Nonperturbative Yang-Mills from supersymmetry and strings

Or, in the Jungles of Strong Coupling

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☆
Unlike models whose relevance to nature is? QCD will stay with us

QCD is extremely rich:

- Nuclear Physics
- Regge behavior
- QGM: high-T/high \( \mu \) (neutron stars)
- Richness of the hadronic world:
  - chiral;
  - light & heavy quarkonia;
  - glueballs & exotics;
  - exclusive & inclusive phenomena;
  - interplay between strong forces & weak interactions...

That's why I do not expect **FULL** analytic solution to QCD to be found
Give us .... 1973
Give us .... 1979
Give us .... 1985
please...
We beg .... 1991
for a WEAK parameter

Take What's available !!!
SUSY Strings???
QCD

Orientifold

\[ \mathcal{L} = - \frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \frac{1}{g^2} \bar{\Psi}_{[ij]} (i \not\! D) \Psi^{[ij]} \]

Orbifold

\[ L = - \frac{1}{4g^2} \left\{ (G^a_{\mu\nu} G^{\mu\nu a})_e + (G^a_{\mu\nu} G^{\mu\nu a})_m \right\} \\
+ \bar{\Psi}^i_{j_e} D_{\mu} \gamma^{\mu} \Psi^{j_m}_{i_e} \]

SUSY gluodynamics

\[ \mathcal{L} = - \frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu a} + \frac{i}{g^2} \bar{\lambda}^a_{\dot{\alpha}} D^{\dot{\alpha}\beta} \lambda^{a}_{\beta} \]
QCD

SUSY gluodynamics

Orientifold

Orbifold

 gluon e
  gluon m

Orientifolding/orbifolding;
Large $N$ (planar) limit;
Supersymmetry.
Perturbative planar equivalence proved:

S. Kachru & E. Silverstein, 4-D CONFORMAL THEORIES AND STRINGS ON ORBIFOLDS, 1998
R6 orbifolds + AdS/CFT; from $\mathcal{N}=4$ => distinct (perturbatively) conformal daughters with $\mathcal{N}<4$. Nonpert. hit tachyons!!

A. Lawrence, N. Nekrasov & C. Vafa, ON CONFORMAL FIELD THEORIES IN FOUR-DIMENSIONS, 1998

M. Bershadsky, Z. Kakushadze, Vafa, STRING EXPANSION AS LARGE $N$ EXP. OF GAUGE THEORIES, ‘98


M. Schmaltz, DUALITY OF NONSUPERSYMMETRIC LARGE $N$ GAUGE THEORIES, 1998

A. Armoni and B. Kol, Type-0 String Description of Schmaltz’ conjecture

(Exact planar $\beta$ functions)$^\text{TM}$

The question of non-perturbative planar equivalence (SUSY$\leftrightarrow$non-SUSY)$^\text{TM}$ raised:

M. Strassler, ON METHODS FOR EXTRACTING EXACT NONPERTURBATIVE RESULTS IN NONSUPERSYMMETRIC GAUGE THEORIES, 2001
Nonperturbative equivalence holds if and only if $\mathbb{Z}_2$ symmetry $e \leftrightarrow m$ is NOT broken spontaneously!

But, in fact, $\mathbb{Z}_2$ symmetry $e \leftrightarrow m$ IS broken spontaneously!
Domain walls & vacuum structure:

SU(2N) SYM parent
SU(N) x SU(N) daughter

○ Vacua of the parent theory
× Vacua of the daughter theory

2-wall (parent) -> e-wall+m-wall (daughter)
1-wall (parent) -> e-wall OR m-wall (unstable!)

e-wall OR m-walls decay into ◼ - ○ walls in the twisted sector which has no prototype in the parent theory!

Genuine vacua in the orbifold theory; $T = F_e^2 - F_m^2$

$<T> = +$
$<T> = -$
Gravitational anomaly (mis)match:

$$\partial_\mu A^\mu = -\frac{x}{192\pi^2} R_{\mu\nu\kappa\lambda} \tilde{R}^{\mu\nu\kappa\lambda}$$

If gluon parts of the anomaly are normalized appropriately,

$$x_D/x_P = \sqrt{2}$$

$Z_2$ is broken. The obvious order parameter is $T \equiv (Tr F^2_e - Tr F^2_m)$

Remarkably, there is another, less obvious order parameter (to leading order in $1/N$):

$$\theta^\mu_\mu = -\frac{3N}{32\pi^2} \sum_{\ell=e,m} (F^a_{\mu\nu} F^a_{\mu\nu})_\ell$$
Instanton-antiinstanton pair is topologically stable...
Orientifold: nonperturbative planar equivalence (at N=3 we have one-flavor QCD)

Common Sector: SUSY$\leftrightarrow$Orienti | Glueballs+bifermions+

Orientifold daughter:

✶ N-2 vacua labeled by

$$<\Psi_R \Psi_L> = -6(N-2)\Lambda^3 e^{2\pi ik/(N-2)} + (1/N \text{ corr.})$$

At N=3 the vacuum is unique
(at $\theta=0$): one-flavor QCD

✶ Both theories confine; only composite color-singlet hadrons in the spectra.

✵✵ Orientifold daughter is NOT supersymmetric:

$$m_B(\text{parent})=O(N^0) \text{ while } m_B(\text{daughter})=O(N^1).$$
Consequences of planar equivalence for orienti at $N = \infty$:

Usually in non-SUSY $\epsilon_{\text{vac}} \sim N^2$;

in orienti $\epsilon_{\text{vac}} \sim N^1$

Infinite number of degeneracies: e.g. $0^+ \& 0^- \mid 1^- \& 0^+ \mid \ldots$;

“BPS” domain walls;

Lightness of $\sigma$; $m_\sigma^2 = m_\eta^2 (1 + O(1/N))$;

Calculable quark condensate.
More generally:

- **Parent:** $k$ “flavors” of adjoint Majoranas
- **Daughter:** $k$ flavors of $\psi[ij]$’s

A new “orientifold” large $N$ expansion

\[ \frac{\Gamma_{\text{gl}}}{\Gamma_{\text{qu}}} \sim N^{-1} \]

\[ \frac{\Gamma_{\text{gl}}}{\Gamma_{\text{qu}}} \sim N^{0} \]

_The same at $N=3$!_
one-flavor QCD

N=3

O(1/3)

orienti

‘t Hooft

pure YM

Remnants of SUSY in pure Yang-Mills?
Conclusions:

★ SUSY gluodynamics is planar equivalent to non-SUSY orienti;

★ At $N=3$ we get one-flavor QCD;

★ Analytic predictions: spectral degeneracies, condensates, $\epsilon_{\text{vac}} \sim N^1$

★ Orientifold large-$N$ expansion (some ideas regarding diquarks; still to be explored!)
Conclusions (second): It's the right time to start ...