Interaction between kink and radiation in ϕ^4 Model

Negative radiation pressure and fractals

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Outline

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Let us consider real scalar field theory in 1+1 dim:

$$\ddot{\phi} - \phi'' + 2\phi\left(\phi^2 - 1\right) = 0.$$

The well known kink solution:

$$\phi_s(x) = \tanh x$$

can be perturbed with a small field $\phi(x,t) = \phi_s(x) + \xi(x,t)$.
We can write an equation for ξ:

$$\ddot{\xi} + \hat{L}\xi + N(\xi) = 0,$$

where

$$\hat{L} = -\frac{\partial^2}{\partial x^2} + 2\left(3\tanh^2 x - 1\right)$$

and

$$N(\xi) = 6\phi_s\xi^2 + 2\xi^3.$$

Neglecting the nonlinear part we seek solutions in the form $\xi = e^{i\omega_k t}\eta_k(x)$, where $\omega_k^2 = k^2 + 4$.

Spectral structure of the solutions

- one translational mode $\phi_s(x+\delta x) = \phi_s(x) + \delta x \eta_t(x) + \mathcal{O}(\delta x^2)$

$$\eta_t(x) = \frac{1}{\cosh^2 x}, \quad \omega = 0, \ (k = 2i)$$

- one discrete mode

$$\eta_d(x) = \frac{\tanh x}{\cosh x}, \quad \omega_d = \sqrt{3}, \ (k = i)$$

- continues spectrum of scattering modes (radiation):

$$\eta_k(x) = e^{ikx}\left(3\tanh^2 x - 3ik\tanh x - 3 - k^2\right)$$

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- We can excite the oscillational mode causing
  - radiation and decay of this mode for small amplitude [Manton]
  - creation of two kinks and radiation
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Let the perturbation ξ has a form:

$$\xi(x,t) = \frac{1}{2} A\eta_q(x) e^{i\omega_q t} + c.c.$$

Question

How will the kink behave?

Answer (?) The kink will be pushed by the radiation

We simulate numerically the partial equation with conditions:

$$\phi(x,t=0) = \phi_s(x), \quad \dot{\phi}(x,t=0) = 0$$

$$\phi(-L,t) = 0, \quad \phi(L,t) = A\sin\omega_q t$$

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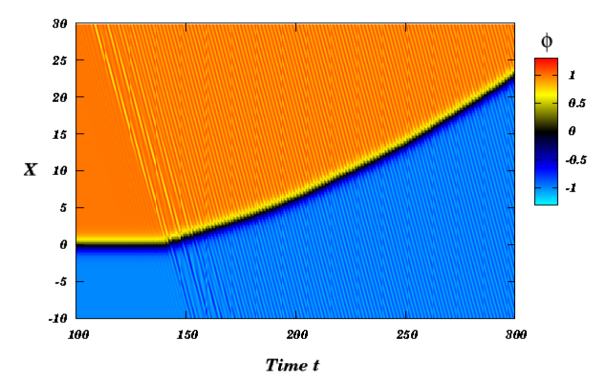
Kink motion in the field of radiation

$A = 0.20$, $\omega_q = 4.0$



Surprise!

The kink is going toward the source of radiation!

We have ”negative radiation pressure”.

The same happens to sine-Gordon model.

But this is not a generic feature. Most models behave ”properly”:

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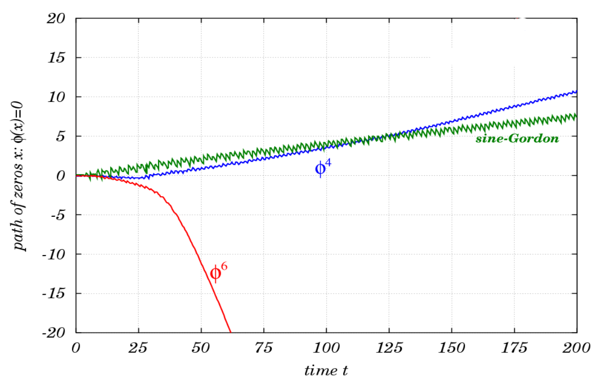

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What is so special in these theories?

