Large SU(N) study of super Yang–Mills quantum mechanics.

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- Super Yang-Mills quantum mechanics (SYMQM)
- Cutoff
- D=2, SU(2)
- D=4, SU(2)
- A comment on D=10, SU(2)
- D=2, SU(3)
- D=2, SU(N).
- Summary

1.SYMQM

Super Yang Mills field theories in a point
(dimmensional reduction from D=d+1 to 0+1 dimensions)

- supersymmetric in gauge invariant sector
- inherit O(d)
- zero volume YM = Y.M.Q.M Lüscher, Münster 83’
- solution for SYMQM, SU(2), D=2 – Claudson, Halpern 85’
- supermembrane = SYMQM – Bergshoef, Sezgin, Townsend 88’
- continuous spectrum – de Wit, Lüscher, Nicolai 89’
- solution for SYMQM, SU(N), D=2 – Samuel 97’
- B.F.S.S. : $U(N \to \infty)$, d=9, S.Y.M.Q.M ≡ M(atrix) theory
  Banks, Fisher, Shenker, Susskind 97’

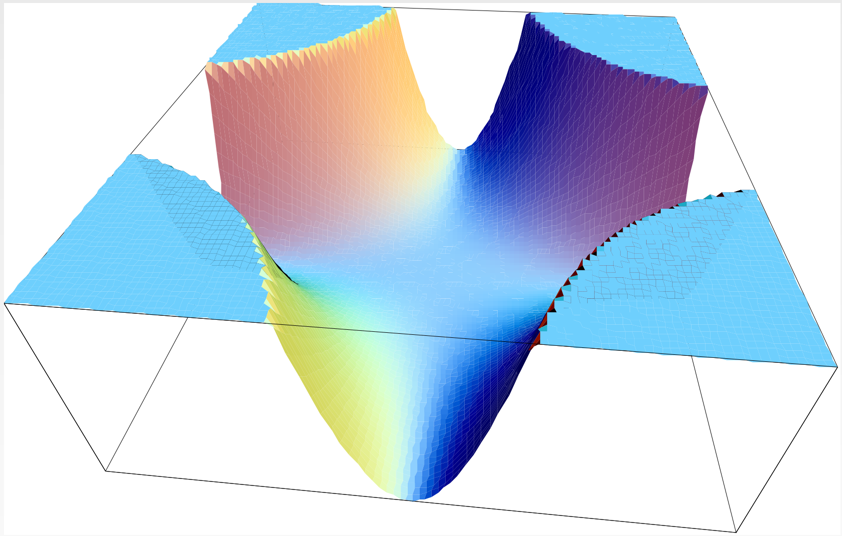
$$H = Tr(P_i P_i + \frac{1}{2}[X_i, X_j][X_i, X_j] + g\Theta^T \Gamma_i [\Theta, X_i])$$

-Feynman last board

$$P_i = p_i^a T_a, \quad X_i = x_i^a T_a, \quad \Theta_i = \vartheta_i^a T_a, \quad i = 1, \ldots, d$$

Γ_i - $(d+1) \times (d+1)$ Dirac matrices, ϑ_i - Majorana spinors

term $Tr([X_i, X_j][X_i, X_j]) \propto r^4$



- $n_F = 0$ – discrete
- $n_F \neq 0$ – continuous & discrete
- B.F.S.S: bound state on the threshold of the continuous spectrum – the supergraviton

Cutoff

-truncate the Hilbert space to a maximum number of quanta

$$n_B = a_i a_i^+ \leq n_{B\,\mathbf{max}}$$

-compute matrix elements of H and diagonalize

$$E_m^{n_B} = E_m + O(e^{-n_B}) \quad \text{-discrete}$$

$$E_m^{n_B} = O(n_B^{-1}) \quad \text{-continuous}$$

Claim

$$m(N) = const.\sqrt{N} \Leftrightarrow \quad E_{m(N)}^{N} \xrightarrow{N\to\infty} E$$

Should work always when one can define p asymptotically

Argument

$$\hat{p} \to \hat{p}^{(n_B)} \to p_m^{n_B} \leftrightarrow Hermite \approx \frac{\pi m}{\sqrt{n_B}}$$

M.T.,Wosiek 03'

M.T 03'

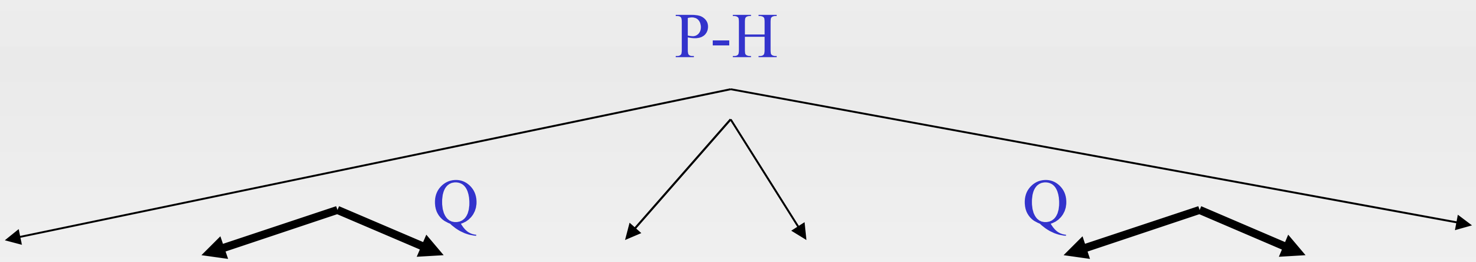
2. D=2, SU(2) Campostrini, Wosiek 02'

$$H^{D=2} = \frac{1}{2} p_a p_a \qquad a = 1,2,3$$

Gauge invariant states: $\rightarrow \quad \delta_{ab}, \varepsilon_{abc}, a_b^+, f_b^+$

$$(aa) = a_b^+ a_b^+ , \qquad (aff) = \varepsilon_{abc} a_a^+ f_b^+ f_c^+ ,$$

$$(af) = a_b^+ f_b^+ , \qquad (fff) = \varepsilon_{abc} f_a^+ f_b^+ f_c^+ ,$$



$n_B \backslash n_F$	0	1	2	3
0	$\vert 0\rangle$	——	——	$(fff)\vert 0\rangle$
1	——	$(af)\vert 0\rangle$	$(aff)\vert 0\rangle$	——
2	$(aa)\vert 0\rangle$	——	——	$(aa)(fff)\vert 0\rangle$
3	——	$(aa)(af)\vert 0\rangle$	$(aa)(aff)\vert 0\rangle$	——

