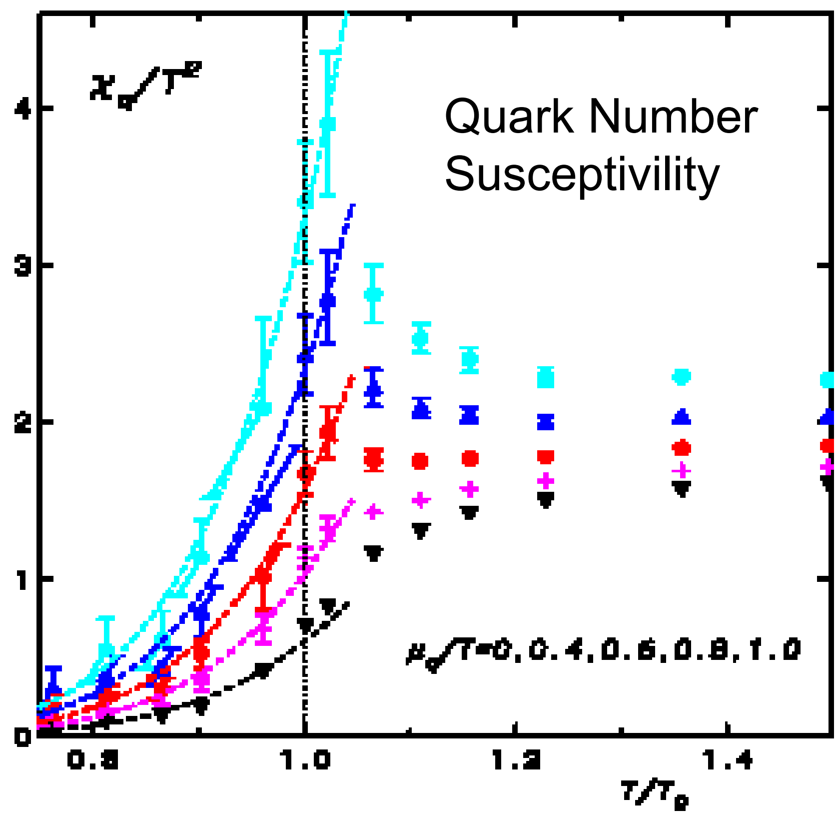
  ...

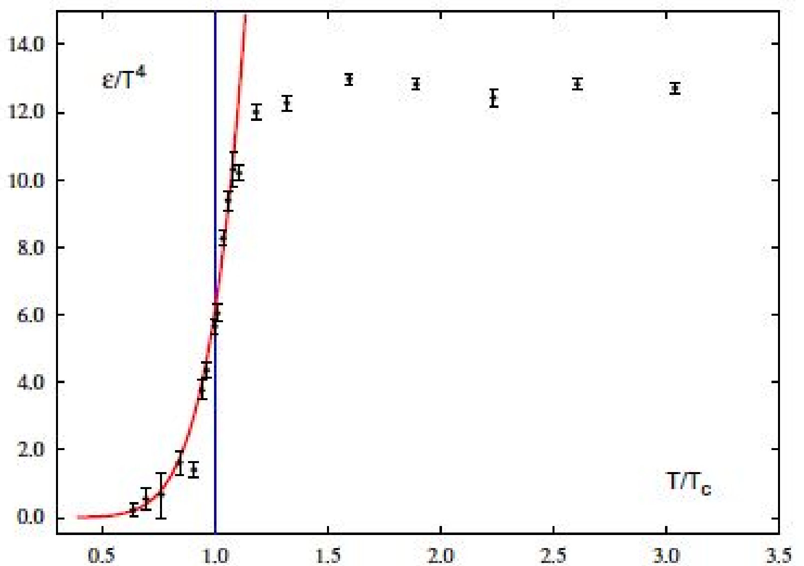
  Part III Quark-Gluon Plasma in Relativistic Heavy Ion ollisions

  ...



