Symmetric and Non-Symmetric Saturation

Leszek Motyka

DESY Theory, Hamburg & Jagellonian University, Kraków

motyka@mail.desy.de

Overview

BFKL Pomeron and Balitsky Kovchegov formalism

Field theoretical formulation

Symmetry and self-duality

Symmetry breaking

Down to 0-dimensions

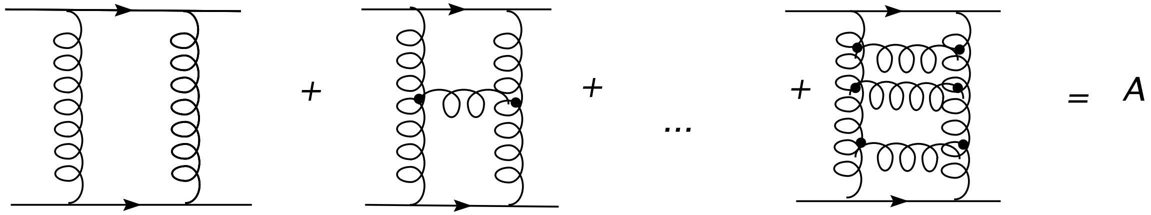
Superselection of the target

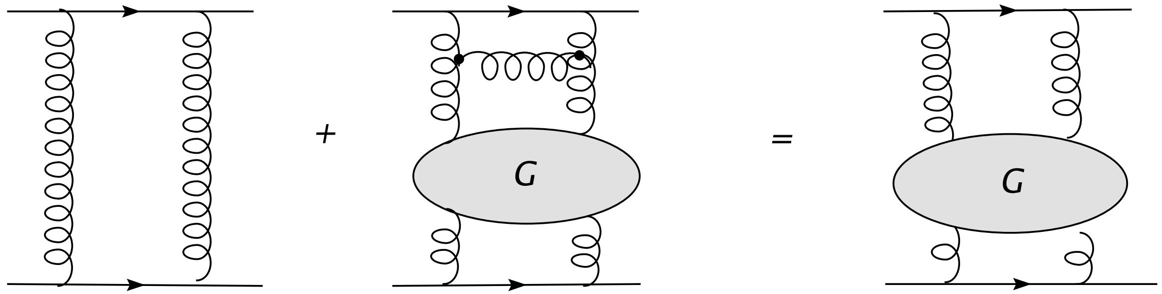
Based on work done with Sergey Bondarenko

BFKL resummation of gluonic ladders

Balitsky, Fadin, Kuraev, Lipatov: 1975–2005

$$A(s) = A_0 + A_1 \alpha_s \log(s) + \ldots + A_k (\alpha_s \log(s))^n + \ldots ; \qquad \alpha_s \log(s) \sim 1$$





$$\Phi_0(x,k) + \int_x^1 \frac{dx'}{x'} \int d^2k' \, \hat{K}(x,x';k,k') \, \Phi(x',k') = \Phi(x,k)$$

BFKL equation

$$\Phi(x,\boldsymbol{k};\boldsymbol{q}) = \Phi_A(\boldsymbol{k};\boldsymbol{q}) + \frac{3\alpha_s}{2\pi^2}\int_x^1 \frac{dx'}{x'}\int \frac{d^2\boldsymbol{k}'}{(\boldsymbol{k}'-\boldsymbol{k})^2}\left\{\left[\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_1'^2} + \frac{\boldsymbol{k}_2^2}{\boldsymbol{k}_2'^2} - q^2\frac{(\boldsymbol{k}'-\boldsymbol{k})^2}{\boldsymbol{k}_1'^2\boldsymbol{k}_2'^2}\right]\Phi(x',\boldsymbol{k}';\boldsymbol{q})\right.$$

$$\left. - \left[\frac{\boldsymbol{k}_1^2}{\boldsymbol{k}_1'^2 + (\boldsymbol{k}'-\boldsymbol{k})^2} + \frac{\boldsymbol{k}_2^2}{\boldsymbol{k}_2'^2 + (\boldsymbol{k}'-\boldsymbol{k})^2}\right]\Phi(x',\boldsymbol{k};\boldsymbol{q})\right\}$$

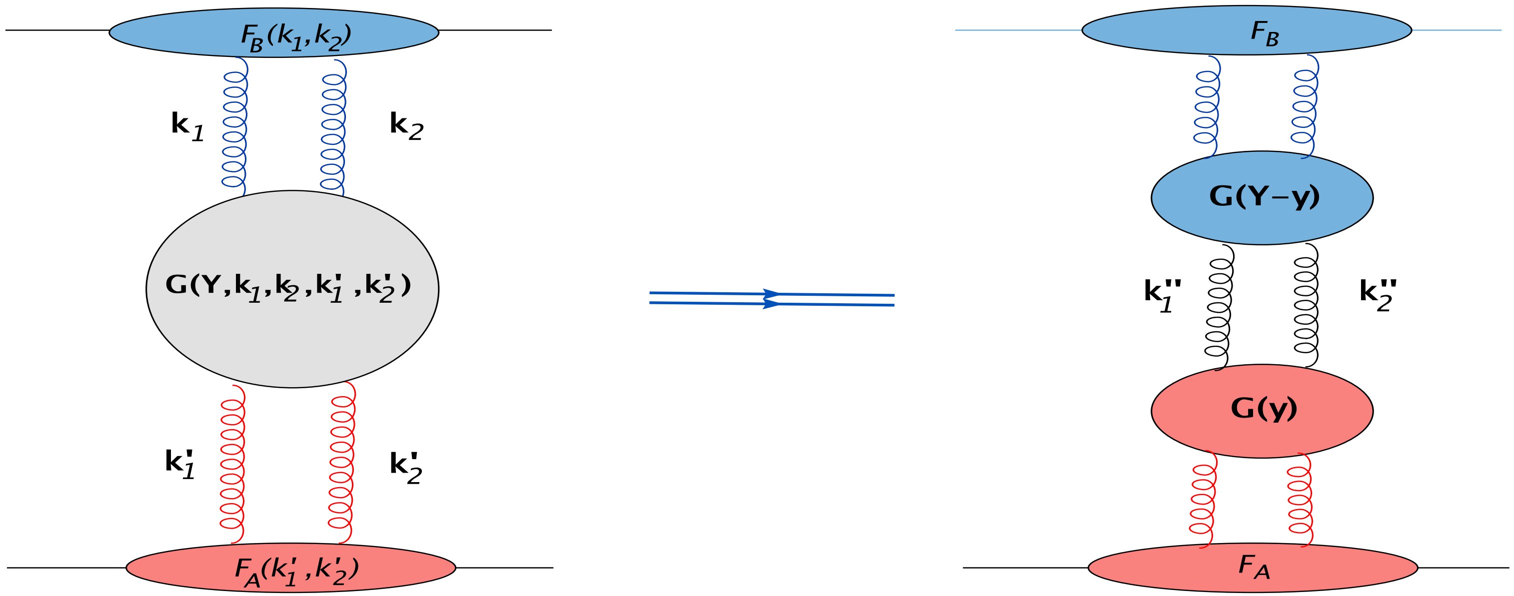
Differential form (LL) $$\frac{\partial\Phi(y,\boldsymbol{k};\boldsymbol{q})}{\partial y} = \int d^2\boldsymbol{k}'\,\hat{K}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q})\,\Phi(y,\boldsymbol{k}';\boldsymbol{q})$$

Formal solution: $$\boldsymbol{\Phi}(Y,\boldsymbol{q}) = \exp[\hat{K}(\boldsymbol{q})\,Y] \otimes \boldsymbol{\Phi}_A(\boldsymbol{q})$$

Two-step evolution $$\boldsymbol{\Phi}(Y,\boldsymbol{q}) = \exp[\hat{K}(\boldsymbol{q})\,y] \otimes \exp[\hat{K}(\boldsymbol{q})\,(Y-y)] \otimes \boldsymbol{\Phi}_A(\boldsymbol{q})$$

BFKL amplitude and Green's function

$$\mathcal{A}(Y) = \int \frac{d^2\boldsymbol{k}'}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2} \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} \Phi_A(\boldsymbol{k}';\boldsymbol{q}) \times \mathcal{G}(Y,\boldsymbol{k}',\boldsymbol{k};\boldsymbol{q}) \times \Phi_B(\boldsymbol{k};\boldsymbol{q})$$



$$\mathcal{A}(Y) = \int \frac{d^2\boldsymbol{k}'}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2} \int \frac{d^2\boldsymbol{k}}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} \int \frac{d^2\boldsymbol{k}''}{\boldsymbol{k}''^2(\boldsymbol{q}-\boldsymbol{k}'')^2} \times$$

$$\Phi_A(\boldsymbol{k}';\boldsymbol{q}) \times \mathcal{G}(y,\boldsymbol{k}',\boldsymbol{k}'';\boldsymbol{q}) \times \mathcal{G}(Y-y,\boldsymbol{k}'',\boldsymbol{k};\boldsymbol{q}) \times \Phi_B(\boldsymbol{k};\boldsymbol{q}) =$$

$$= \int \frac{d^2\boldsymbol{k}''}{\boldsymbol{k}''^2(\boldsymbol{q}-\boldsymbol{k}'')^2} \quad \Phi_A(y,\boldsymbol{k}'';\boldsymbol{q}) \times \Phi_B(Y-y,\boldsymbol{k}'';\boldsymbol{q})$$

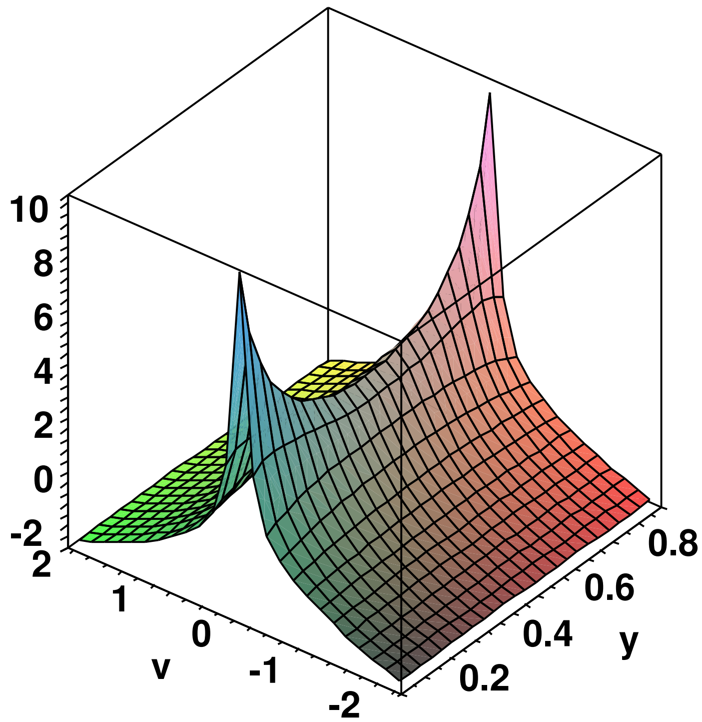
Solutions to BFKL

Unintegrated gluon densities:

$$\Phi_A(y, \boldsymbol{k}, q=0) \longrightarrow f_A(y, k^2), \qquad \Phi_B(Y-y, \boldsymbol{k}, q=0) \longrightarrow f_B^\dagger(Y-y, k^2)$$

$$LL(1/x): \qquad f_A(y,k^2) \simeq \int \frac{d\gamma}{2\pi i}\, f_A^{(0)}(\gamma)\, k^{2\gamma}\, \exp[\bar{\alpha}_s\, y\, \chi(\gamma)]$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$