Witten index $I_W = \sum (-)^{n_F} \exp(-TE_n) = 0$ but $I_W^{0,1} = I_W^{2,3} = \frac{1}{2}$

Index

$I_W^{0,1}$

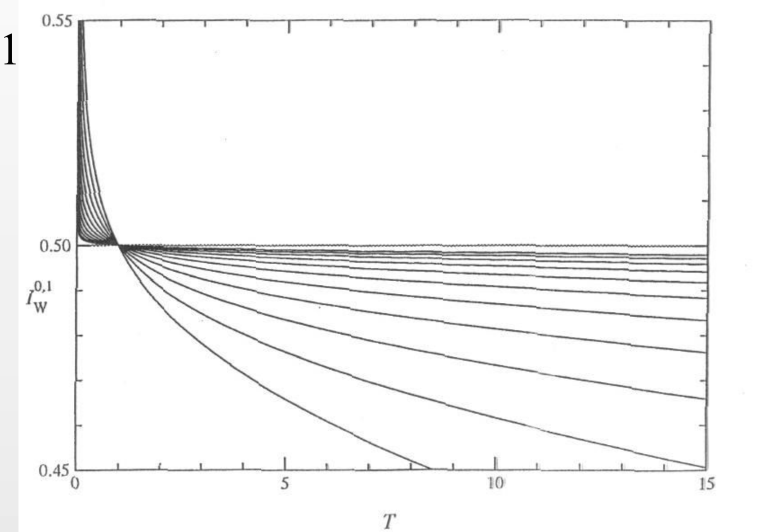
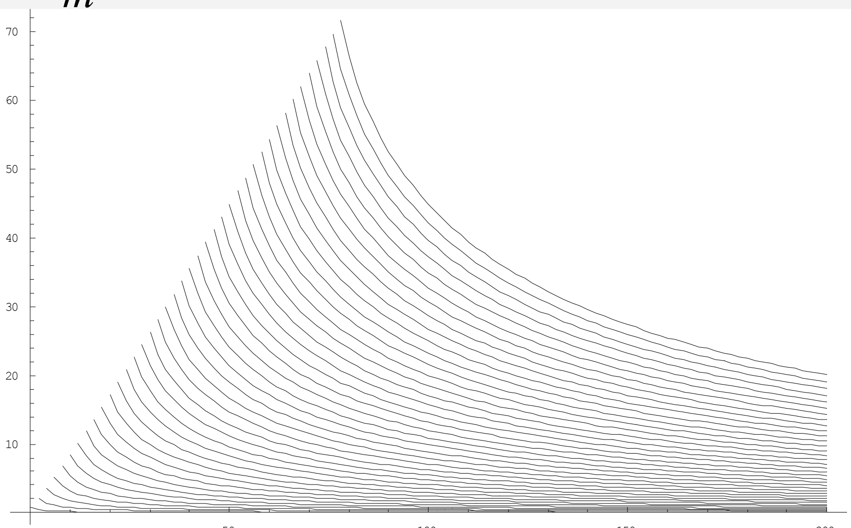


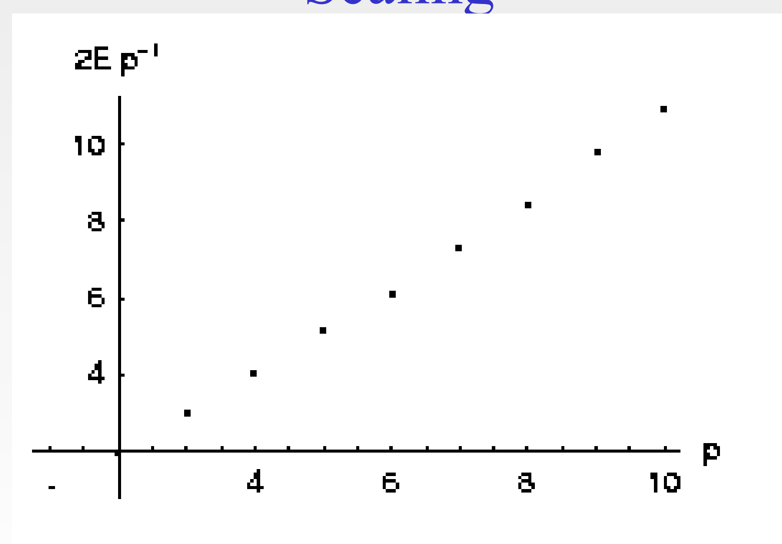
Figure 1: The restricted Witten index computed for values of N_{cut} ranging from 125 to 128000.