Comparison of Lattice with Resonance Gas Model





Karsch, Redlich and Tawfik

Phys.Lett. B571 (2003) 67

Masses in the model are modified to fit Lattice data.

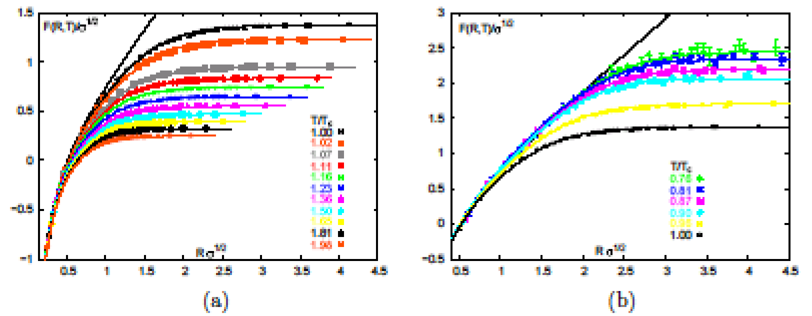
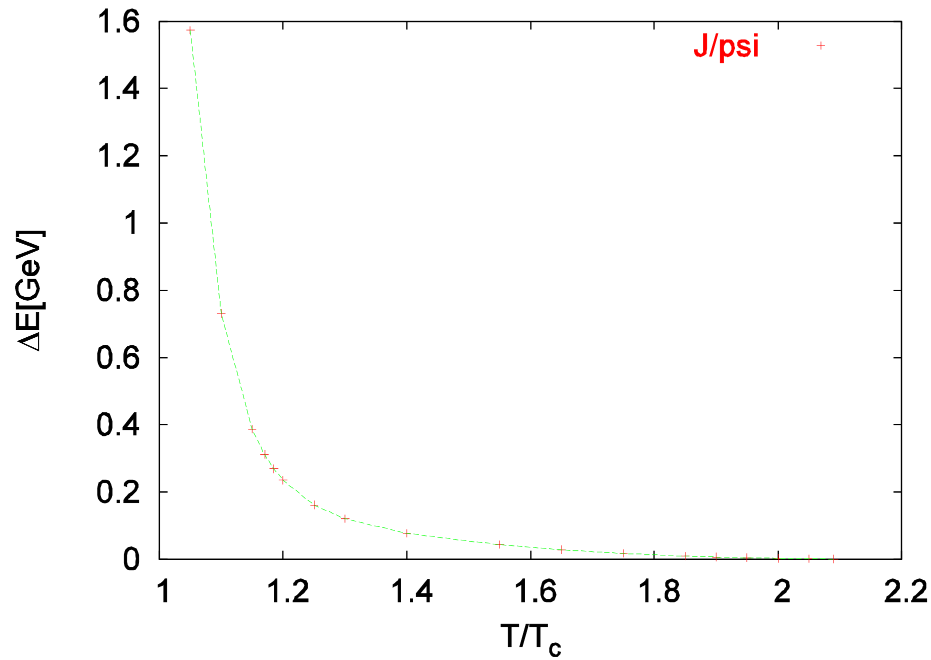


Figure 16: Screening fits to the $Q\bar{Q}$ free energy $F(r,T)$ for $T \geq T_c$ (left) and $T \leq T_c$ (right) [28]