Diffusion approximation for

$y \gg \log(Q^2/Q_0^2) \Rightarrow$

saddle point at $\gamma = 1/2$

$$f_A(y,k^2) \sim k k_0 \, \exp(\omega_0\, y) \, \exp\left[\frac{-\log^2(k^2/k_0^2)}{2D\, y}\right]$$

Rapidity dependence:

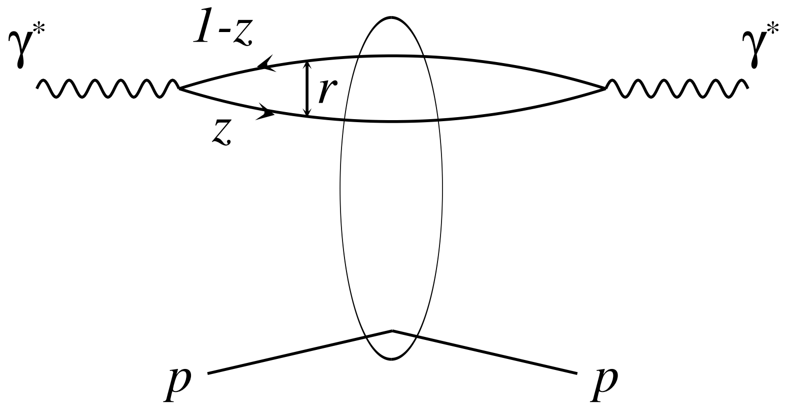
$\exp(\omega_0\, y)$ with $\omega_0 = 4\bar{\alpha} \log 2$

$\omega_0 \sim 0.5$ (too large)

Colour Dipole Representation

At very high energies and the LL approximation description of high energy scattering in the position representation is possible

- long-living fluctuations: colour dipoles
- short interaction time
- parton energy $\sim z$ is conserved
- parton transverse positions do not change
- conservation of parton helicity



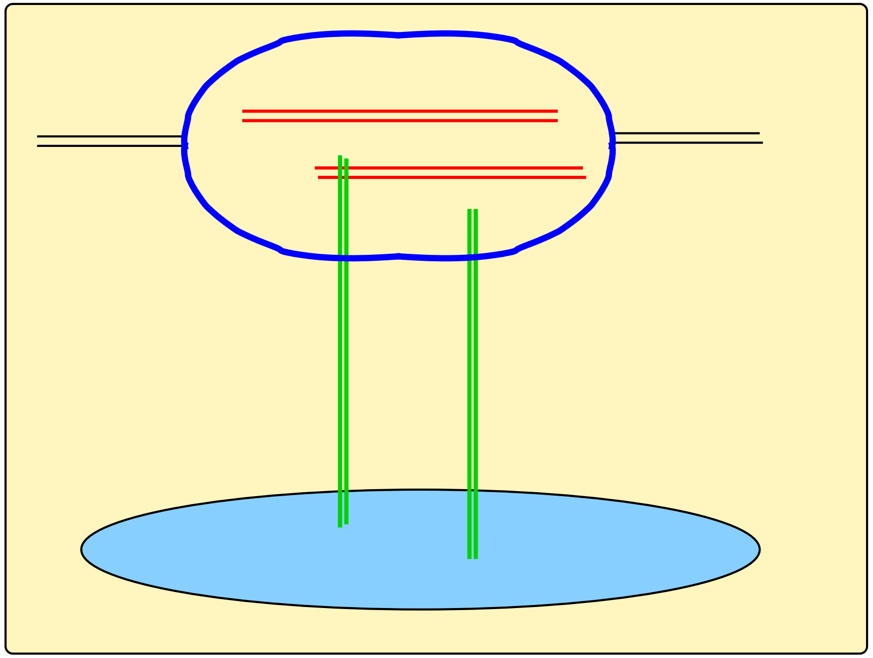
Colour dipole cascade model **[Mueller]**

$N_c \gg 1$ gluon emissions
$\longrightarrow$ dipole splittings

Splitting: BFKL kernel

Scattering of dipoles: two gluon exchange

Single scattering $\propto$ dipole multiplicity $\longrightarrow$ linear BFKL



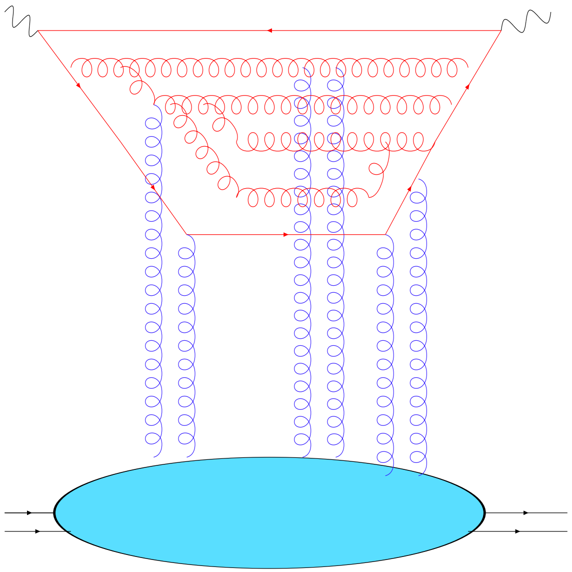
Dipole rescattering / gluon recombination

Assume dominance of single ladder exchange ⇒ gluon growth $\sim e^{\lambda y} \Rightarrow \mathrm{Im T} \sim e^{\lambda y}$

Diffractive cross section $\sim |T|^2 \sim e^{2\lambda y}$ would eventually surpass the total cross section

Violation of S-matrix unitarity

Target frame



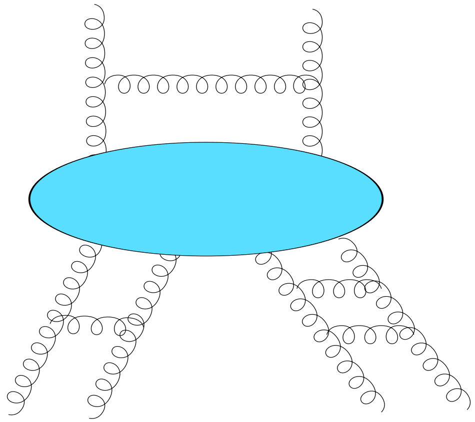
Exponential growth of dipole density $\longrightarrow$
Rescattering
Neglecting correlations:

$$\sigma = \sigma_1 - \sigma_1^2/2 + \ldots \sim \sigma_0[1 - \exp(-\sigma_1/\sigma_0)]$$

Projectile frame

$f_A(y, k^2) \sim e^{\lambda y}$

Triple Pomeron Vertex



allows for gluon recombination

with rate $\sim f_A^2(y, k^2)$ **[Bartels]**

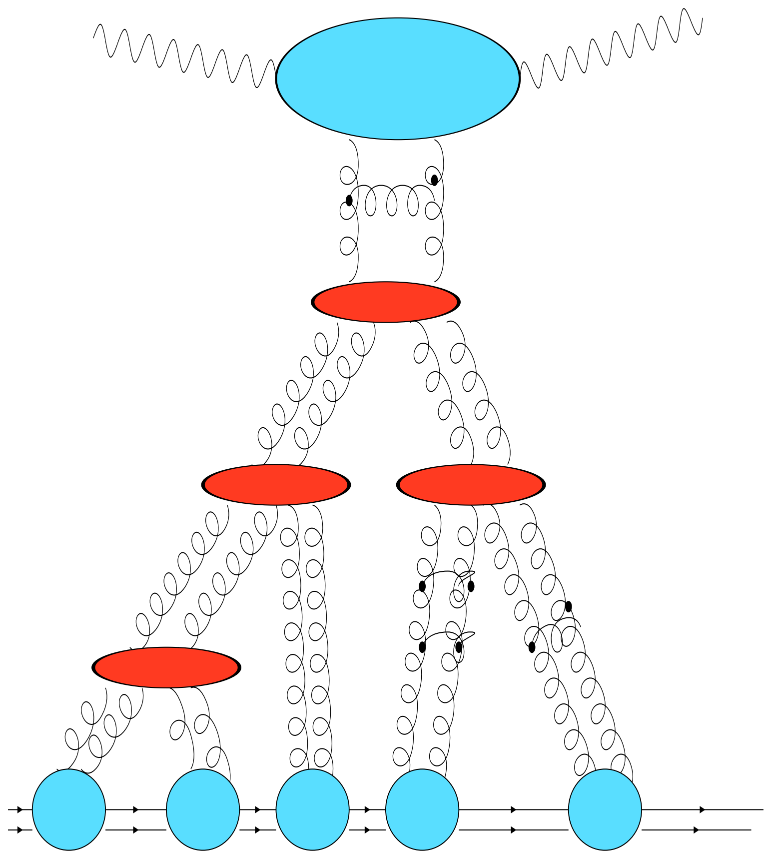
Theoretical basis: JIMWaLK/BK equation

[Jalilian-Marian,Iancu,McLerran, Leonidov,Weigert],[Balitsky,Kovchegov]

Propagation of a dilute projectile through a dense target at large energy – at the LL$1/x$: BFKL evolution of amplitude tempered by unitarity corrections

Enhancement + Correlations + Rescattering

Target frame: in the large N_c limit BFKL pomeron fan diagrams – gluon recombination at large density



$$\frac{\partial N(\boldsymbol{x},\boldsymbol{y};Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int d^2z\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}\times$$

$$[N(\boldsymbol{x},\boldsymbol{z};Y)+N(\boldsymbol{z},\boldsymbol{y};Y)-N(\boldsymbol{x},\boldsymbol{y};Y)-N(\boldsymbol{x},\boldsymbol{z};Y)N(\boldsymbol{z},\boldsymbol{y};Y)]$$

BK equation in momentum space

$$\frac{\partial N(k,Y)}{\partial Y} = \chi_{\rm BFKL}\left(1+k^2\partial_{k^2}\right)N(k,Y) - N^2(k,Y)$$

For large k / small r – BFKL-like growth with y.
For small k / large r – saturation.

