Nonperturbative Yang-Mills from supersymmetry and strings

Or, in the Jungles of Strong Coupling

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with Adi Armoni

Unlike models whose relevance to nature is ? QCD will stay with us

QCD is extremely rich:

- ★ Nuclear Physics
  - ★ Regge behavior
    - ★ QGM: high-T/high μ (neutron stars)
      - ★ Richness of the hadronic world:

- ★ chiral;
- ★ light & heavy quarkonia;
- ★ glueballs & exotics;
- ★ exclusive & inclusive phenomena;
- ★ interplay between strong forces & weak interactions...

That's why I do not expect **FULL** analytic solution to QCD to be found

Give us 1973

Give us 1979

Give us.... 1985

please... 1991

We beg 2001

for a WEAK parameter

Take What's available !!!

SUSY Strings???

QCD

Orientifold

$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} + \frac{1}{g^2} \bar{\Psi}_{[ij]} \, (i \not{D}) \, \Psi^{[ij]}$$

Orbifold

$$L = -\frac{1}{4g^2} \left\{ \left(G^a_{\mu\nu} G^{\mu\nu\, a} \right)_e + \left(G^a_{\mu\nu} G^{\mu\nu\, a} \right)_m \right\}$$
$$+ \bar{\Psi}^{j_m}_{i_e} D_\mu \gamma^\mu \Psi^{i_e}_{j_m}$$

SUSY gluodynamics

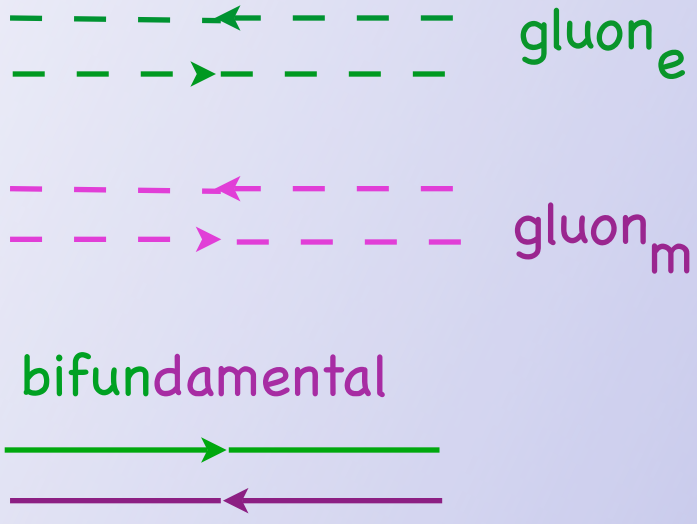
$$\mathcal{L} = -\frac{1}{4g^2} G^a_{\mu\nu} G^{\mu\nu\, a} + \frac{i}{g^2} \bar{\lambda}^a_{\dot{\alpha}} D^{\dot{\alpha}\beta} \lambda^a_\beta$$

- QCD

- SUSY gluodynamics

- Orientifold

- Orbifold



- ☆ Orientifolding/orbifolding;
- ☆ Large N (planar) limit;
- ☆ Supersymmetry.

Perturbative planar equivalence proved:

S. Kachru & E. Silverstein, **4-D CONFORMAL THEORIES AND STRINGS ON ORBIFOLDS, 1998**

R6 orbifolds + AdS/CFT; from $\mathcal{N}$=4 => distinct (perturbatively) conformal daughters with $\mathcal{N}$<4. Nonpert. hit tachyons!!