Potential *V*

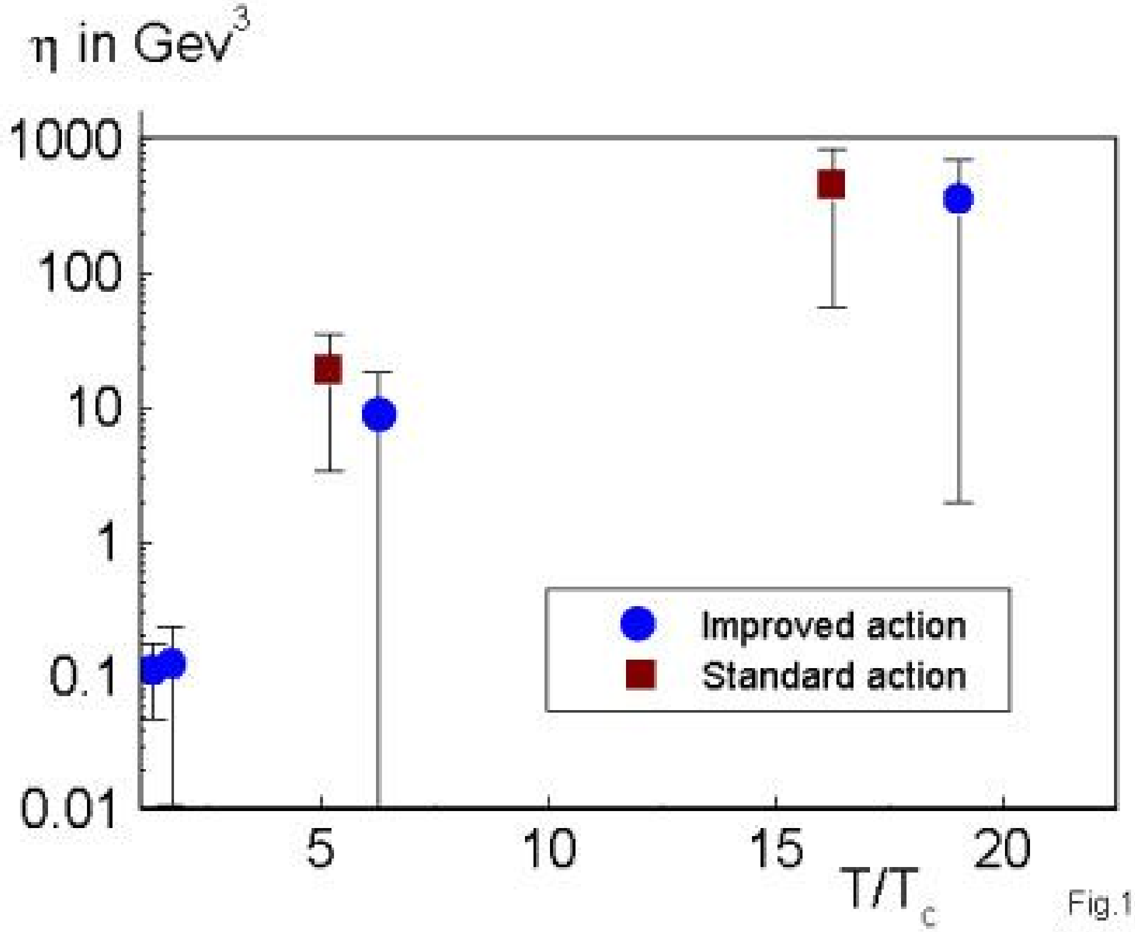
Schroedinger Eq.



T-dependence of binding energy for J/Psi.

H.Satz, hep-ph/0512217

Very high Temperature



Entropy Density

$$F = fV$$

$$f = -p$$

$$U - TS = -T\log Z = F$$

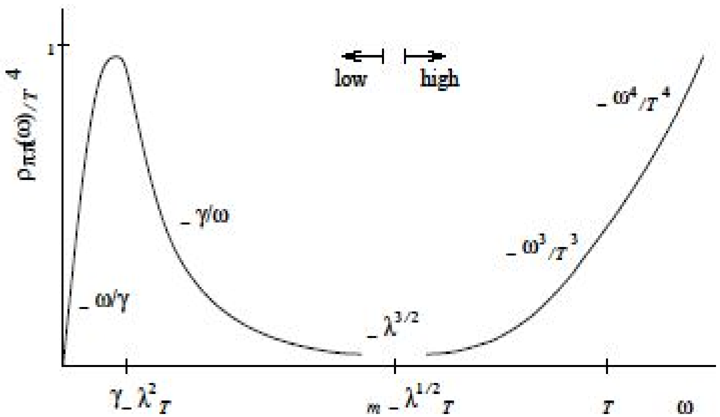
$$\Rightarrow \quad s = \frac{S}{V} = \frac{\varepsilon + p}{T}$$

$$\left.\frac{p}{T^4}\right|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \frac{d}{d\beta'}\frac{p}{T^4}$$