Unitarity is not violated

Kwieciński–Kutak formulation of BK

Dipole density in momentum space:

$$\tilde{N}(y,k^2) \;=\; \int \frac{d^2\boldsymbol{r}}{2\pi}\;\exp(-i\boldsymbol{k}\cdot\boldsymbol{r})\,\frac{N(y,r)}{r^2}$$

Dipole scattering amplitude $\quad N(y,r) \;\propto\; \int \frac{d^2k}{k^4}\, f(y,k^2)\,[1-\exp(-i\boldsymbol{k}\boldsymbol{r})]$

$$f(y,k^2) \;\propto\; k^4\,\nabla_k^2\,\tilde{N}(y,k^2)$$

Inverse transform: $\quad \tilde{N}(y,k^2) \;\propto\; \int_{k^2} \frac{da^2}{a^4}\, f(y,a^2)\;\log\left(\frac{a^2}{k^2}\right)$

BK equation $\quad \partial_y f(y,k^2) = \bar{\alpha}_s\, k^2 \int \frac{da^2}{a^2}\left[\frac{f(y,a^2)-f(y,k^2)}{|a^2-k^2|} + \frac{f(y,k^2)}{[4a^4+k^4]^{\frac{1}{2}}}\right]$

$$-2\pi\alpha_s^2\,\left[k^2\int_{k^2}\frac{da^2}{a^4}\,f(y,a^2)\int_{k^2}\frac{db^2}{b^4}\,f(y,b^2) \;+\; f(y,k^2)\int_{k^2}\frac{da^2}{a^4}\;\log\left(\frac{a^2}{k^2}\right) f(y,a^2)\right]$$

Solutions of BK / geometric scaling

BK equation generates saturation scale $Q_s(y) \sim \exp(\lambda(y)$

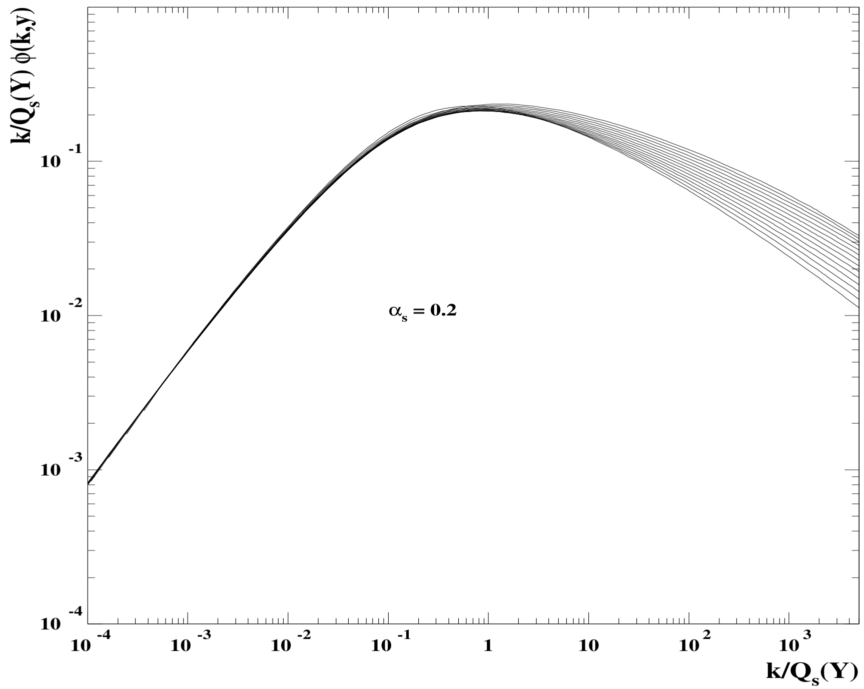
At large y solutions of BK depend on $\xi = (k/Q_s(y))^2$

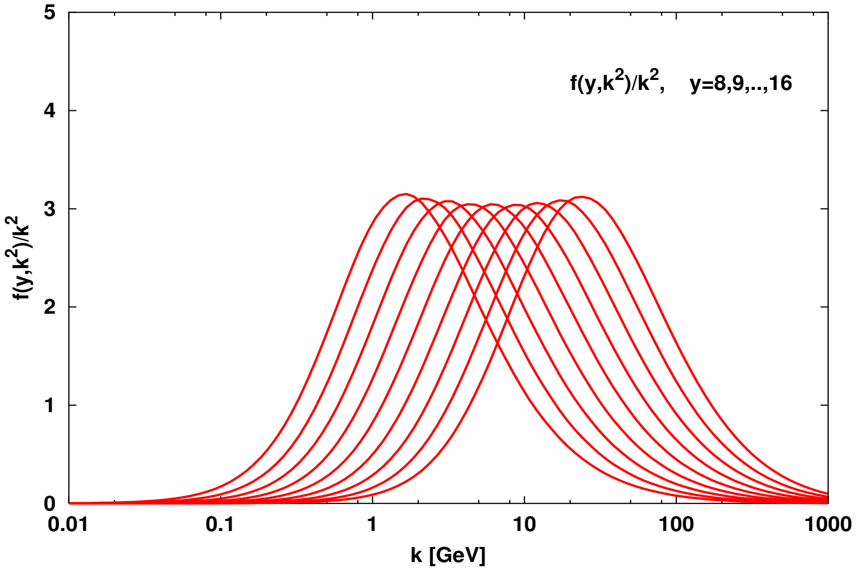
$$\tilde{N}(y,k) \sim -\log(\xi), \quad \xi \ll 1$$

$$\tilde{N}(y,k) \sim \xi^{-\gamma_0}, \quad \xi \gg 1$$

$$f(y,k^2)/k^2 \sim \xi, \quad \xi \ll 1$$

$$f(y,k^2)/k^2 \sim \xi^{-\gamma_0} \quad \xi \ll 1$$





Geometric Scaling

[A. Staśto, K. Golec-Biernat and J. Kwieciński]

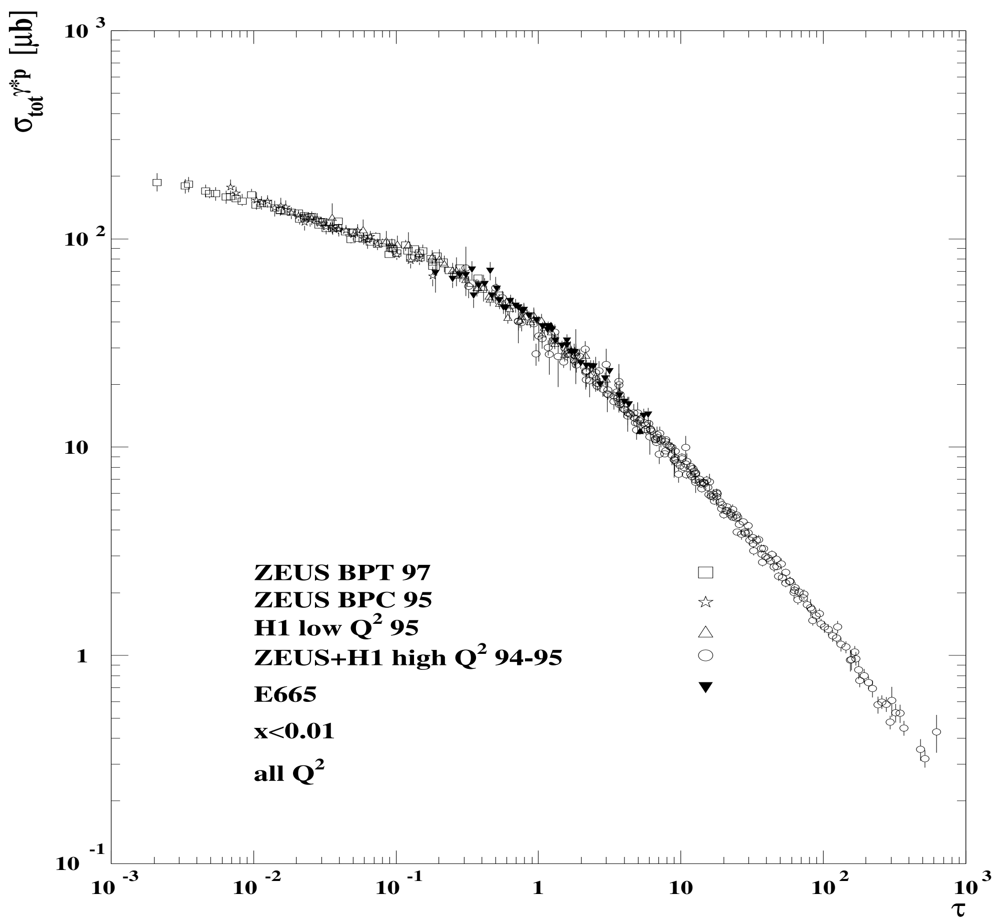
$$\sigma_i^{\gamma^* p}(Q^2, W^2) = \int_0^1 dz \int d^2\boldsymbol{r} \underbrace{|\Psi_i(z, \boldsymbol{r})|^2}_{Q^2} \underbrace{\hat{\sigma}(x, r^2)}_{R_s(x)}$$

Cross section should depend only on the variable

$$\tau = Q^2 R_s^2(x)$$

Data for $0.045\ \mathrm{GeV}^2 < Q^2 < 450\ \mathrm{GeV}^2$ and $x < 0.01$

Warning! Open problem of the contribution of charm



Geometric Scaling and Universality

[Munier,Peschanski]

Linear term in BK equation may be expanded around $\chi(1/2)$

$$\frac{\partial N(k,Y)}{\partial Y} \simeq \left[\lambda + \frac{D}{2}\left(\frac{\partial}{\partial \log(k^2)} + 1/2\right)^2\right] N(k,Y) - N^2(k,Y)$$

Universality class of Kolmogorov-Petrovsky-Piscounov equation:

$$\partial_t u(x,t) = \partial_x^2 u(x,t) + u(x,t) - u^2(x,t)$$

The equation describes evolution of a system from a repulsive fixed point $(u = 0)$ to an atractive fixed point $(u = 1)$

The equation admits a travelling wave front solution with

$$Q_s(\tau) = v_g \tau - b \log \tau - \frac{c}{\sqrt{\tau}} + O(1/\tau)$$

with v_g, b and c being independent on the form of nonlinearity.

Interesting: results of analysis of JIMWaLK equation on the lattice conform with results of BK equation [Rummukainen,Weigert]

Generic case: saturation scale moving with energy and approximate geometric scaling

Limitations of BK and need to improve

BK is well founded for very asymmetric scattering like γ^* – Nucleus, DIS

Quantum effects (pomeron loops) are absent

Need to describe multiple scattering and production in symmetric situation like heavy ion collisions at RHIC and pp scattering at the LHC

Three complementary ways to go beyond BK:

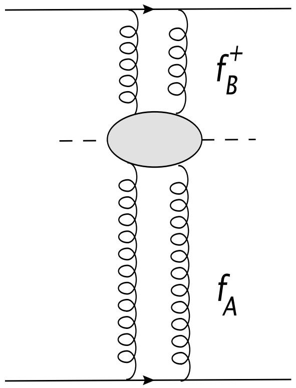
Effective Field Theory approach $\longrightarrow$ Path integral: $\int [D\phi_\alpha] \exp(-S[\phi_\alpha, \partial\phi_\alpha])$

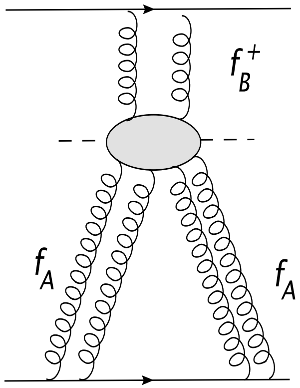
Stochastic formulation $\longrightarrow$ Langevin equation: $\partial_Y N = A(N) + B(N)\nu(Y)$

Hamiltonian formulation $\longrightarrow$ Generating functional: $\partial_Y \Psi[Y;u] = (\mathcal{H} \otimes \Psi)[Y;u]$

Field theoretical formulation

[Bondarenko,LM]





Amplitudes of the diagrams:

$$\int \frac{d^2k}{k^4}\, f_A(y,k^2)\, f_B^\dagger(y,k^2)$$

$$\int \frac{d^2k}{k^4} \int \frac{d^2a}{a^4} \int \frac{d^2b}{b^4}\, V_{3P}(\boldsymbol{k};\boldsymbol{a},\boldsymbol{b})\, f_B^\dagger(y,k^2)\, f_A(y,a^2)\, f_A(y,b^2)$$