$E_m^{n_B}$ Cut spectrum



n_B

$E^{n_B}_{m(n_B,p)}$ Scaling T



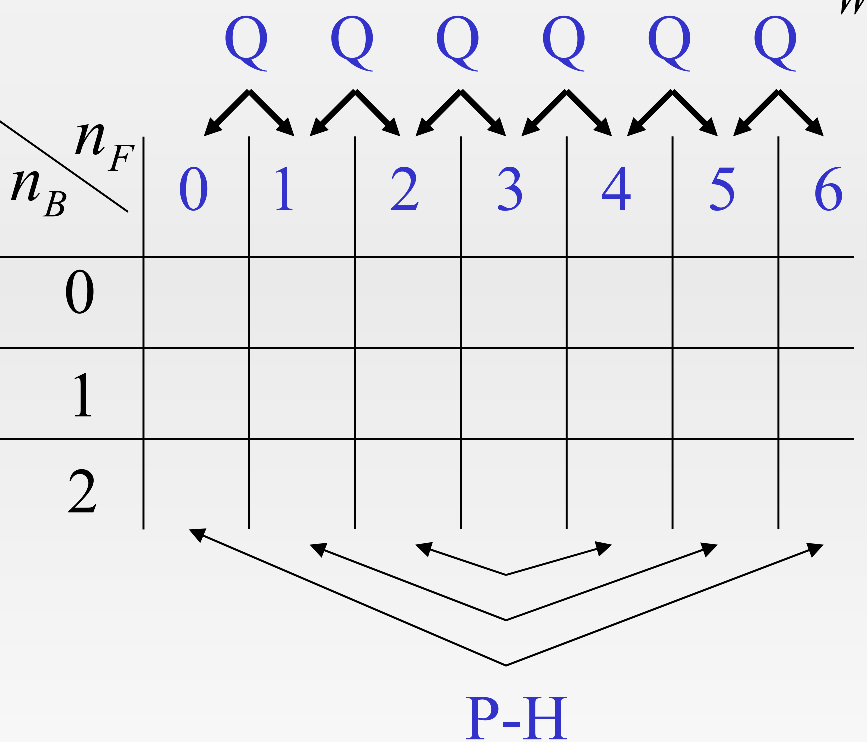
p

2. D=4, SU(2) Campostrini, Wosiek 04'

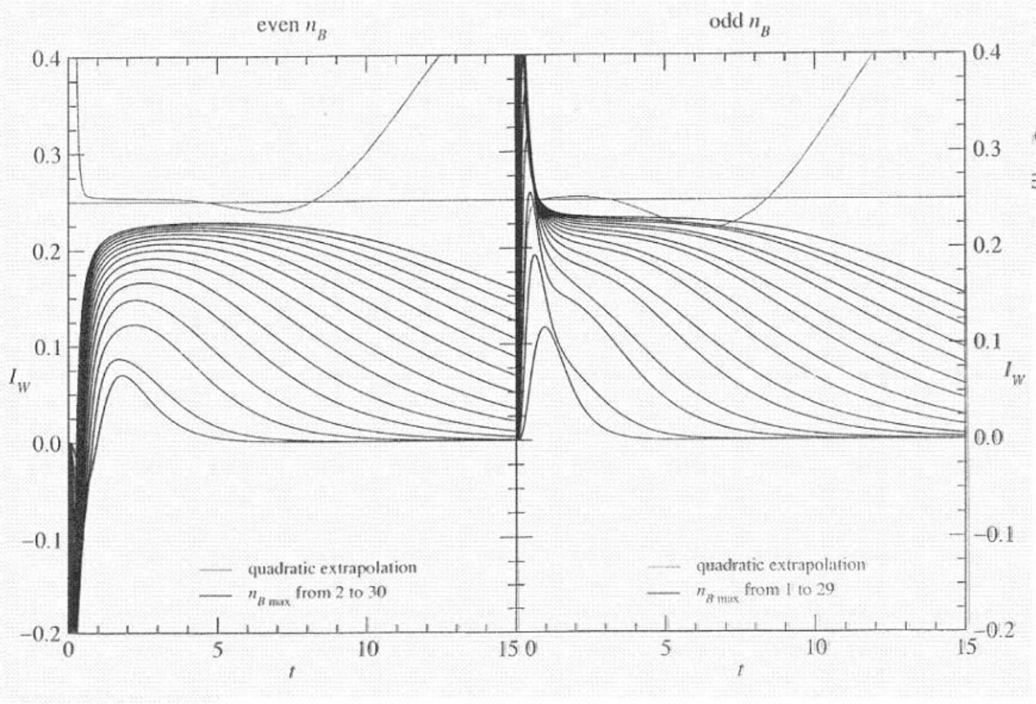
$$H \overset{D=4}{=} \frac{1}{2}\vec{p}_i \circ \vec{p}_i + \frac{1}{2}\sum_{i,j}(\vec{x}_i \times \vec{x}_j)^2 + H_F$$

a,b,c=1,2,3
m=1,2 i=1,2,3

Gauge invariant states: $\rightarrow \delta_{ab}, \varepsilon_{abc}, a^{i+}_b, f^{m+}_b$



$I_W^{0,1}$ Index

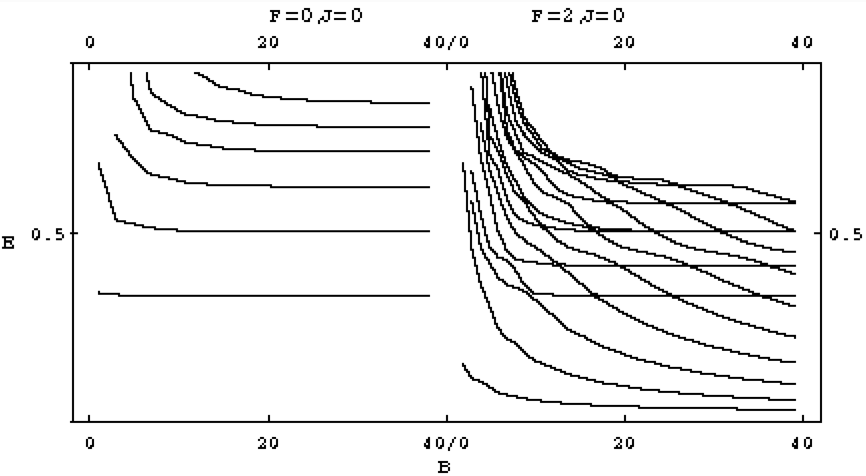


T

$\rightarrow$ Sethi,Stern 97' $I_W = I_M + I_{\partial M} = \frac{1}{4} - \frac{1}{4} = 0$