We reconstruct *p* from Raw-Data by CP-PACS
(Okamoto et al., Phys.Rev.D (1999) 094510)

Spectral Function by Aarts and Resco

$$\rho(\omega) = \rho^{\text{low}}(\omega) + \rho^{high}(\omega)$$



$$\frac{\rho^{\text{low}}(\omega)}{T^4} = x\frac{b_1 + b_2x^2 + \cdots}{1 + c_1x^2 + c_2x^4 + \cdots}$$

$$x \S \frac{\omega}{T}$$

$$\rho^{high}(\omega) = \theta(\omega - 2m_{th})\frac{(N_c^2 - 1)(\omega^2 - 4m_{th}^2)^{5/2}}{80\pi^2\omega}[n(\omega) + 0.5]$$

Fitting with three parameters, $b_1 \ c_1 \ m$

$\Rightarrow$ $c_1 < 0$?

Effect of High-Frequency part

$$\rho = \rho^{BW} + \rho^{high}$$

$$\frac{\rho^{low}(\omega)}{T^4} = x\frac{b_1 + b_2x^2 + \cdots}{1 + c_1x^2 + c_2x^4 + \cdots} \qquad x \equiv \frac{\omega}{T}$$

$$\rho^{BW} = \frac{A}{\pi}\left(\frac{\gamma}{(m-\omega)^2 + \gamma^2} + \frac{\gamma}{(m+\omega)^2 + \gamma^2}\right)$$

β=3.3

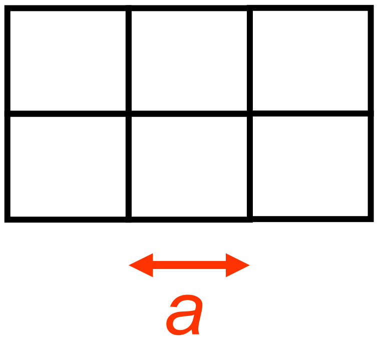
ηa^3	m_{th}
0.00225(201)	∞
0.00223(191)	5.0
0.00194(194)	3.0
0.00126(204)	2.0

$m_{th} = 1.8$

ρ^{high} contribution is larger than ρ^{BW} at t=1.

Why they are so noisy ?

- RG improved action helps lot.
  - Noise from Lattice Artifact ?
    (Finite *a* correction ?)
  - Once we checked that there is not so much difference between



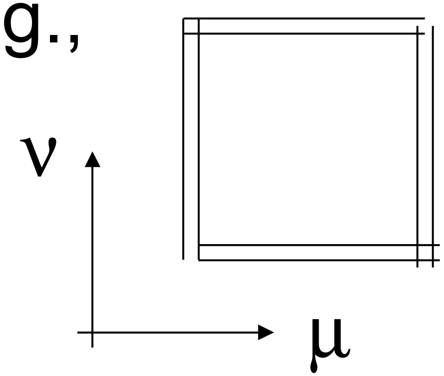
$$F_{\mu\nu} = \left(U_{\mu\nu} - U_{\mu\nu}^{\dagger}\right)/2i \text{ and } F_{\mu\nu} = \log U_{\mu\nu}/i$$

for SU(2). But we should check it again.