BFKL Green's function

$$\int \frac{d^2k}{k^4}\frac{d^2k'}{k'^4}\, f_A(k'^2)\, \mathcal{G}(Y;k'^2,k^2)\, f_B^\dagger(k^2) = \int \frac{d^2k}{k^4}\frac{d^2k'}{k'^4}\, f_A(k'^2)\frac{1}{\partial_y - \hat{K}_0(k'^2,k^2)}\, f_B^\dagger(k^2)$$

Effective action $\quad \mathcal{A}[f,f^\dagger;Y] \;\propto\; \int_0^Y dy\, \left\{\mathcal{L}_0[f,f^\dagger] + \mathcal{L}_3[f,f^\dagger] + \mathcal{L}_E[f,f^\dagger]\right\}$

$$\mathcal{L}_0[f,f^\dagger] = \frac{1}{2}\left[f(y)\otimes \overleftrightarrow{\partial}_y \otimes f^\dagger(y)\right] \;+\; f^\dagger(y)\;\otimes\;\mathcal{K}_0\;\otimes\; f(y)$$

$$\mathcal{L}_3[f,f^\dagger] = -2\pi\alpha_s^2\,\langle f^\dagger(y)\,|\,V_{3P}\,|\,f(y)\otimes f(y)\rangle$$

$$\mathcal{L}_E[f,f^\dagger] = f_A^\dagger(y)\otimes f(y) + f^\dagger(y)\otimes f_B(y)$$

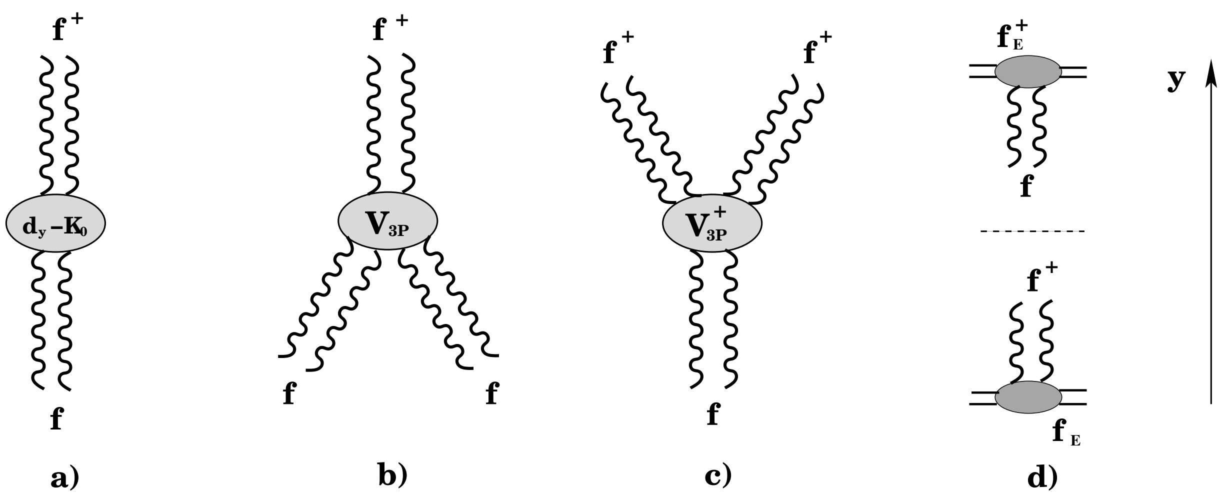
Canonical structure — Gribov fields

$$\frac{\delta \mathcal{L}_0[f, f^\dagger]}{\delta(\partial_y f^\dagger(y,k^2))} = \frac{1}{k^4} f(y,k^2), \qquad \frac{\delta \mathcal{L}_0[f, f^\dagger]}{\delta(\partial_y f(y,k^2))} = -\frac{1}{k^4} f^\dagger(y,k^2)$$

Canonical commutator $\quad [-if(y,k^2), -if^\dagger(y,k'^2)] = k^4 \delta(k^2 - k'^2)$

Gribov fields $\quad \frac{-if(y,k^2)}{k^2} \longrightarrow \psi, \qquad \frac{-if^\dagger(y,k^2)}{k^2} \longrightarrow \psi^\dagger$

Feynman rules



Target – projectile symmetry

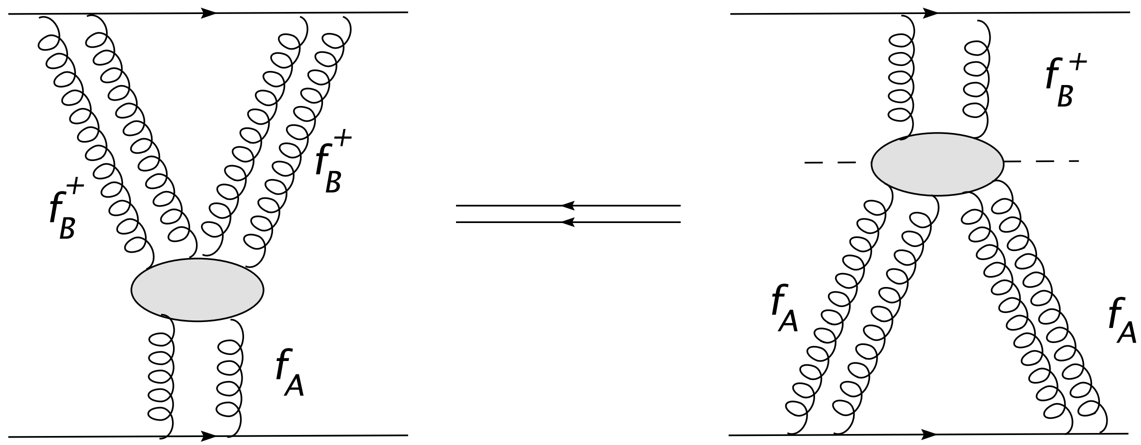
[Kovner,Lublinsky]

Direction of rapidity evolution is arbitrary

Free to choose target and projectile

Action symmetric w.r.t. target $\longleftrightarrow$ projectile

$$y \;\rightarrow\; -y, \qquad f(y,k^2) \;\longleftrightarrow\; f^\dagger(Y-y,k^2)$$



$$\langle\, f^\dagger(y) \otimes f^\dagger(y) \,|\, V_{3P}^\dagger \,|\, f(y)\,\rangle \;=\; \langle\, f^\dagger(y) \,|\, V_{3P} \,|\, f(y) \otimes f(y)\,\rangle^\dagger$$

f and $f^\dagger$ are canonically conjugated $\Rightarrow$ Self-duality

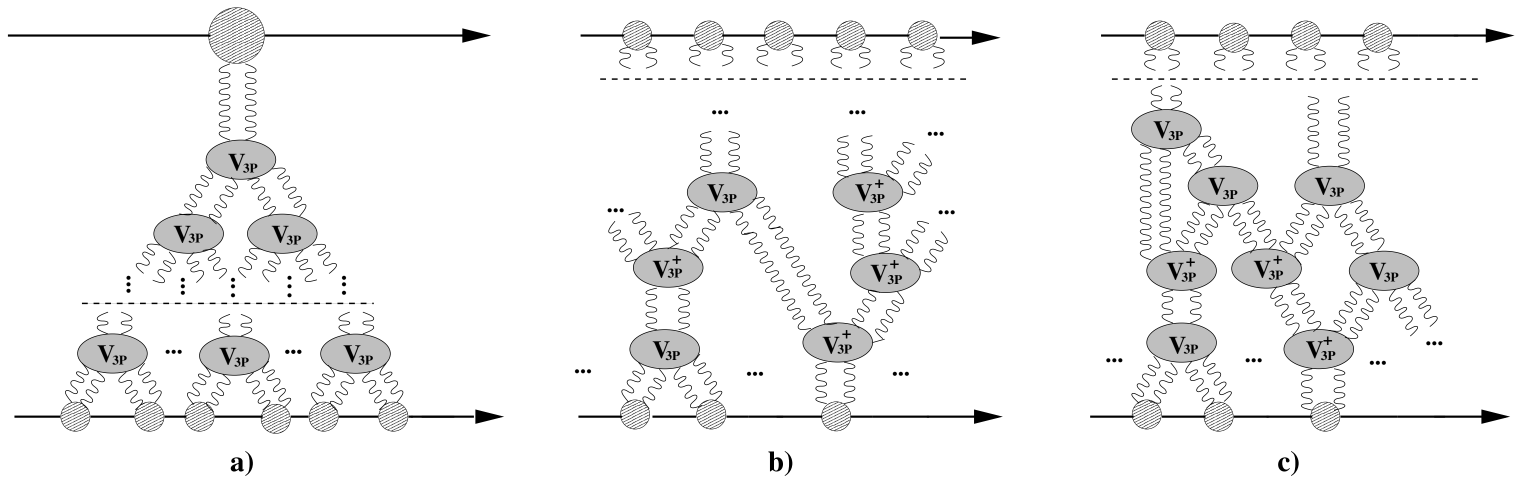
Semi-classical limit

Effective action $\mathcal{A}[f, f^\dagger; Y] \;\propto\; \int_0^Y dy\; \left\{ \mathcal{L}_0[f, f^\dagger] + \mathcal{L}_3[f, f^\dagger] + \mathcal{L}_3^\dagger[f, f^\dagger] + \mathcal{L}_E[f, f^\dagger] \right\}$

S − matrix: $S(Y; f_A, f_B^\dagger) \;=\; \int_{f_A, f_B^\dagger} [Df\, Df^\dagger]\; \exp(-\mathcal{A}[f, f^\dagger; Y])$

Classical trajectories $\bar{f}(y, k^2)$ and $\bar{f}^\dagger(y, k^2)$: $\delta\mathcal{A}[\bar{f}, \bar{f}^\dagger; Y] \;=\; 0$

$$S(Y; f_A, f_B^\dagger) \;=\; \sum_\alpha \Delta_\alpha\; \exp(-\mathcal{A}[\bar{f}_\alpha, \bar{f}^\dagger{}_\alpha; Y])$$



Equations of motions – Braun equations in KK representation

Mutual absorption of the fields

$$\partial_y f(y,k^2) \;=\; \mathrm{BK}(f,k^2) - 2\pi\alpha_s^2\,\left[2\int_0^{k^2}\frac{da^2}{a^4}\,a^2 f(y,a^2)\int_{a^2}\frac{db^2}{b^4}\,f^\dagger(y,b^2)+\right.$$

$$\left.+f(y,k^2)\int_{k^2}\frac{da^2}{a^4}\,\log\left(\frac{a^2}{k^2}\right)f^\dagger(y,a^2)+\int_0^{k^2}\frac{da^2}{a^4}\,f(y,a^2)\,f^\dagger(y,a^2)\,\log\left(\frac{k^2}{a^2}\right)\right]$$

and

$$-\partial_y f^\dagger(y,k^2) \;=\; \mathrm{BK}(f^\dagger,k^2) - 2\pi\alpha_s^2\,\left[2\int_0^{k^2}\frac{da^2}{a^4}\,a^2 f^\dagger(y,a^2)\int_{a^2}\frac{db^2}{b^4}\,f(y,b^2)+\right.$$