A.Lawrence, N.Nekrasov & C.Vafa, **ON CONFORMAL FIELD THEORIES IN FOUR-DIMENSIONS, 1998**

M.Bershadsky, Z.Kakushadze, Vafa, **STRING EXPANSION AS LARGE *N* EXP. OF GAUGE THEORIES, '98**

M.Bershadsky, a. Johansen, **LARGE *N* LIMIT OF ORBIFOLD FIELD THEORIES, 1998**

M.Schmaltz, **DUALITY OF NONSUPERSYMMETRIC LARGE *N* GAUGE THEORIES, 1998**

A. Armoni and B. Kol, **Type-0 String Description of Schmaltz' conjecture**

(Exact planar ß functions)™

The question of non-perturbative planar equivalence (SUSY<->non-SUSY)™ raised:

M.Strassler, **ON METHODS FOR EXTRACTING EXACT NONPERTURBATIVE RESULTS IN NONSUPERSYMMETRIC GAUGE THEORIES, 2001**

Z_2 Orbifold

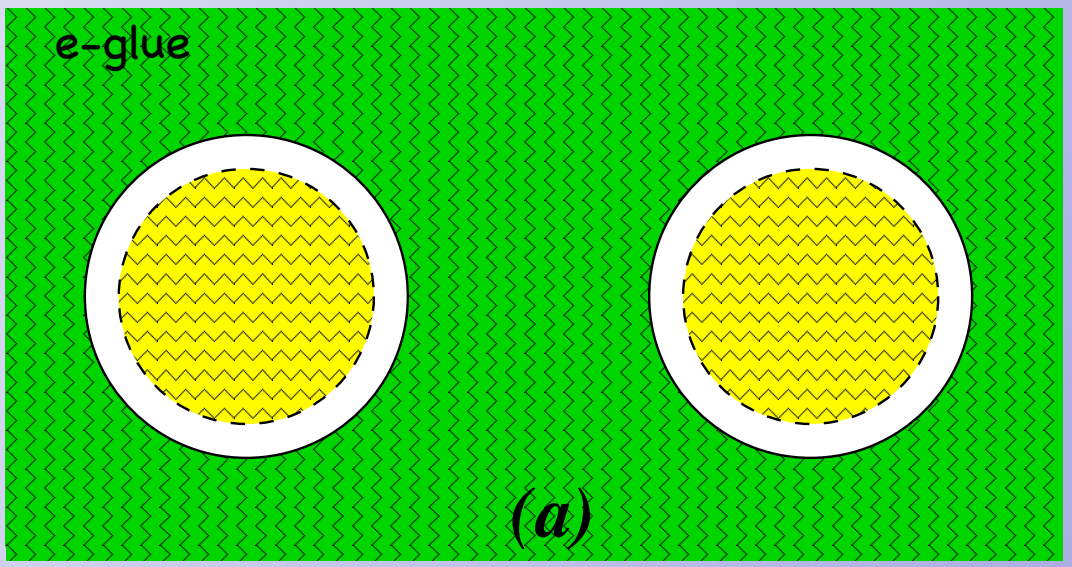
(Z_N Orbifolds are nonchiral!)

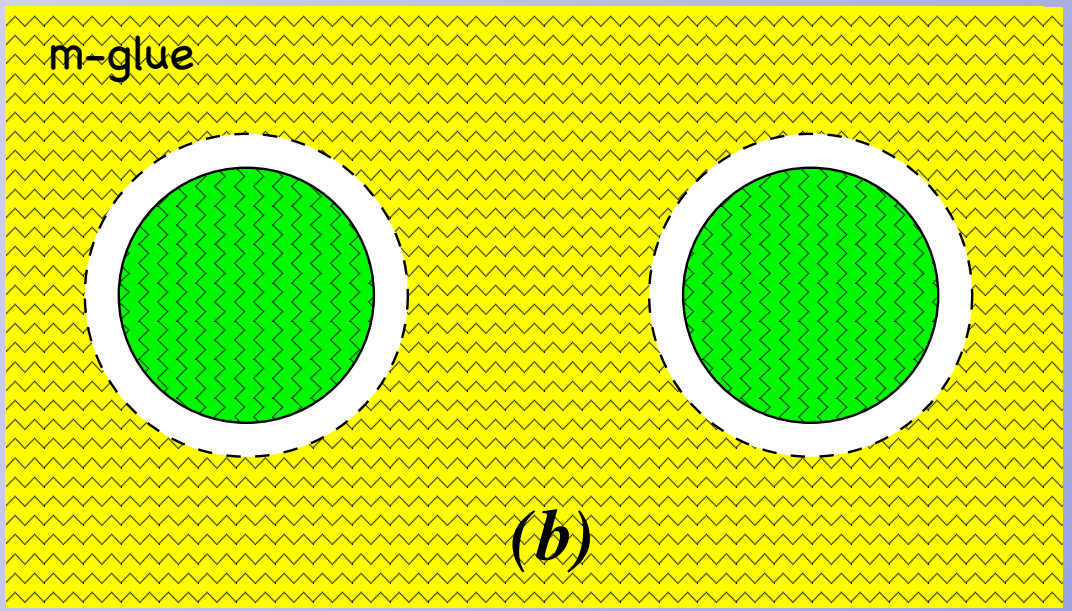
Nonperturbative equivalence holds if and only if Z_2