Cut spectrum F=0,2 J=0

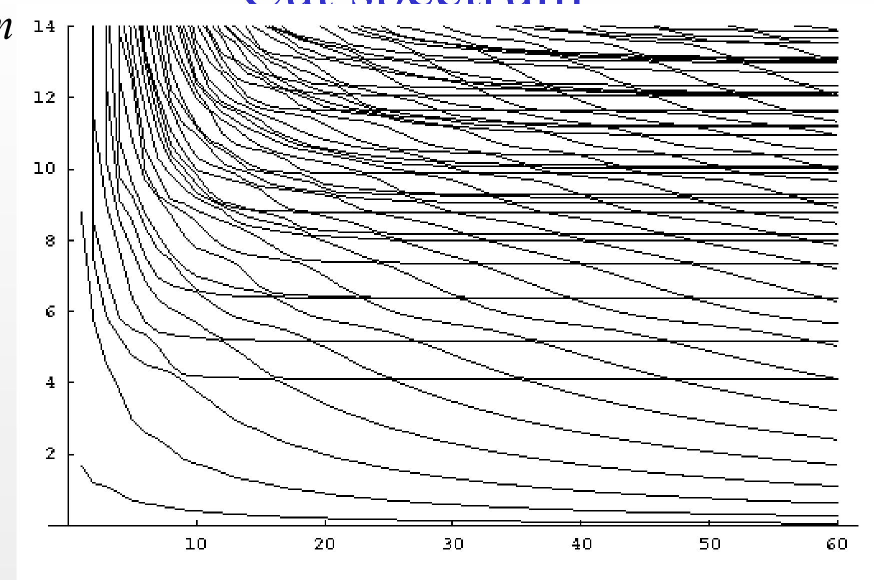
$E_m^{n_B}$



n_B

$E_m^{n_B}$

Cut spectrum

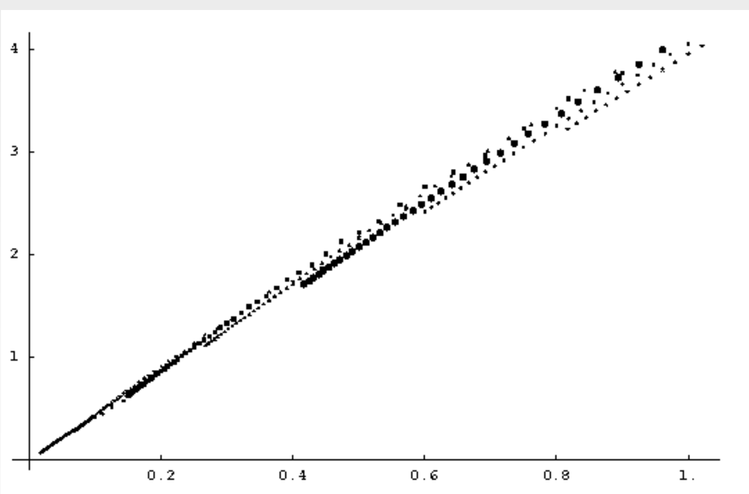


Supermultiplets-diamonds

n_B

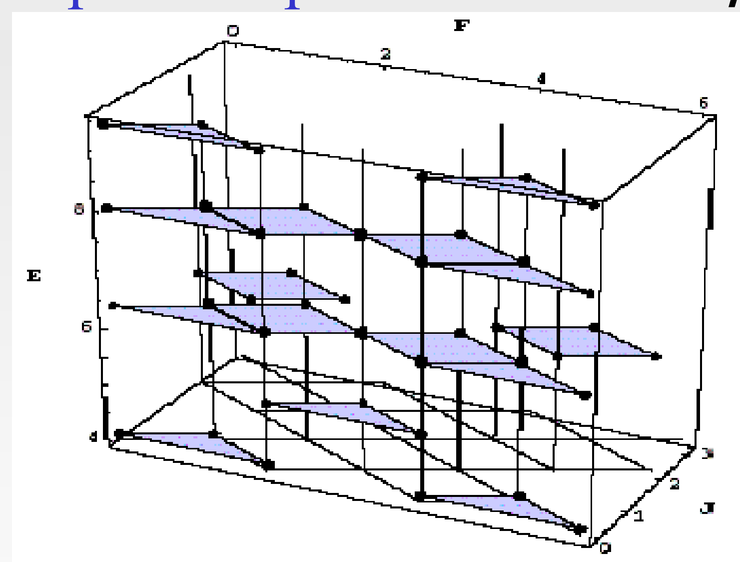
$E_{m(n_B,p)}^{n_B}$

Scaling



p^2/2

Kotański



→ gluino condensate

Shifman, Vainshtain 88’

2.A comment on D=10, SU(2)   Wosiek 05'

$$H \overset{D=10}{=} \frac{1}{2}\vec{p}_i \circ \vec{p}_i + \frac{g^2}{4}\sum_{i,j}(\vec{x}_i \times \vec{x}_j)^2 + H_F$$

i=1,2,…,9

$$\vec{x}_i = (x_i^1, x_i^2, x_i^3)$$

SO(9) $\rightarrow$ four planes eg. (12),(34),(56),(78) $\rightarrow$ four Casimirs

states: $|m_1, m_2, m_3, m_4, j_1, j_2, j_3, j_4\rangle$

$$J^2 = \sum_{i<k} J_{ik} \qquad J_{ik} = x_a^i p_a^k - x_a^k p_a^i - \frac{1}{2}\psi_a^+ \Sigma^{ik} \psi_a$$

surprise $J^2|0\rangle = 78|0\rangle$ $\rightarrow$???????

Empty state $|0_B, 0_F\rangle$ is not invariant under rotations!!!

SO(3) singlet is made out of 132132 Fock states (F=12).