The scattering modes have no reflection part

$$\eta_k(x) = e^{ikx}\left(3\tanh^2 x - 3ik\tanh x - 3 - k^2\right) \approx Be^{ikx} \ (x \to \pm\infty)$$

kinks are transparent in linear approximation.
The same for sine-Gordon equation (but there is no discrete oscillational mode).

Reflectionless [Bordag]

All reflectionless spectra for potentials

$$V(x) = N(N+1)\tanh^2 x.$$

$N = 2$ for ϕ^4 and $N = 1$ for s-G
$N - 1$ - number of discrete modes

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We move up to the second order and solve the equation for ξ.
We seek the solution in the form of a series:

$$\xi(x,t) = A\xi^{(1)}(x,t) + A^2\xi^{(2)} + \cdots ,$$

where $\xi^{(1)} = e^{i\omega_q t}\eta_q(x) + c.c.$ The equation for $\xi^{(2)}$ is

$$\ddot{\xi}^{(2)} + \hat{L}\xi^{(2)} + \frac{3}{4}\phi_s\left(\eta_q^2 e^{2i\omega_q t} + 2\eta_q\eta_{-q} + \eta_{-q}e^{-2i\omega_q t}\right) = 0.$$

This is inhomogeneous linear partial equation. We can find a solution in a form

$$\xi^{(2)} = \xi^{(2)}_{+2}(x)e^{2i\omega_q t} + \xi^{(2)}_{-2}(x)e^{-2i\omega_q t} + \xi^{(2)}_0(x,t).$$

Note that

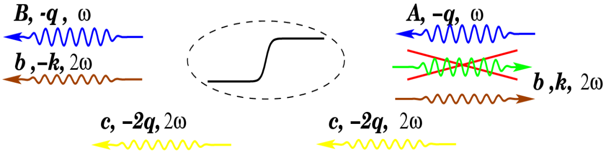
$$\langle \phi_s \eta_q \eta_{-q} | \eta_t \rangle = \langle \phi_s \eta_q^2 | \eta_t \rangle = \langle \phi_s \eta_{-q}^2 | \eta_t \rangle = 0$$

(because of reflectionless spectrum).

We are interested only in a time dependent part, and since we already know the solutions of homogeneous equation we can construct the Green's function.
It is quite easy to get the asymptotic form for large $|x|$:

$$\xi^{(2)}_{\pm\infty}(x,t) = b_{\pm}(\omega_q)\cos(2\omega_q t \mp kx \pm \delta_1) + c(2\omega_q)\cos(2\omega_q t + 2qx \pm \delta_2),$$

where $k = k(q) = \sqrt{(2\omega_q)^2 - 4}$.
The expressions for b and c are complicated but can be calculated analytically.



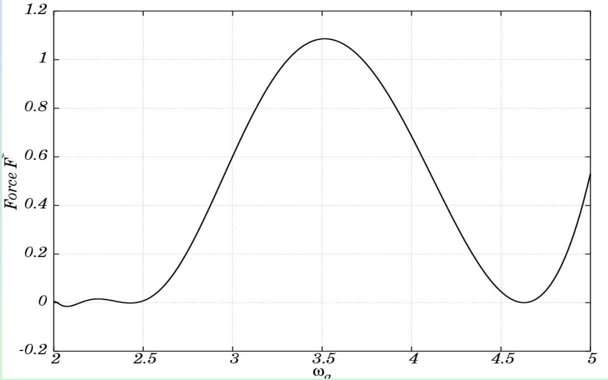
Having these we can write the conservation laws for energy and momentum inside the large segment and averaged over a period:

$$q\omega_q A^2 - q\omega_q B^2 - 2k\omega_q \left(b_-^2 + b_+^2\right) = \frac{dE}{dt} = 2M^* \frac{d\gamma}{dt}$$