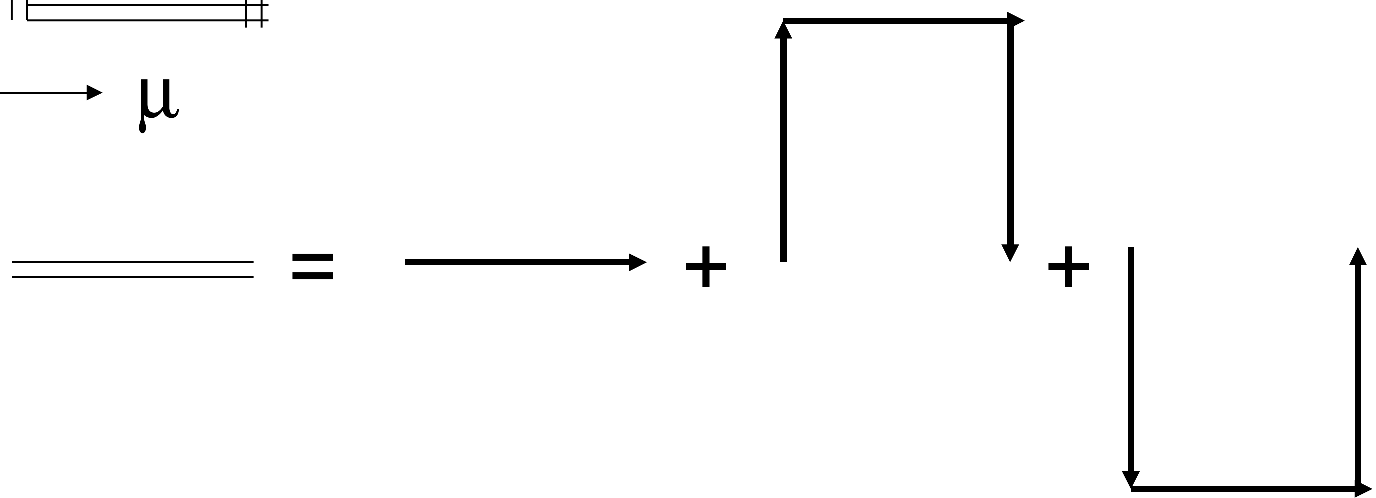
The situation reminds us Glue-Ball Case. (I thank Ph.deForcrand for discussions on this point.)