symmetry e<->m is NOT broken spontaneously!

But, in fact, Z_2

symmetry e<->m IS broken spontaneously!

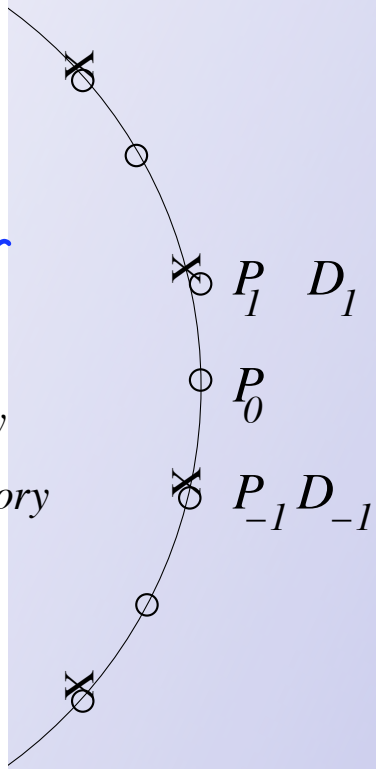




Domain walls & vacuum structure:

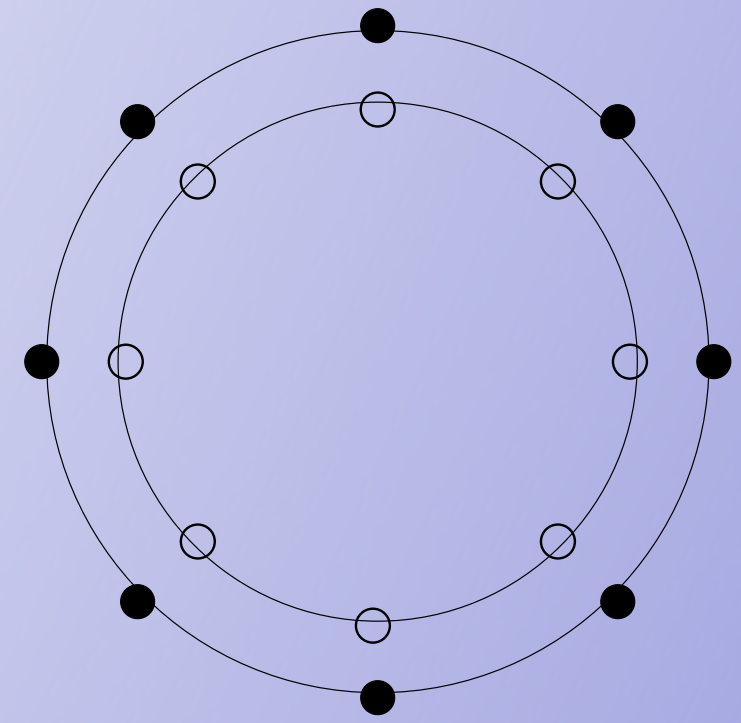
Genuine vacua in the orbifold theory; $T = F_e^2 - F_m^2$

SU(2N) SYM parent
SU(N)xSU(N) daughter



○ *Vacua of the parent theory*

✕ *Vacua of the daughter theory*



2-wall (parent) -> e-wall+m-wall (daughter)
1-wall (parent) -> e-wall OR m-wall (unstable!)