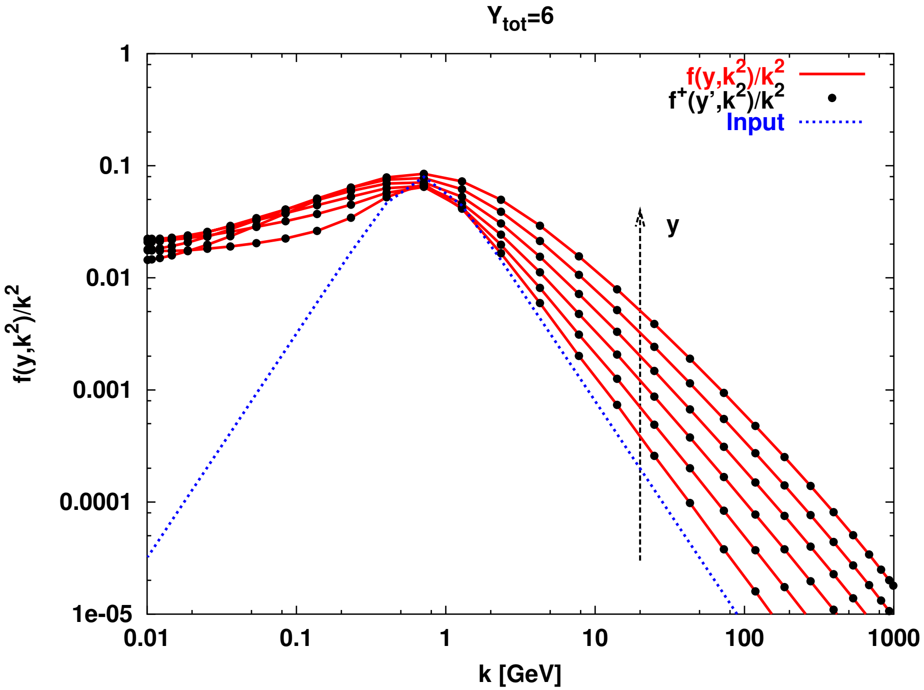
$$\left.+f^\dagger(y,k^2)\int_{k^2}\frac{da^2}{a^4}\,\log\left(\frac{a^2}{k^2}\right)f(y,a^2)+\int_0^{k^2}\frac{da^2}{a^4}\,f^\dagger(y,a^2)\,f(y,a^2)\,\log\left(\frac{k^2}{a^2}\right)\right]$$

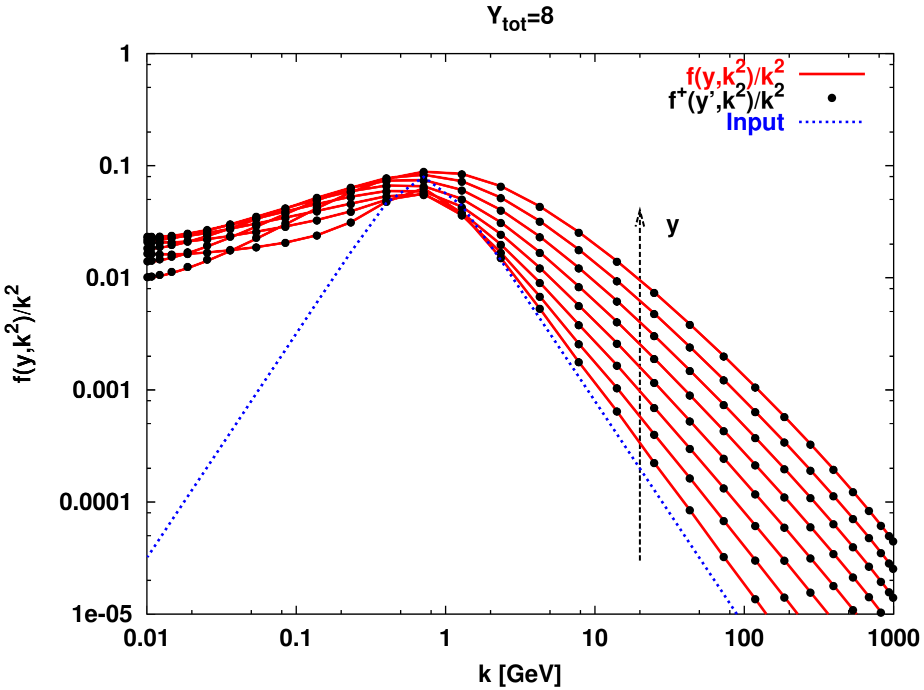
Two-point boundary conditions: (symmetric!)

$$f(y=0,k^2)=f_A(k^2), \qquad f^\dagger(y=Y,k^2)=f_B^\dagger(k^2)=f_A(k^2)$$

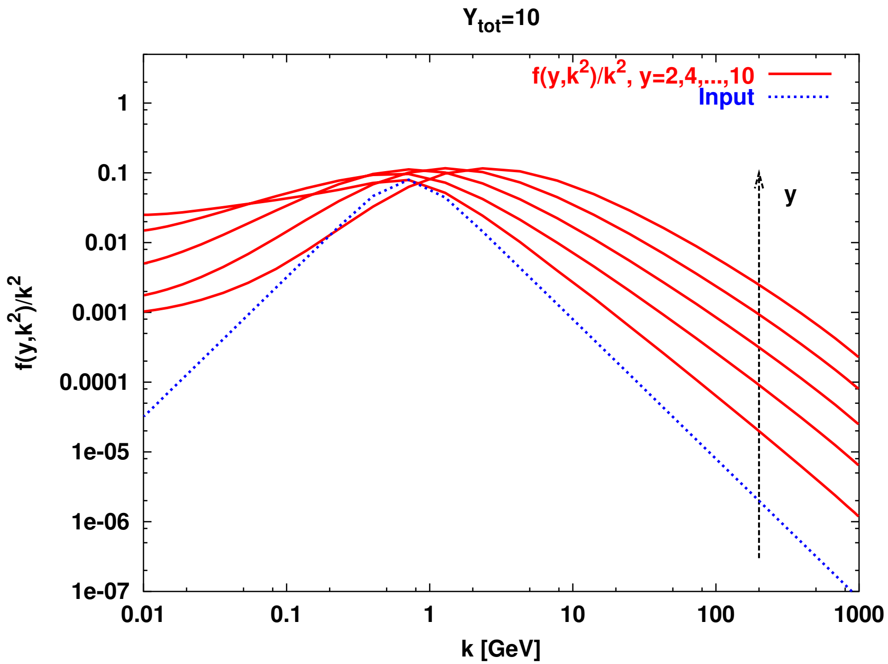
Symmetric solutions to Braun equations

$f(y,k^2)/k^2 = f^{\dagger}(Y-y,k^2)/k^2$ for $Y=6$ and $Y=8$





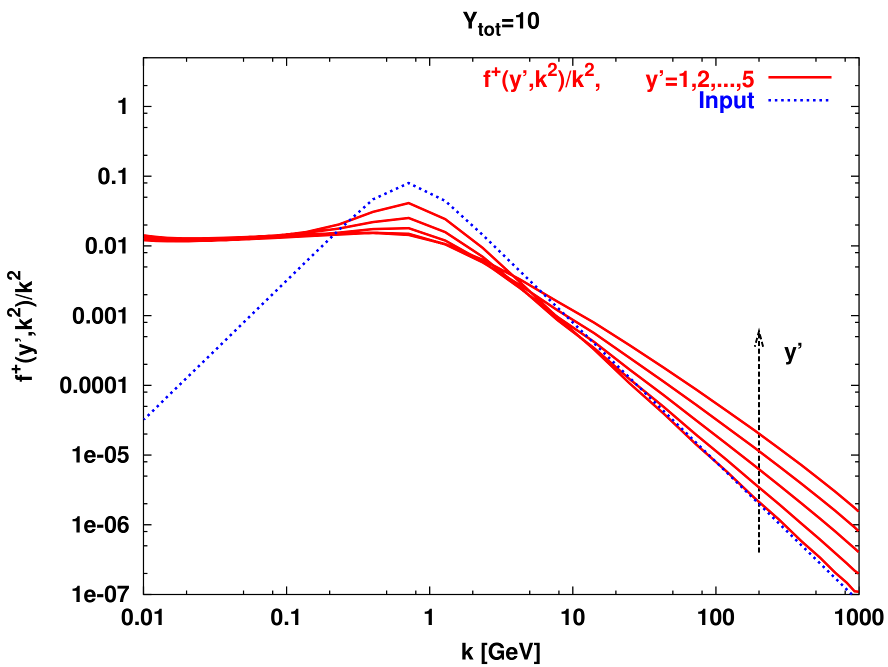
Above critical rapidity Y_c

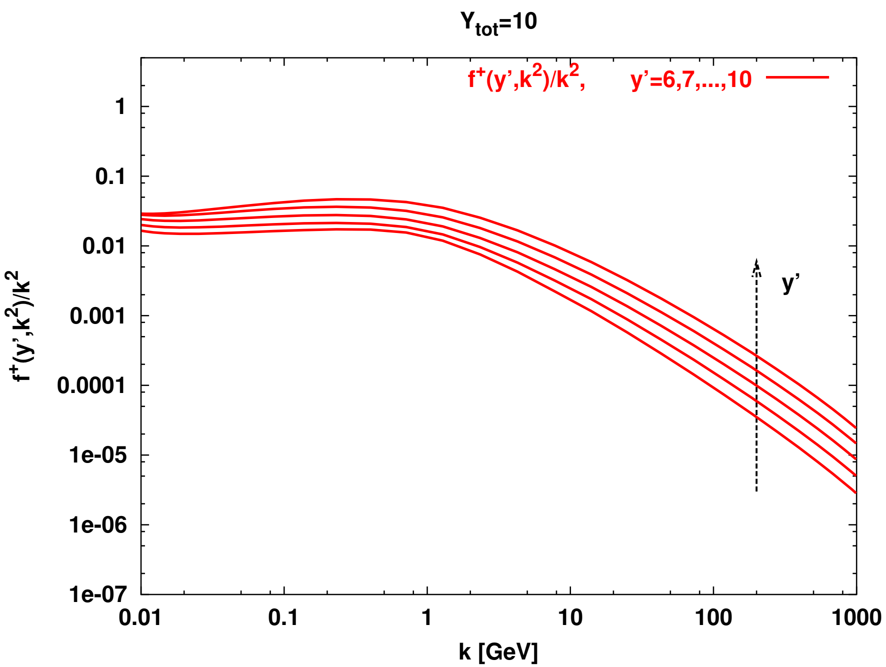


For $Y > Y_c$

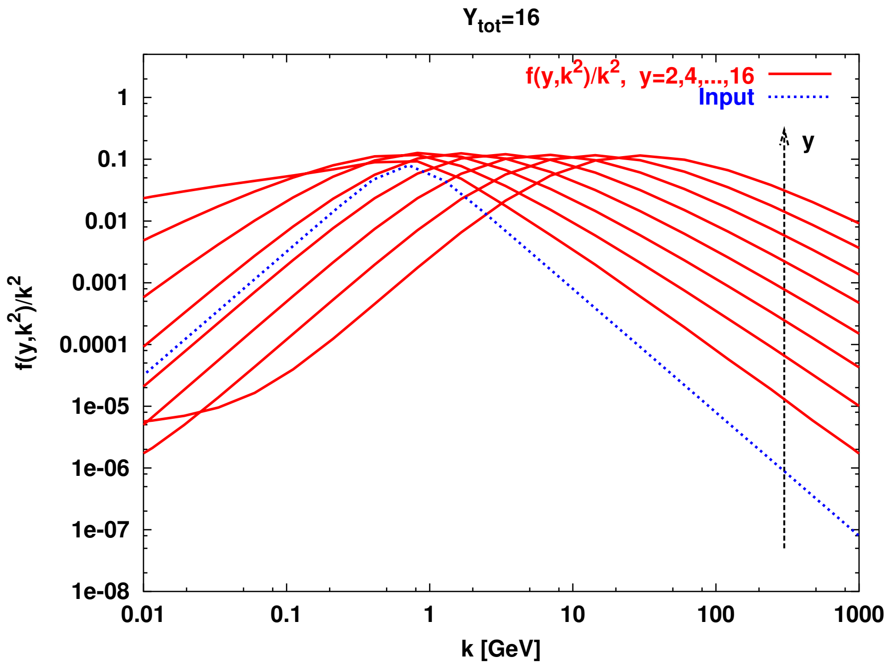
$f(y, k^2)$ (left) — different from

$f^{\dagger}(Y - y, k^2)$ (down)