3. D=2, SU(3)

SU(N) invariant states

$$SU(2) \longleftrightarrow \varepsilon_{abc}, \delta_{ab}, a_b^+, f_b^+$$

$$SU(N) \longleftrightarrow f_{abc}, d_{abc}, \delta_{ab}, a_b^+, f_b^+ \qquad a,b,c = 1,\ldots,N^2-1$$

hance in general

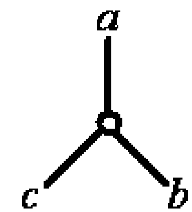
$$A_{b_1\ldots b_n a_1\ldots a_m} a_{b_1}^+ \ldots a_{b_n}^+ f_{a_1}^+ \ldots f_{a_m}^+ \left|0\right\rangle, \qquad m \le N^2-1$$

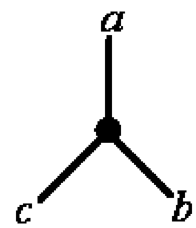
$$A_{b_1\ldots b_n a_1\ldots a_m} \longleftrightarrow \delta_{ab}, d_{abc}, f_{abc},$$

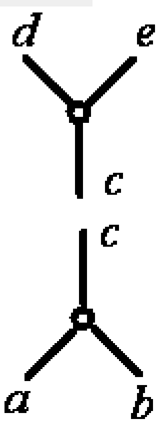
+ d,f identities

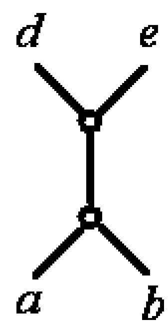
BIRDTRACS

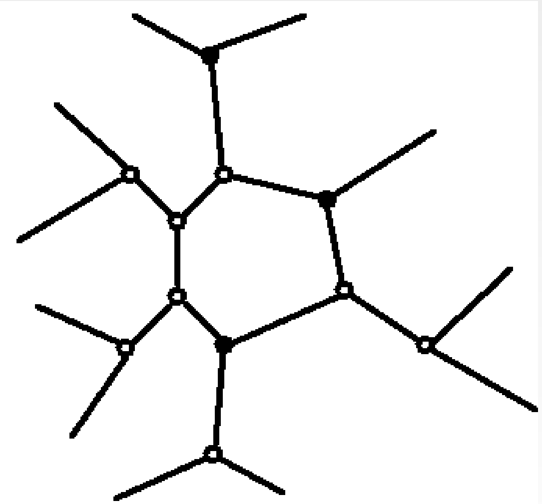
Cvitanović 75'

d_{abc} $\longleftrightarrow$ 

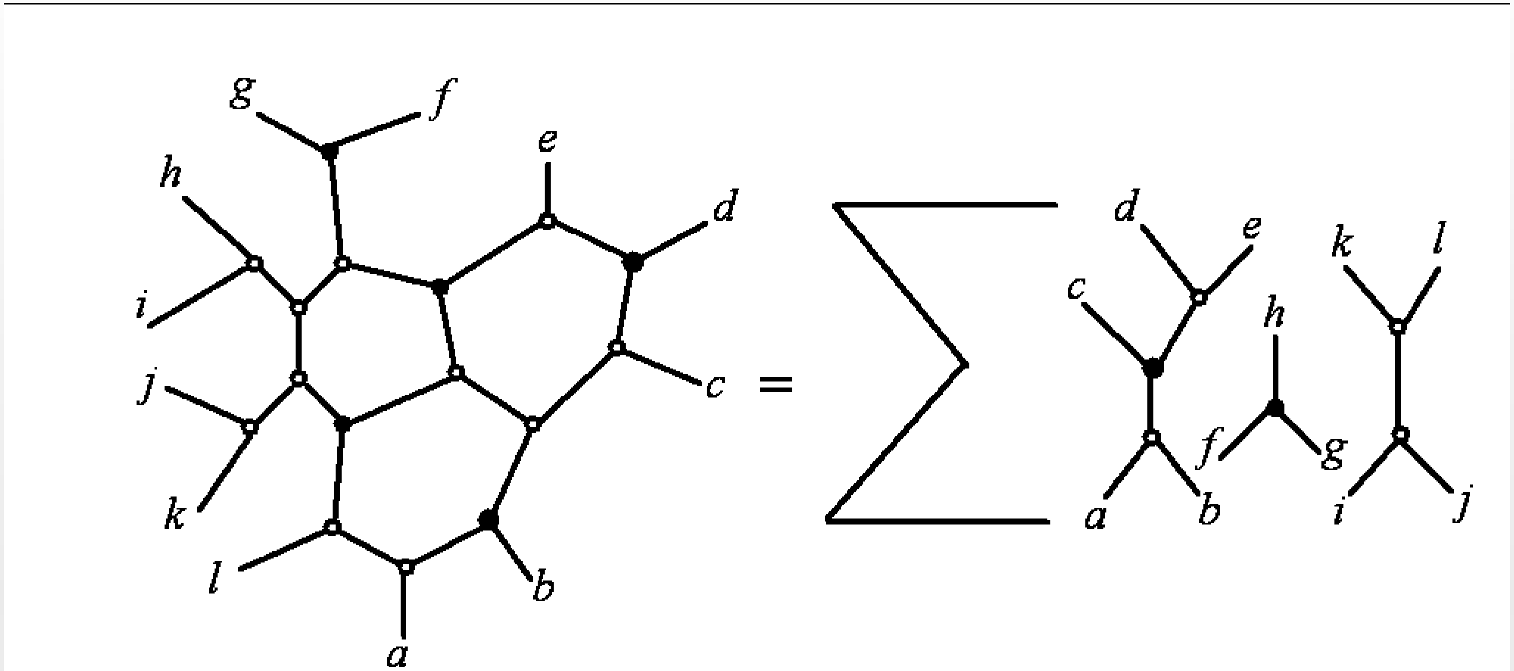
f_{abc} $\longleftrightarrow$ 

$d_{abc}d_{cde}$ = 

= 

dfddfdf... = 

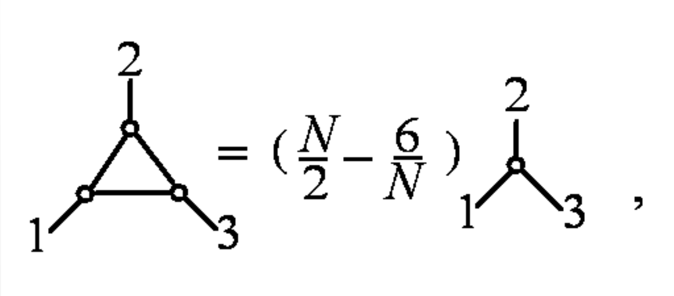
Theorem. There are no loops and the trees are small!