● $\langle T \rangle = +$ ○ $\langle T \rangle = -$

e-wall OR m-walls decay into ●-○ walls in the twisted sector which has no prototype in the parent theory!

T is the tachyonic operator in dual type-0 string theory

Gravitational anomaly (mis)match:

$$\partial_\mu A^\mu = -\frac{x}{192\pi^2} R_{\mu\nu\kappa\lambda}\tilde{R}^{\mu\nu\kappa\lambda}$$

If gluon parts of the anomaly are normalized appropriately,

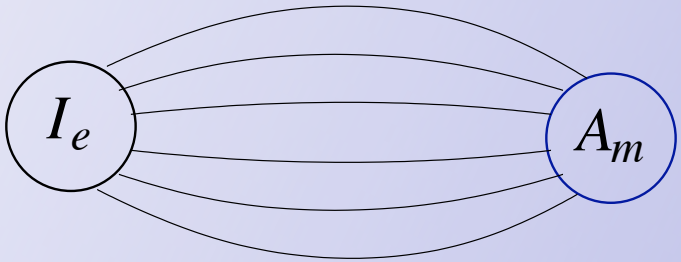
$$x_D/x_P = \sqrt{2}$$

Z_2 is broken. The obvious order parameter is $T \equiv \left(Tr F_e^2 - Tr F_m^2\right)$

Remarkably, there is another, less obvious order parameter (to leading order in 1/N):

$$\theta_\mu^\mu = -\frac{3N}{32\pi^2} \sum_{\ell=e,m} \left(F_{\mu\nu}^a F_{\mu\nu}^a\right)_\ell$$

Side remark:



Instanton-antiinstanton pair is topologically stable...

Orientifold: nonperturbative planar equivalence (at N=3 we have one-flavor QCD)

Common Sector: SUSY$\leftarrow\rightarrow$Orienti | Glueballs+bifermions+...

Orientifold daughter:

✵ N-2 vacua labeled by

$\langle \Psi_R \Psi_L \rangle = -6(N-2)\Lambda^3 e^{2\pi i k/(N-2)} + (1/N \text{ corr.})$

At N=3 the vacuum is unique (at $\theta=0$): one-flavor QCD

✵ Both theories confine; only composite color-singet hadrons in the spectra.

✵✵ Orientifold daughter is NOT supersymmetric: $m_B(\text{parent}) = O(N^0)$ while $m_B(\text{daughter}) = O(N^1)$.

Consequences of planar equivalence for orienti at $N = \infty$:

Usually in non-SUSY $\epsilon_{vac} \sim N^2$;

in orienti $\epsilon_{vac} \sim N^1$

Infinite number of degeneracies: e.g. 0^+ & 0^- | 1^- & 0^+ | ...;
"BPS" domain walls;

Lightness of σ; $m_\sigma^2 = m_\eta^2 (1+O(1/N))$;

Calculable quark condensate.

More generally:

- Parent: k "flavors" of adjoint Majoranas
- Daughter: k flavors of $\Psi^{[ij]}$'s