Spontaneous Symmetry Breaking

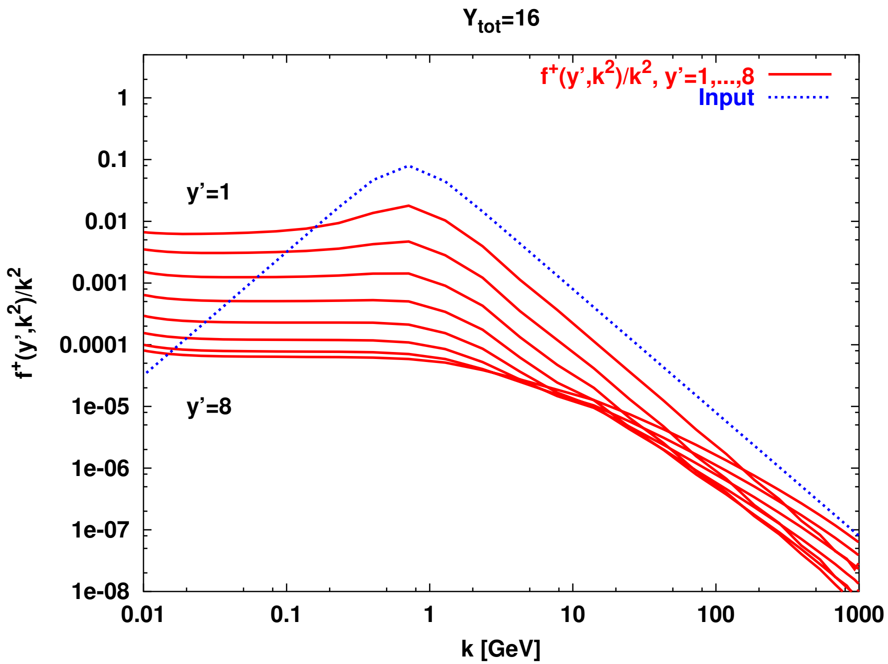


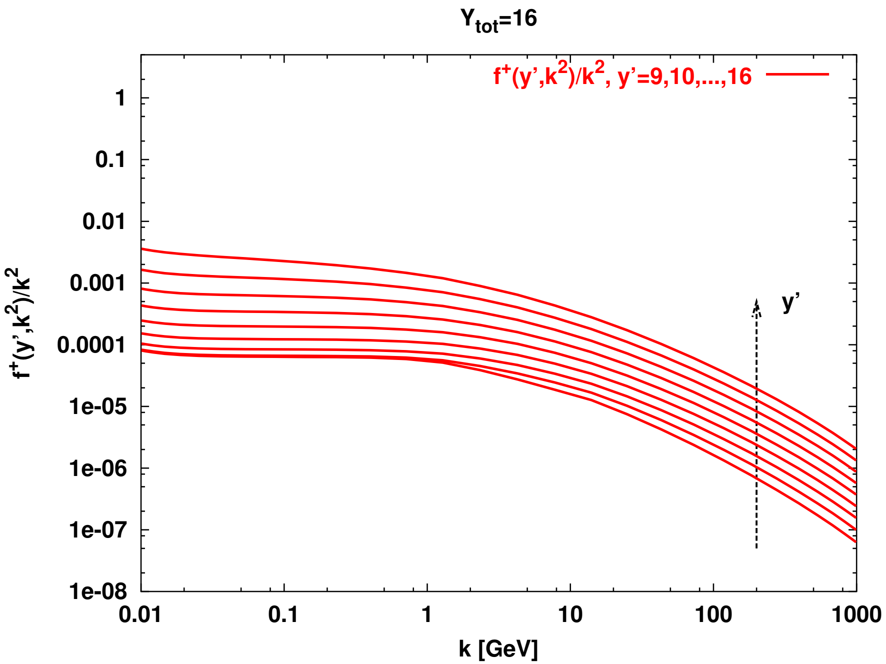
At $Y = 16$

$f(y, k^2)$ (left) larger than

$f^{\dagger}(Y - y, k^2)$ (down)