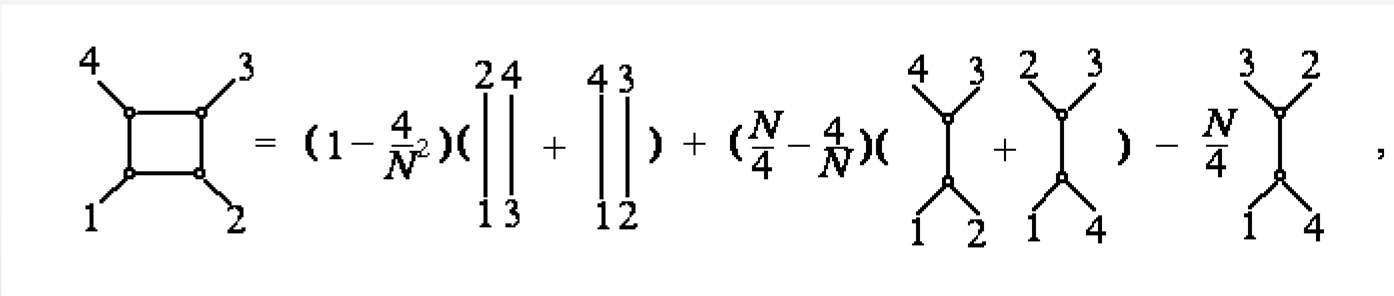


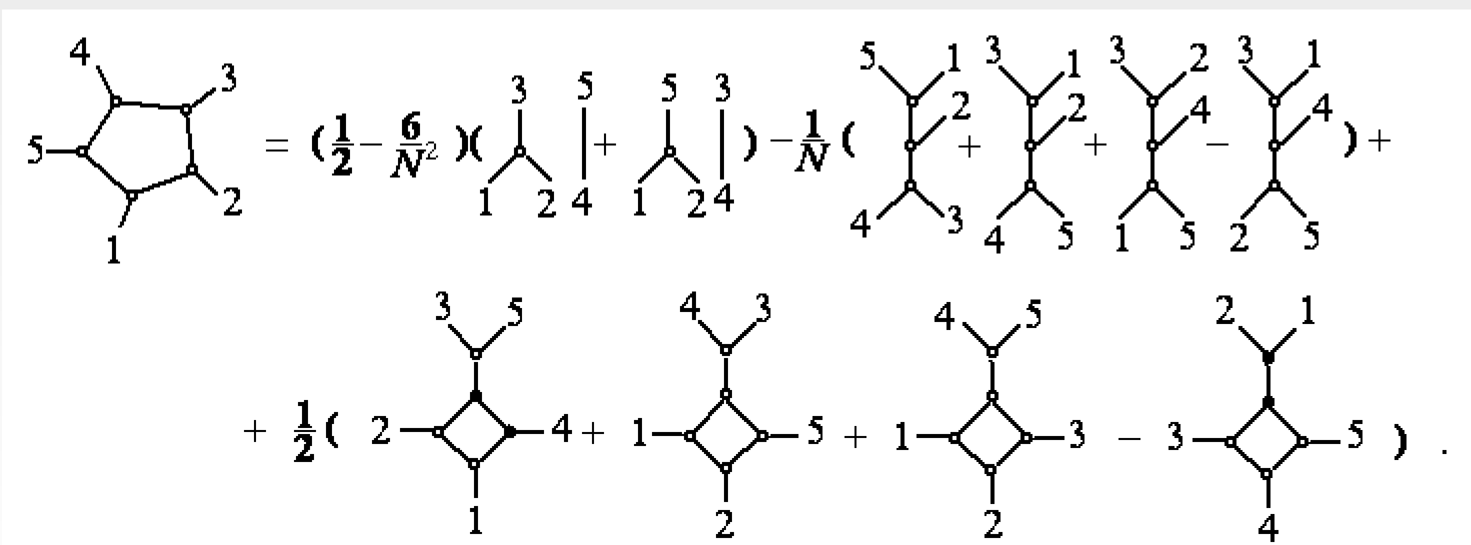
=

$$= Tr(\lambda_a \lambda_b \lambda_c) Tr(\lambda_d \lambda_e \lambda_f \lambda_g) Tr(\lambda_h \lambda_i \lambda_j \lambda_k \lambda_l) + \ldots$$

Examples







M.T 05'

Cayley-Hamilton theorem

$$A^3 + p_1 A^2 + p_2 A + p_3 = 0, \qquad A \in GL(3, F)$$

→ There exists a finite set of operators (bricks) that generate the basis of gauge sector.

Notation: $a = a_i^+ T_i, \qquad f = f_i^+ T_i$

$$(aa) = Tr(a^+ a^+) = a_i^+ a_j^+ Tr(T_i T_j)$$

$$(aaf) = Tr(a^+ a^+ f^+) = a_i^+ a_j^+ f_k^+ Tr(T_i T_j T_k)$$

ect.

$$|state\rangle = \sum (aa)(aafa)(aaa)\ldots|0\rangle \quad \leftarrow$$ Samuel claim 97'

Irreductible SU(3) brics

Q Q Q Q Q Q Q Q

n_B \ n_F	0	1	2	3	4	5	6	7	8	B	F
0	1 1	0	0	(*fff*) 1	0	1	0	0	1	1	1
1	0	(*af*) 1	(*aff*) 1	(*afff*) 1	(*affff*) 2	1	1	1	0	4	4
2	(*aa*) 1	(*aaf*) 1	(*aaff*) 1	(*aafff*) 3	(*aaffff*) 4	3	1	1	1	8	8
3	(*aaa*) 1	1	(*aafaf*) 3	(*aaffaf*) 5	(*afafaff*) 5	5	3	1	1	12+1	12
4	1	2	2	5	(*aaffafaf*) 9	5	2	2	1	14+1	14
5	1	2	4	(*aafaafaf*) 8	10	8	4	2	1	20	20
6	2	2	4	(*aafaafaaf*) 10	13	10	4	2	2	24+1	24
	....	....	....	....	....					26+1	26
										32	32
										36+1	36
										38+1	38
										44	44

P-H

-vacuum in 0,8 and 4!