A new "orientifold" large N expansion

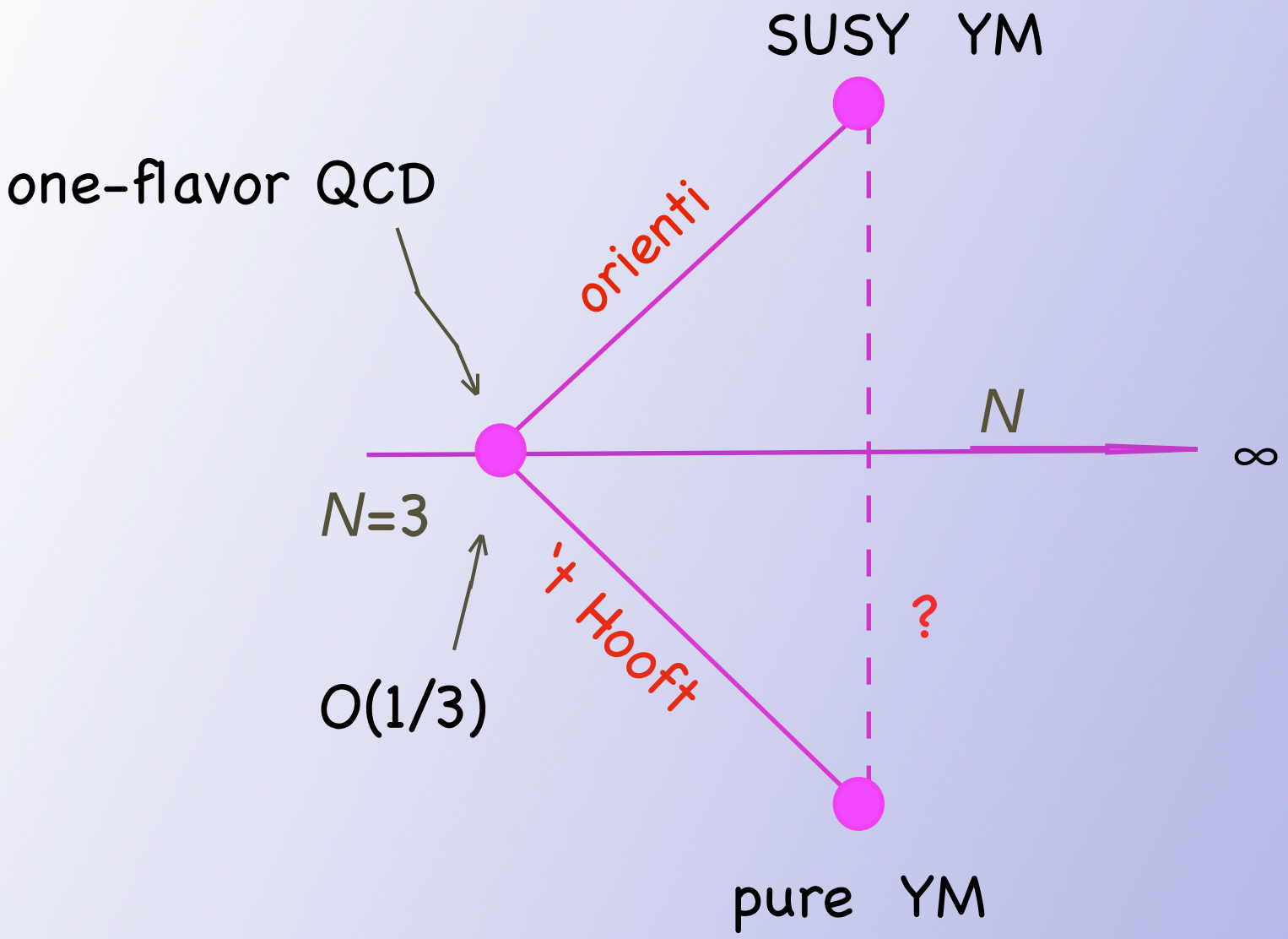
't Hooft: fundamental Dirac quarks at all N