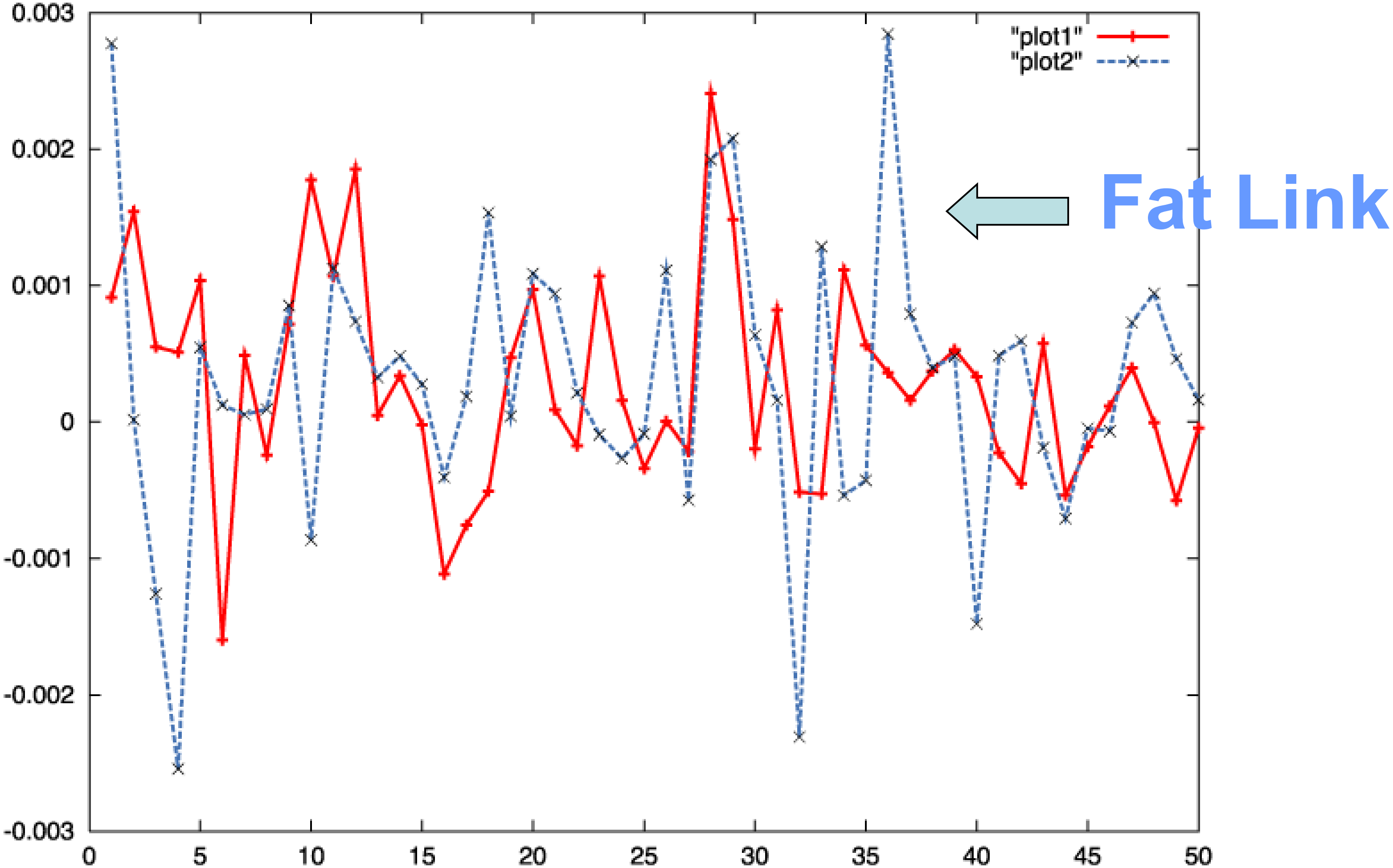
- Glue-Ball Correlators = $\left\langle \Box(\tau)\Box(0) \right\rangle$