-matrix kinematics

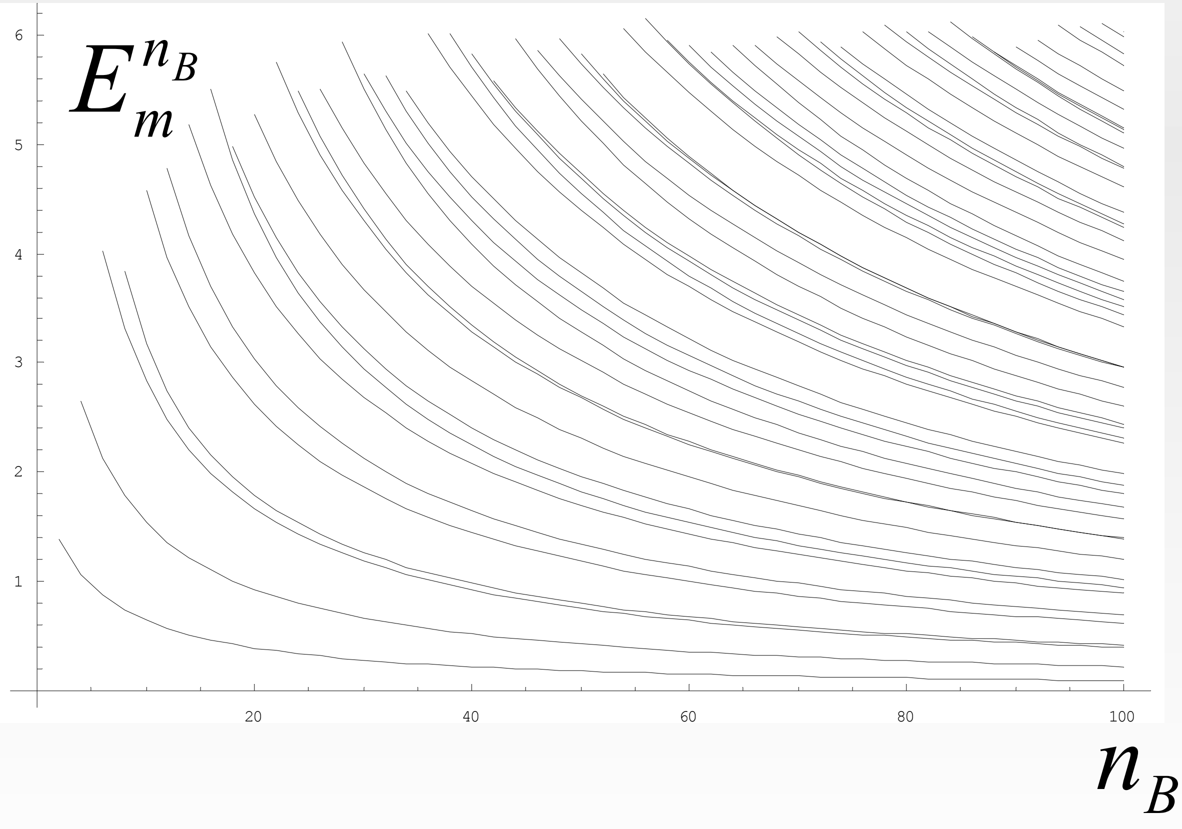
$n_F = 0$,SU(3) basis:

$$|i,j\rangle = (++)^i(+++)^j|0\rangle \rightarrow \qquad S_{i,j}^{i',j'} = \langle i',j'|i,j\rangle$$

$\rightarrow$ commutation rules between bricks

$\rightarrow$ recurrence on $S_{i,j}^{i',j'} \rightarrow H \rightarrow H_{ort}$ $\leftarrow$ nice numbers

$\rightarrow$ representation of hamiltonian, cutoff, spectrum



$\rightarrow$ cutoff breaks SU(3) symmetry

$\rightarrow$ two Casimirs

$$C_1 = \lambda_i \lambda_i$$

$$, C_2 = d_{ijk}\lambda_i\lambda_j\lambda_k$$

$$[C_1^{(n_B)}, H^{(n_B)}] = 0$$

but $[C_2^{(n_B)}, H^{(n_B)}] \neq 0!!!$

$\rightarrow \quad n_F > 0$ Scalar products are functions of S symbol however

$n_F = 1$ 3 recurrences

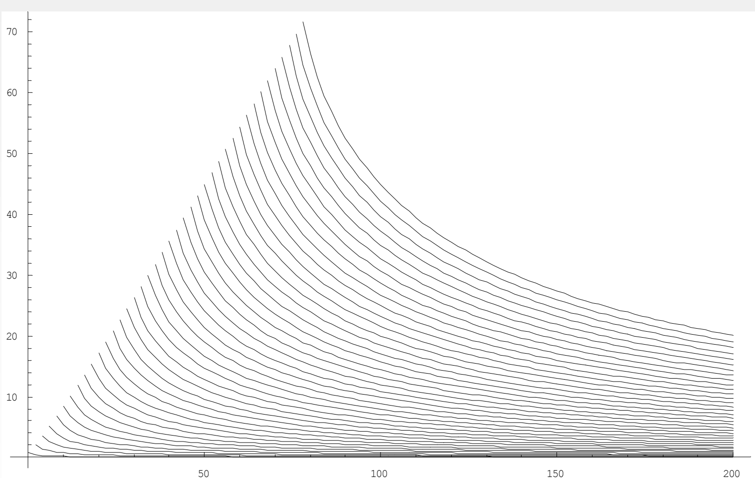
$n_F = 2$ 10 recurrences

$n_F = 3$ 55 recurrences

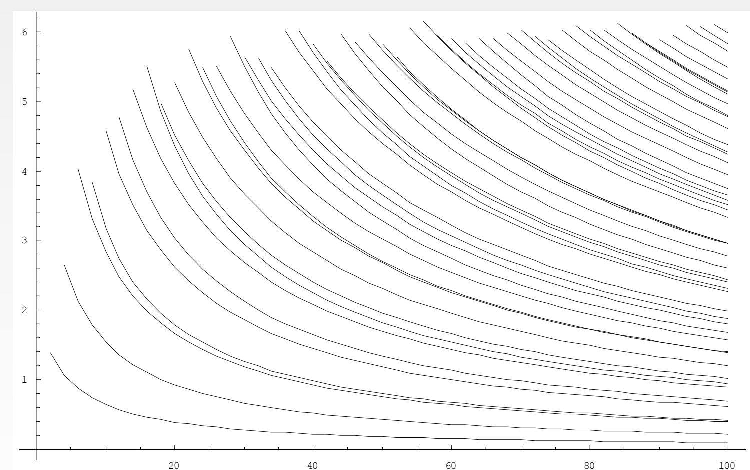
$n_F = 4$ 91 recurrences

\+ 159!!