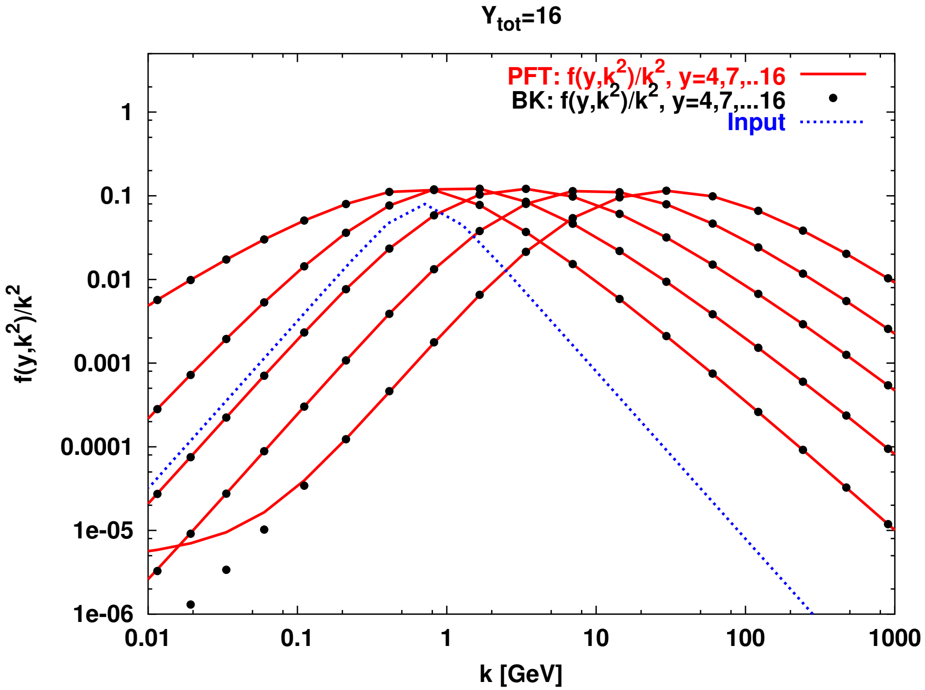
by three orders of magnitude

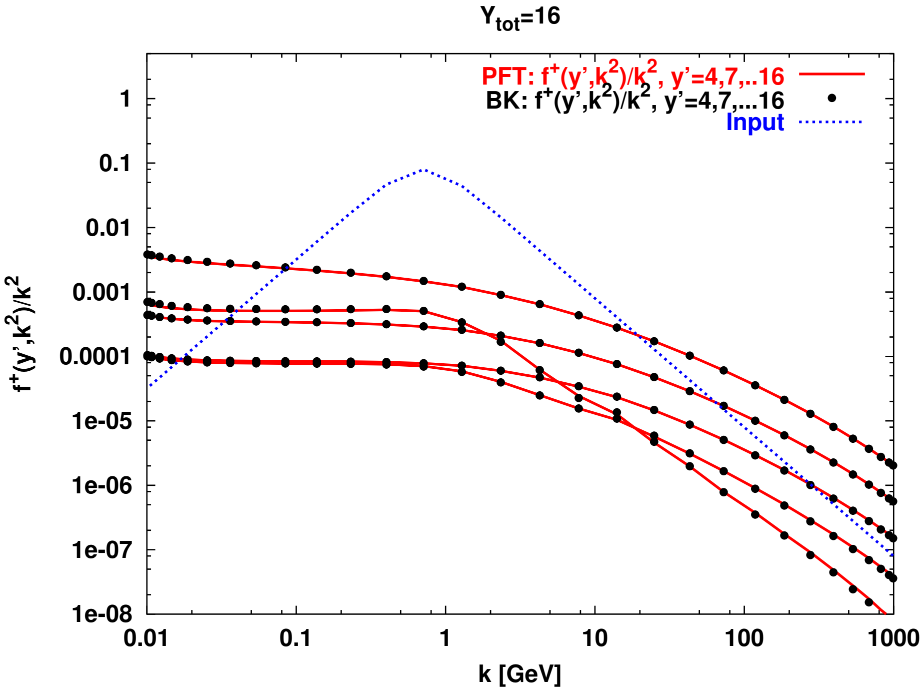


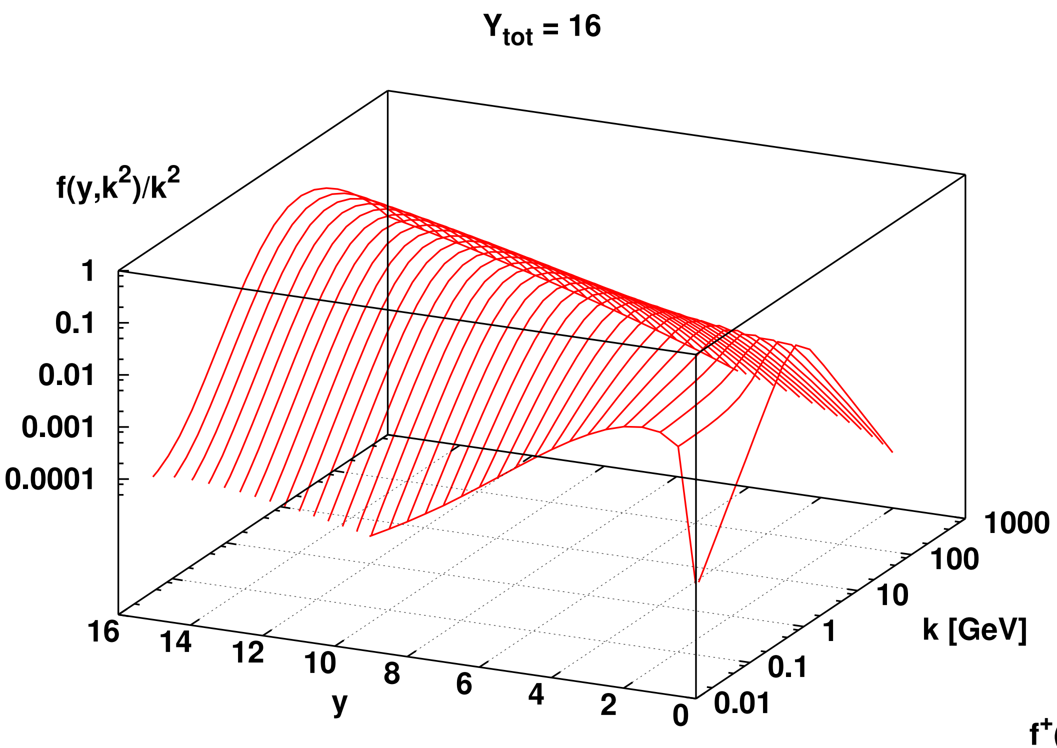


Fan dominance

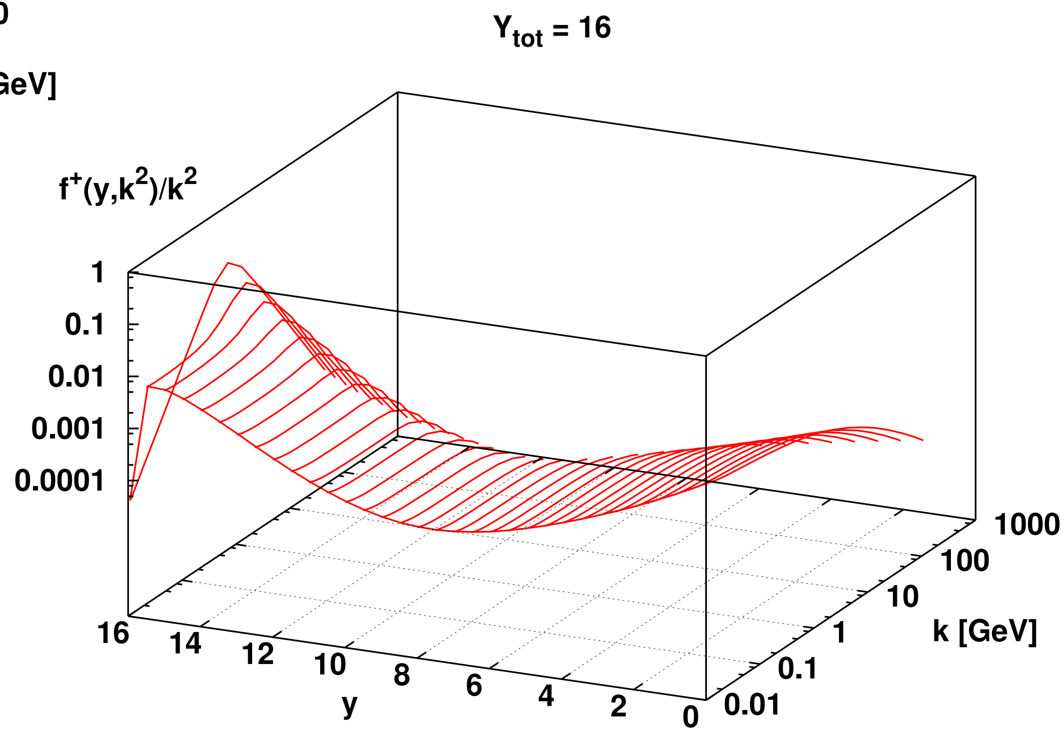
Solutions of Braun equations vs BK solutions







Y_{tot} = 16
$f(y,k^2)/k^2$
1
0.1
0.01
0.001
0.0001
16
14
12
10
8
6
4
2
0
y
0.01
0.1
1
10
100
1000
k [GeV]



Y_{tot} = 16
$f^+(y,k^2)/k^2$
1
0.1
0.01
0.001
0.0001
16
14
12
10
8
6
4
2
0
y
0.01
0.1
1
10
100
1000
k [GeV]

Detour to 0-dimensions

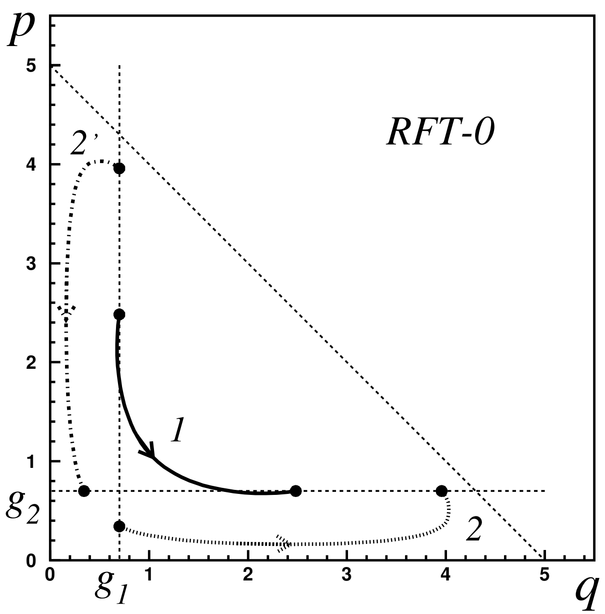
Formulation

$$\mathcal{L}_{\rm RFT-0} \,=\, \frac{1}{2}\, q\, \partial_y p \,-\, \frac{1}{2}\, p\, \partial_y q \,+\, \mu\, q\, p \,-\, \lambda\, q\, (q\,+\,p)\, p \,+\, p(y)\, q_0(y) \,+\, p_0(y)\, q(y)$$

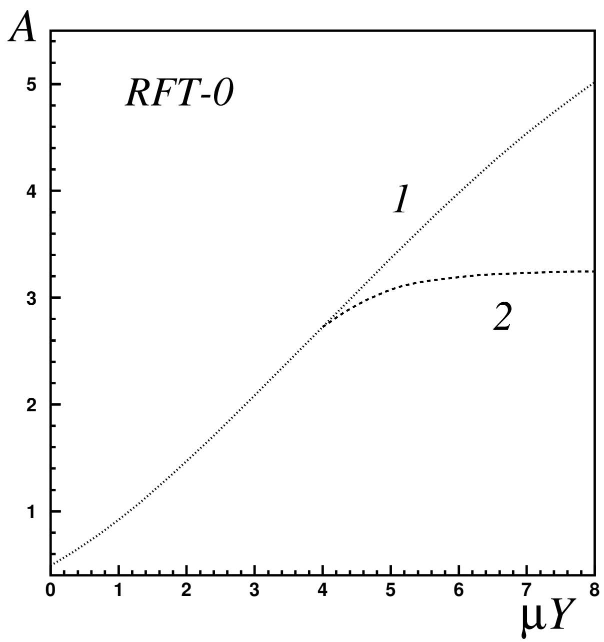
Equations of motion

$$\partial_y q \,=\, \mu\, q \,-\, \lambda\, q^2 \,-\, 2\,\lambda\, q\, p; \qquad -\partial_y p \,=\, \mu\, p \,-\, \lambda\, p^2 \,-\, 2\,\lambda\, q\, p$$

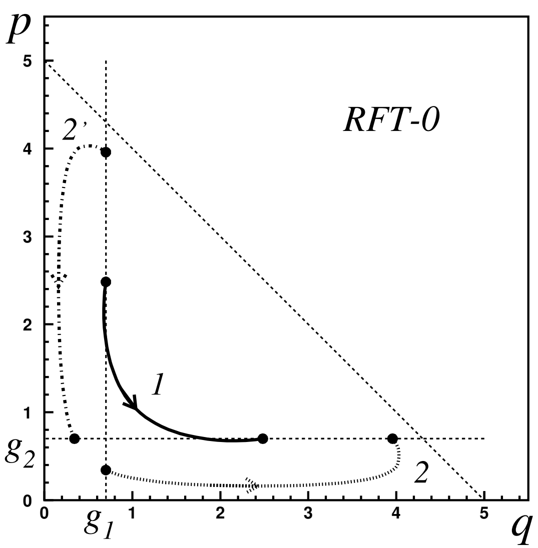
Phase space and trajectories

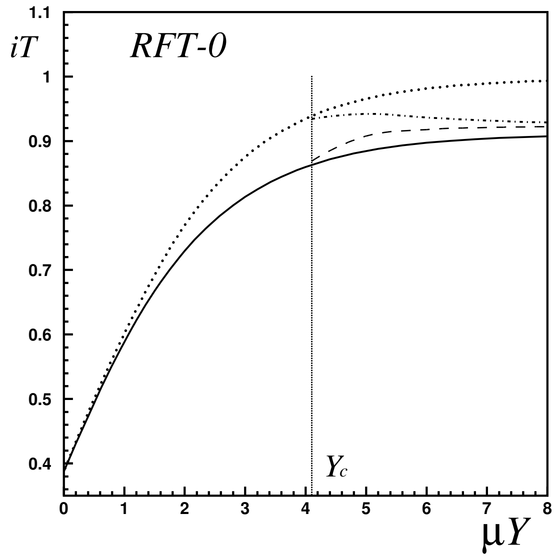


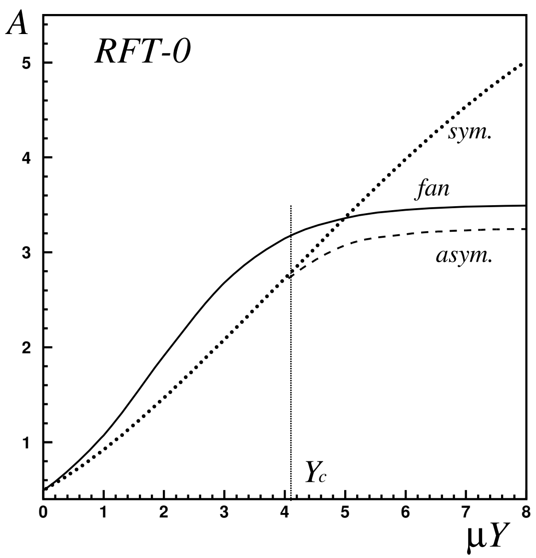
Action

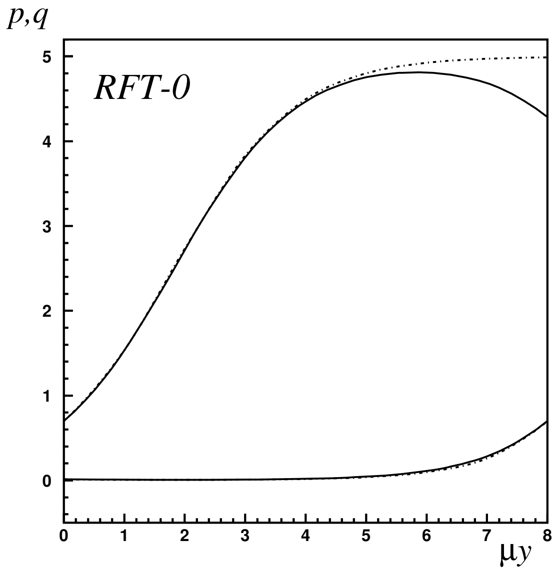


Semi-classical vs exact, fan dominance









Symmetric set of solutions

$$\bar{q}_2'(y; g, g) = \bar{p}_2(Y - y; g, g)$$

$$\bar{p}_2'(y; g, g) = \bar{q}_2(Y - y; g, g).$$

$$S(Y; g_1, g_2) \simeq$$
$$-\exp\{-\mathcal{A}_{RFT-0}[\bar{q}_1, \bar{p}_1; Y]\}$$
$$+\exp\{-\mathcal{A}_{RFT-0}[\bar{q}_2, \bar{p}_2; Y]\}$$
$$+\exp\{-\mathcal{A}_{RFT-0}[\bar{q}_2', \bar{p}_2'; Y]\}$$