- Large (extended) Operators work better, e.g.,



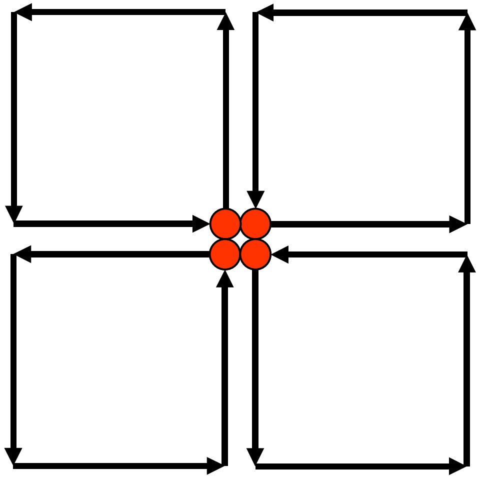
where





- Mmmm… not works …

Another Extended $F_{\mu\nu}$



A Crazy method
Source method + Langevin (Parisi)

Source Method

$$Z(J) = \int D\phi\, e^{-S + J\phi}$$

$$\left\langle \phi(x)\phi(y) \right\rangle = \frac{\delta}{\delta J(x)} \frac{\delta}{\delta J(y)} \log Z(J)$$

Langevin Update

$$\frac{d\phi(x)}{dt} = -\frac{\partial S}{\partial \phi(x)} + \eta$$

Deterministic
No Accept-
Reject step

t : Langevin time,

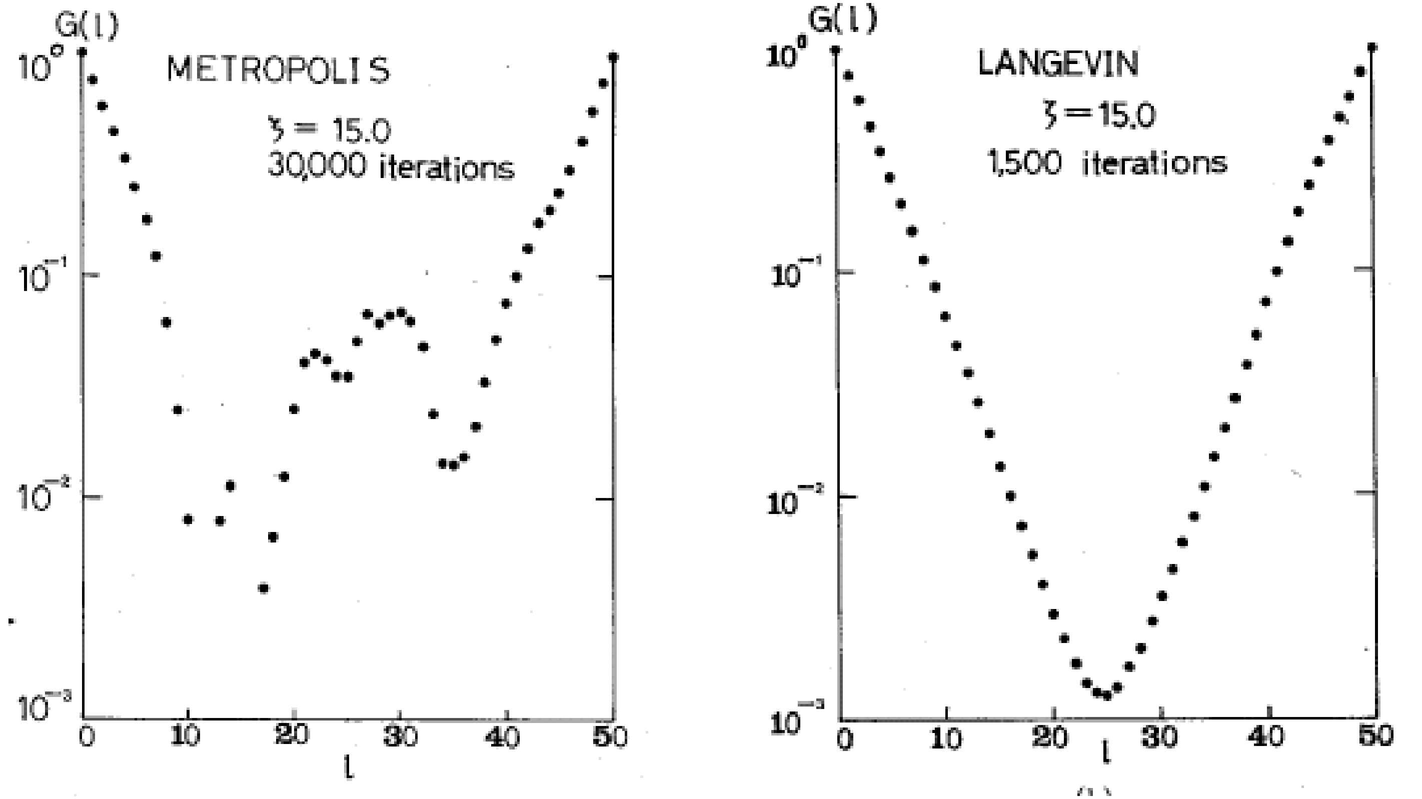
η : Gaussian Random Numbers

$$\left\langle \phi(x)\phi(y) \right\rangle = \frac{\delta}{\delta J(x)} \left\langle \phi(y) \right\rangle_J = \frac{\left\langle \phi(y) \right\rangle_{\varepsilon J} - \left\langle \phi(y) \right\rangle_0}{\varepsilon}$$