SU(2)



SU(3)



3. D=2, SU(N).

$\rightarrow \quad n_F = 0$ sector

$$|n_B\rangle = \sum_{2i+3j=n_B} c_{ij} (aa)^i (aaa)^j |0\rangle \qquad \text{SU(3)}$$

$$|n_B\rangle = \sum_{2i_2+\ldots+Ni_N=n_B} c_{i_2\ldots i_N} (aa)^{i_2} (aaa)^{i_3} \ldots (a\ldots a)^{i_N} |0\rangle \qquad \text{SU(N)}$$

$$\#\{|n_B\rangle, n_B = const., n_F = 0\} \approx \text{P(n)} \approx \frac{1}{4n_B\sqrt{3}} \exp(\pi\sqrt{\frac{2n_B}{3}})$$

$\uparrow$

Hardy, Ramanujan 18’ Uspensky 20’
Rademacher 37’

Lüscher limit versus SU(N) limit.

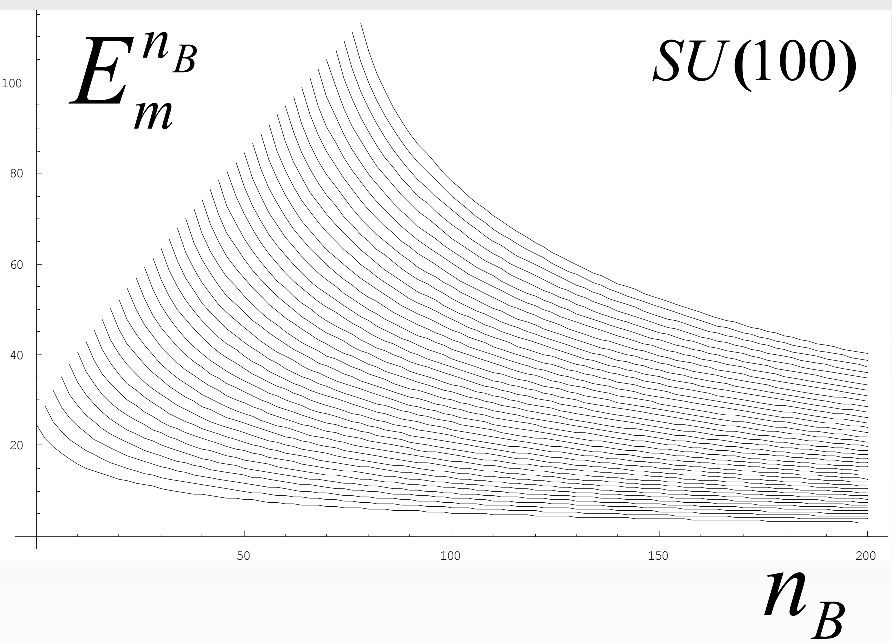
SU(N) basis: $\left|i_2\ldots,i_N\right\rangle = (aa)^{i_2}(aaa)^{i_3}\ldots(\underbrace{a\ldots a}_{N})^{i_N}\left|0\right\rangle$

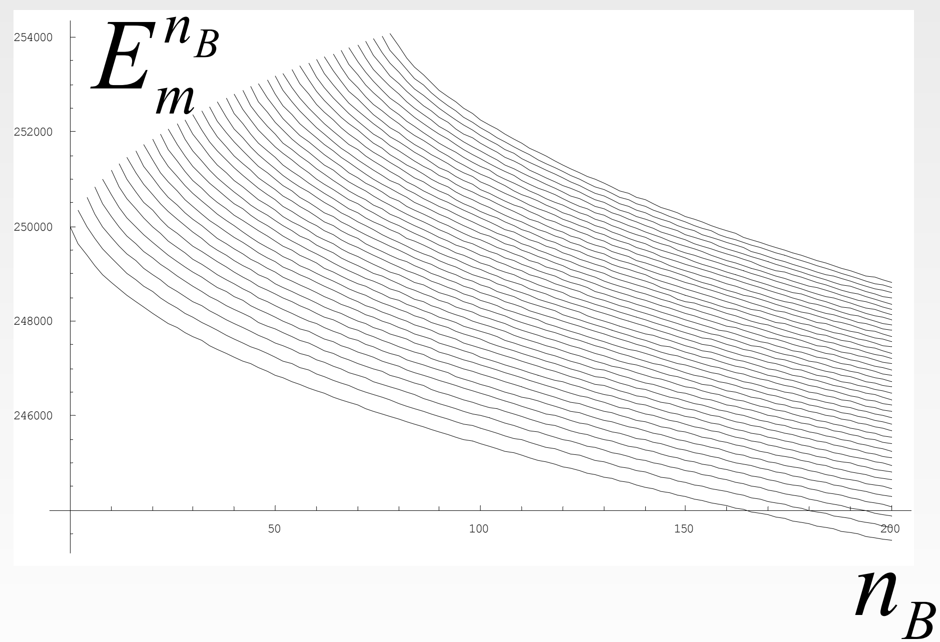
Significant states at large N:

$$(aa)^i\left|0\right\rangle,\ (aaa)\left|0\right\rangle,\quad \ldots,(\underbrace{a\ldots a}_{N})\left|0\right\rangle$$

$\Downarrow$ Lüscher 83’

$$(aa)^i\left|0\right\rangle + \quad \text{D=4 indices}$$





Summary

-SYMQM has many faces- supersymmetry laboratory, QCD, strings

-Cutoff study of hamiltonian gives reachable, exact results (spectrum, wave functions ect.)

-the exact, large SU(N) limit as well as D=10 is not out of question