Asymptotic answer

$$T \sim [1-\exp(-\rho g_1)][1-\exp(-\rho g_2)]$$

Decrease: $\exp(-E_0 y)$

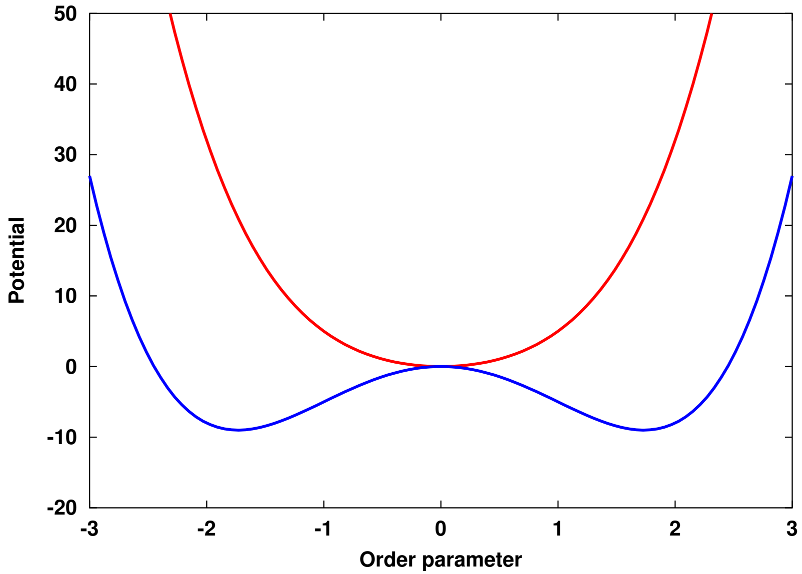
$$E_0 \sim -\exp(-\mu^2/\lambda^2)$$

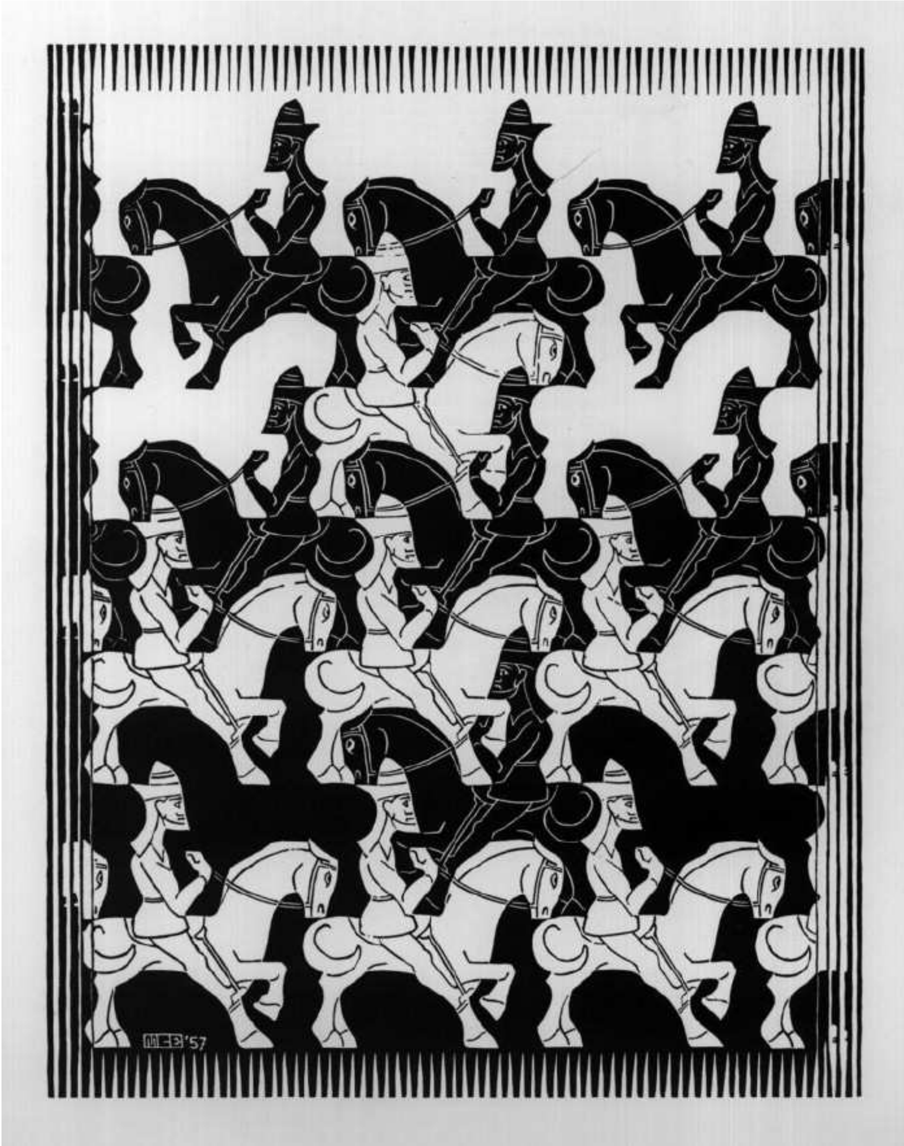
Interpretation

Competition between 2 self-multiplying absorbing each other fields

Instability of symmetric configurations for strong fields

Tempting analogy to second order phase transition





[M.C. Escher]

Superselecting target and projectile

Symmetric set of solutions

$$f_2'(y,k^2) = f_2^\dagger(Y-y;k^2), \qquad f^{\dagger\prime}_2(y,k^2) = f_2^\dagger(Y-y,k^2)$$

Target–projectile symmetry of the S-matrix

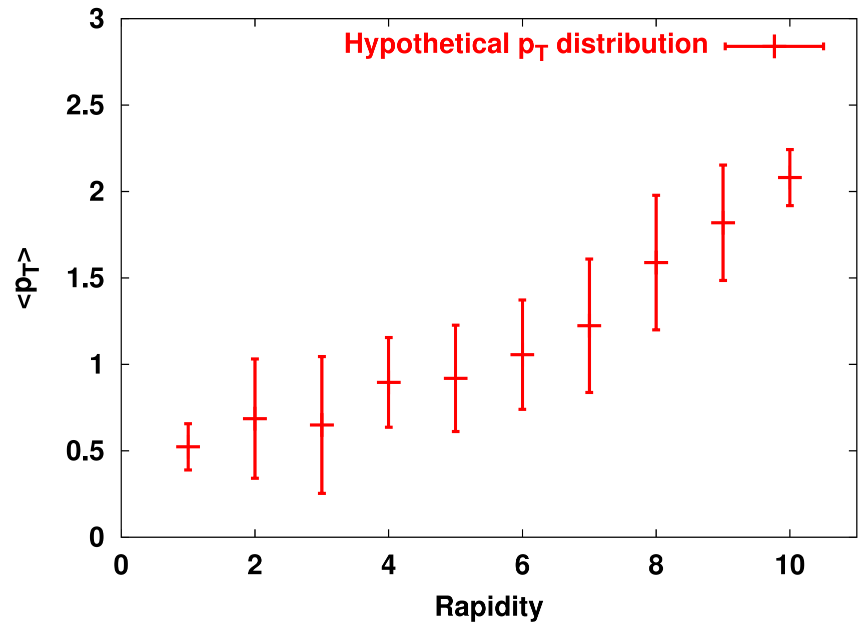
$$S(Y) \simeq -\exp\left\{-\mathcal{A}[f_1, f_1^\dagger; Y]\right\} + \exp\left\{-\mathcal{A}[f_2, f_2^\dagger; Y]\right\} + \exp\left\{-\mathcal{A}[f_2', f^{\dagger\prime}_2; Y]\right\}$$

Classical measurement of the event: for asymmetric trajectories saturation scale $Q_s(y)$ monotonic between target and projectile

Super-selection of asymmetric classical trajectory?

Proposal: Analysis of $\langle p_T(y)\rangle$ on the event-by-event basis

Possible: uncorrelated domains in impact parameter plane



Connection to stochastic formulation

Dipole model – splittings and mergings $\longrightarrow$ Stochastic realization **[Iancu, Mueller, Munier]**

$$\frac{d\tilde{n}}{dY} = \alpha\tilde{n} - \beta\tilde{n}^2 + \sqrt{2[\alpha\tilde{n} - \beta\tilde{n}^2]}\,\nu, \qquad \langle\nu(Y)\rangle = 0, \quad \langle\nu(Y')\nu(Y)\rangle = \delta(Y - Y').$$

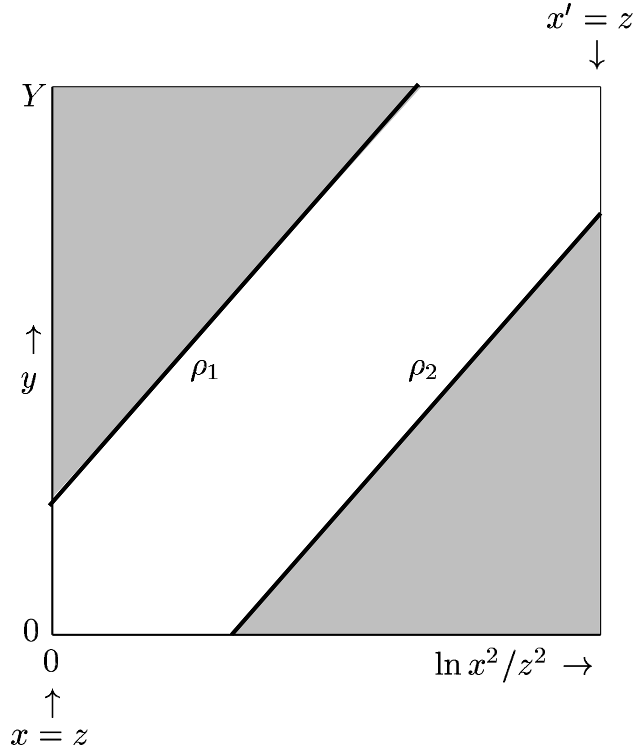
Evolution: Dipole splittings $\longrightarrow$ Fixed Point : $\quad n = \dfrac{\alpha}{\beta}$

Explicit targer-projectile asymmetry

Two absorptive boundaries **[Mueller,Shoshi]**

Similar conclusions from studies of stochatic FKPP **[Enberg, Golec-Biernat, Munier]**

Asymmetric formulation $\longrightarrow$ asymmetric outcome



Conclusions

⟶ Field theoretical formulation of interacting QCD pomerons in momentum space is available incorporating both pomeron mergings and splittings

⟶ Degrees of freedom are pomeron fields related to unintegrated gluon densities and the action is seld dual

⟶ The theory was solved in the semi-classical approximation for symmetric two-side boundary conditions (Braun equations)

⟶ Above critical rapidity Y_c symmetry between target and projectile is spontaneously broken

⟶ Superselection of asymmetric configurations by classical measurements is suggested

⟶ Possible asymmetric particle distributions in heavy ion collisions (event-by-event)

⟶ Suppression of loops?