$$\left\langle \phi(y) \right\rangle_{\varepsilon J}$$

$$\left\langle \phi(y) \right\rangle_0$$

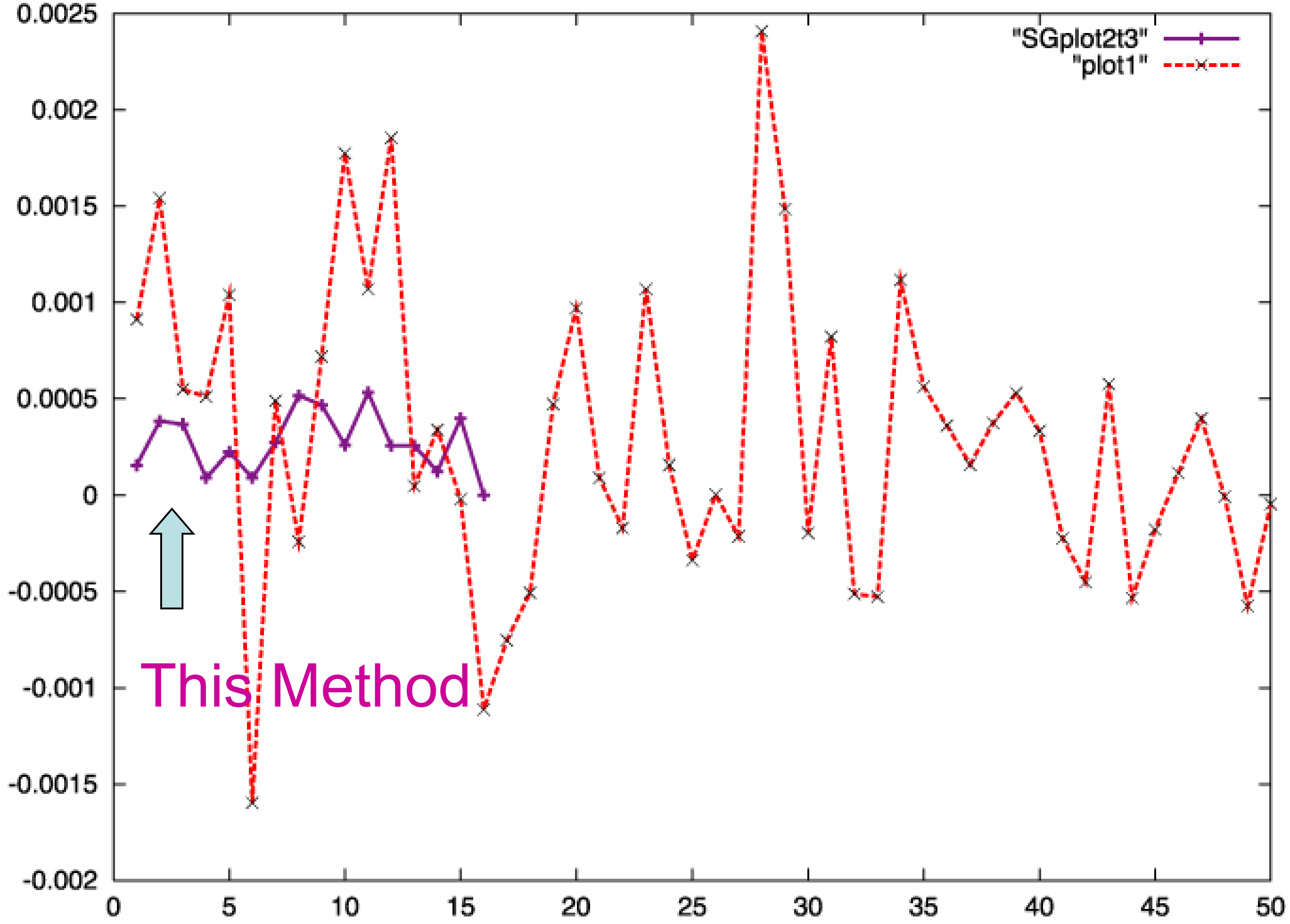
Calculate by Langevin
by the same Random Numbers



Namiki et al., Prog.Theor.Phys. 76 (1986) 501

O(3) Non-linear σ-model

In our case, …
(Very very preliminary)



Anisotropic Lattice ?

- Anisotropic lattice has matured and will help us to get more data points to determine the spectral function.