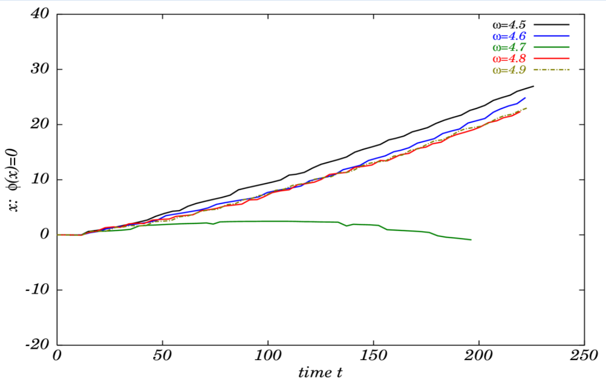
and

$$q^2A^2 - q^2B^2 - k^2\left(b_-^2 - b_+^2\right) = -\frac{dP}{dt} = -F^*$$

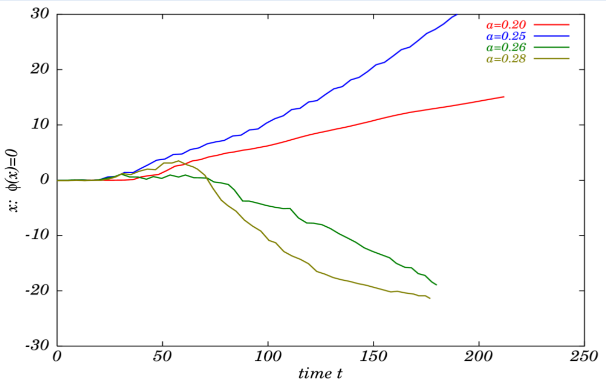
As a solution of this system of equations we obtain the force with which the kink is being pulled by this radiation:



We can test our predictions for the minimum:



But for large amplitudes the higher orders are more important:



Outline

Let us take a closer look at the second possible process:

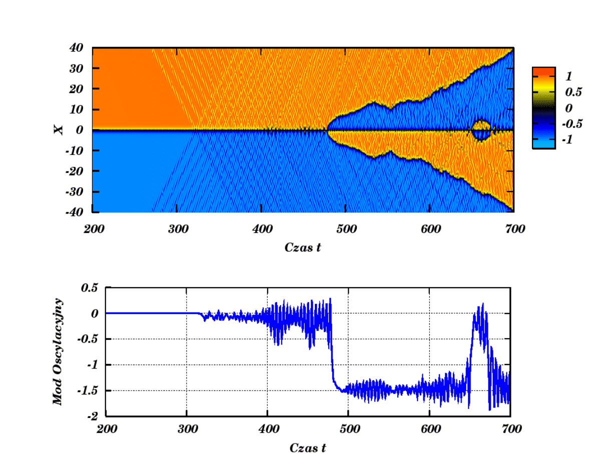
We can simulate this process numerically.
In order to measure the excitation of the osc. mode we calculate the projection onto our mode:

$$A_d(t) = \langle \phi - \phi_s | \eta_d \rangle$$

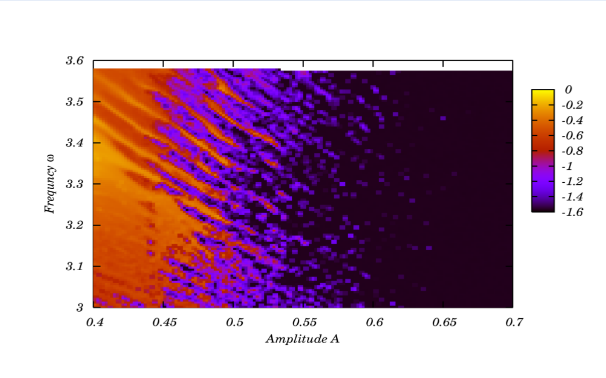
The radiation in linear approx is orthogonal to η_d.
When the creation process occurs instead of a kink an antikink remains and two kinks are radiated out:
What we see is

$$A_d(t) = \langle -2\phi_s | \eta_d \rangle = -\frac{\pi}{2}$$

Example figure:



Finally we can present the figure:

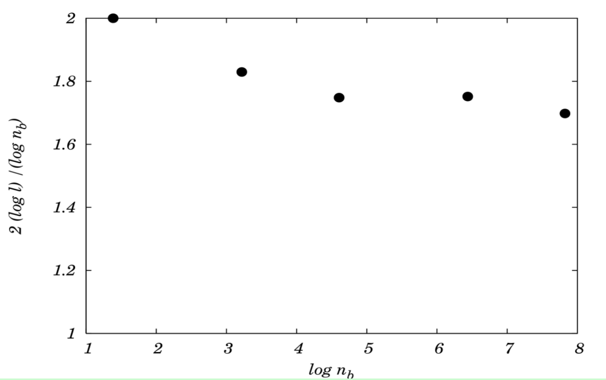


Notice a very complicated (fractal?) critical line for production of defects.

We can measure the fractal diminution:

$$D = \lim_{l \to 0} \frac{2 \log l}{\log n_b},$$

where l - length of a box, n_b -number of boxes containing boundary.



Again let us use the second order equation:

$$\ddot{\xi} - \hat{L}\xi + 6\phi_s\xi^2 = 0$$

but now $\xi(x,t) = Ah_k(x)\cos\omega t + A_d(t)\eta_d(x) + (\eta)$.
We substitute the above to our eq. and project onto the oscillational mode (integrate with η_d):

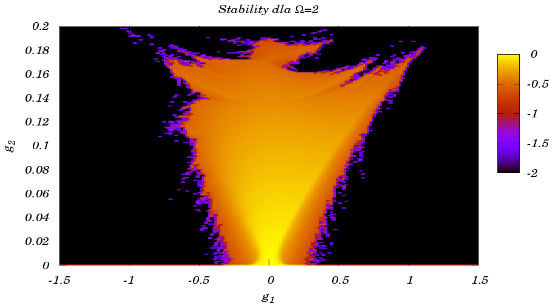
$$\ddot{A}_d + \omega_d^2 A_d + \alpha(k)A^2\cos^2\omega t + \beta(k)AA_d\cos\omega t + \gamma A_d^2 = 0.$$

By rescaling we can see there are only three important coefficients:

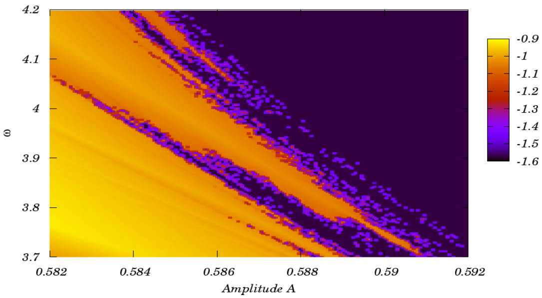
$$\ddot{u} + u + u^2 + g_1 u \cos \Omega t + g_2 \cos^2 \Omega t = 0.$$

The above eq. is similar to the Mathwieu's eq. but with nonlinear term and external force.
The above equations are correct for all double well field th. in 1+1d (the difference is in α and β) where we can expand the potentials into the Taylor's series (exception: compactons).