$\Gamma_{gl}/\Gamma_{qu} \sim N^{-1}$

orineti: Dirac $\psi^{[ij]}$ at all N

$\Gamma_{gl}/\Gamma_{qu} \sim N^{0}$

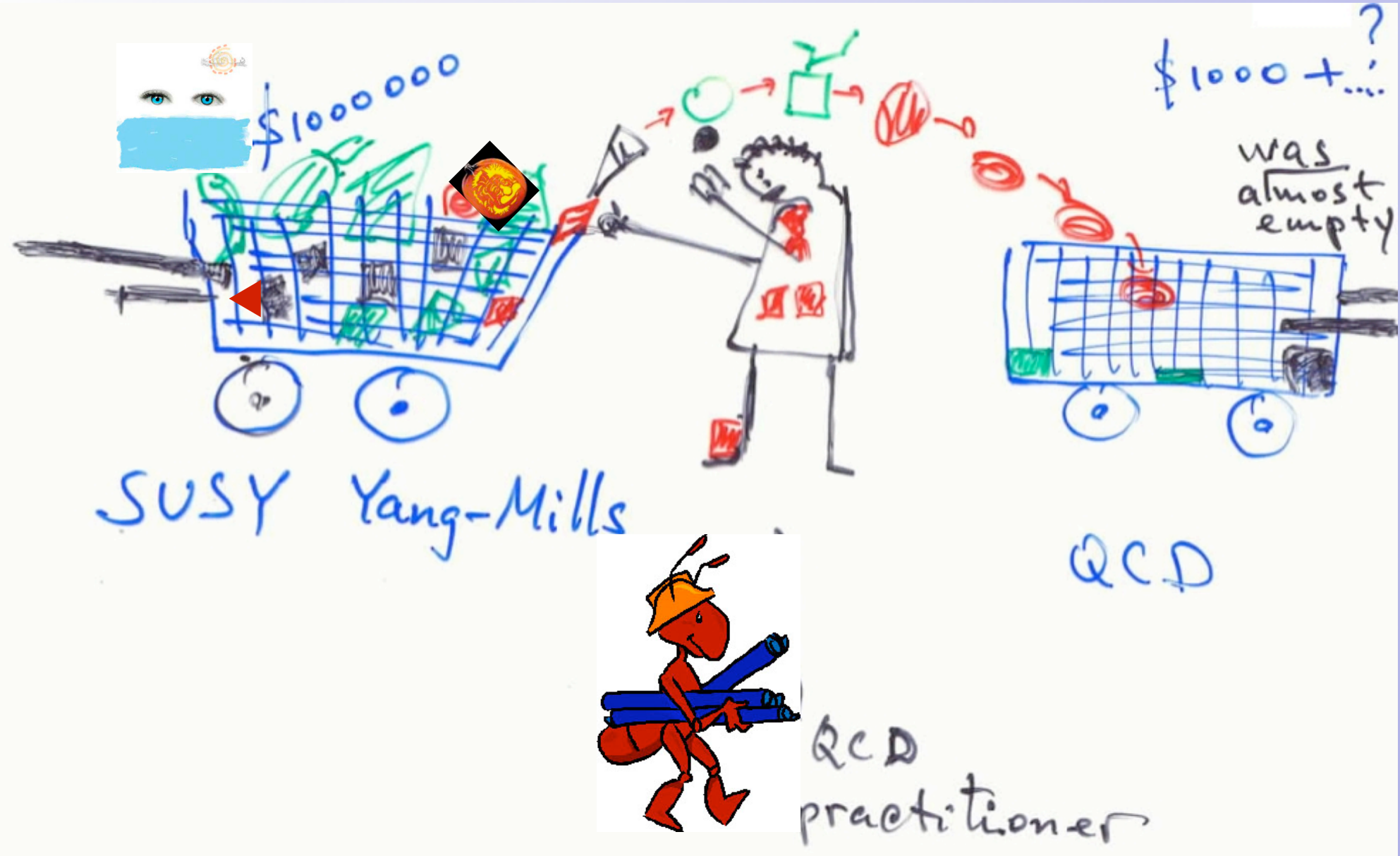
The same at N=3!



Remnants of SUSY in pure Yang-Mills?

Conclusions:

- SUSY gluodynamcs is planar equivalent to non-SUSY orienti;
- At N=3 we get one-flavor QCD;
- Analytic predictions: spectral degeneracies, condensates,... $\epsilon_{vac} \sim N^{1}$
- Orientifold large-N expansion (some ideas regarding diquarks; still to be explored!



Conclusions (second): It's the right time to start ...