Therefore it is interesting to investigate this equation for different g_1 and g_2:



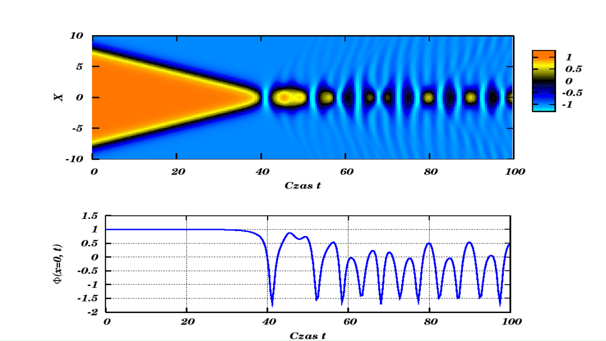
Or finally we can reproduce the figure:



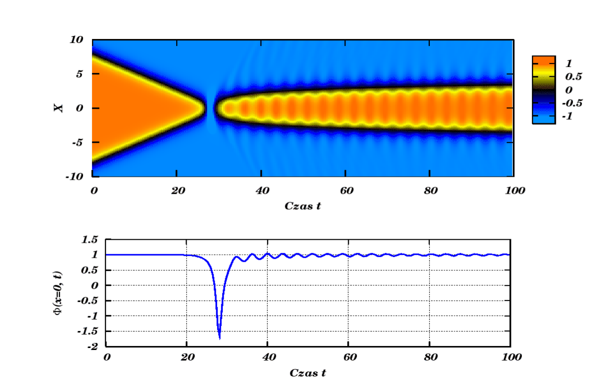
Outline

Matzner studied collision of two kinks.
The kinks can either annihilate creating pseudo-breather (bion, oscillon) or can be scattered back

$$v = 0.17$$



$v = 0.26$


10
5
0
-5
-10
X
0
20
40
60
80
100
Czas t
1
0.5
0
-0.5
-1
1.5
1
0.5
0
-0.5
-1
-1.5
-2
Φ(x=0, t)
0
20
40
60
80
100
Czas t

When v is between 0.18 and 0.26 for certain velocities the kinks can come beck and reflect once more.
We can observe a narrowing series of two-bounce windows
Each two-bounce window is surrounded by series of narrowing three-bounce windows.
Each three-bounce window is...

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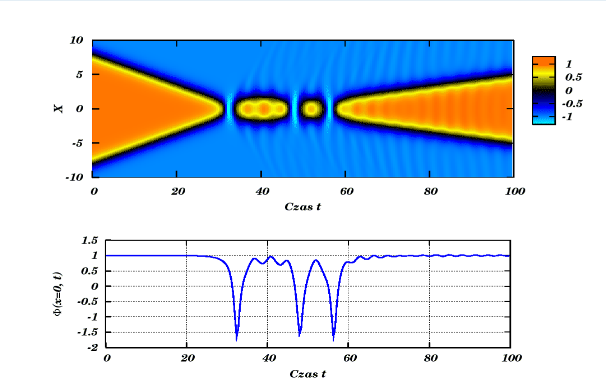
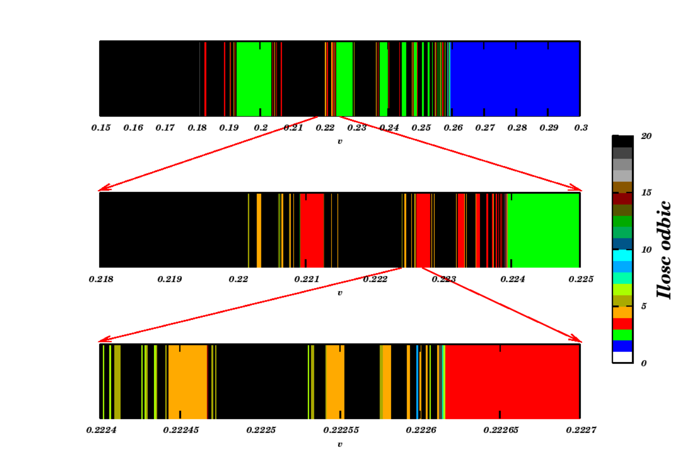
Three-bounce window

$v = 0.221$



nice fractal structure



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