k-Strings from Various Perspectives: QCD, Lattices, String Theory, and Toy Models

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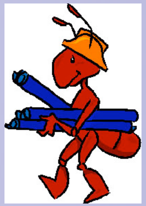
Ringberg Castle

Tegernsee

Understanding Confinement /Zakharovfest

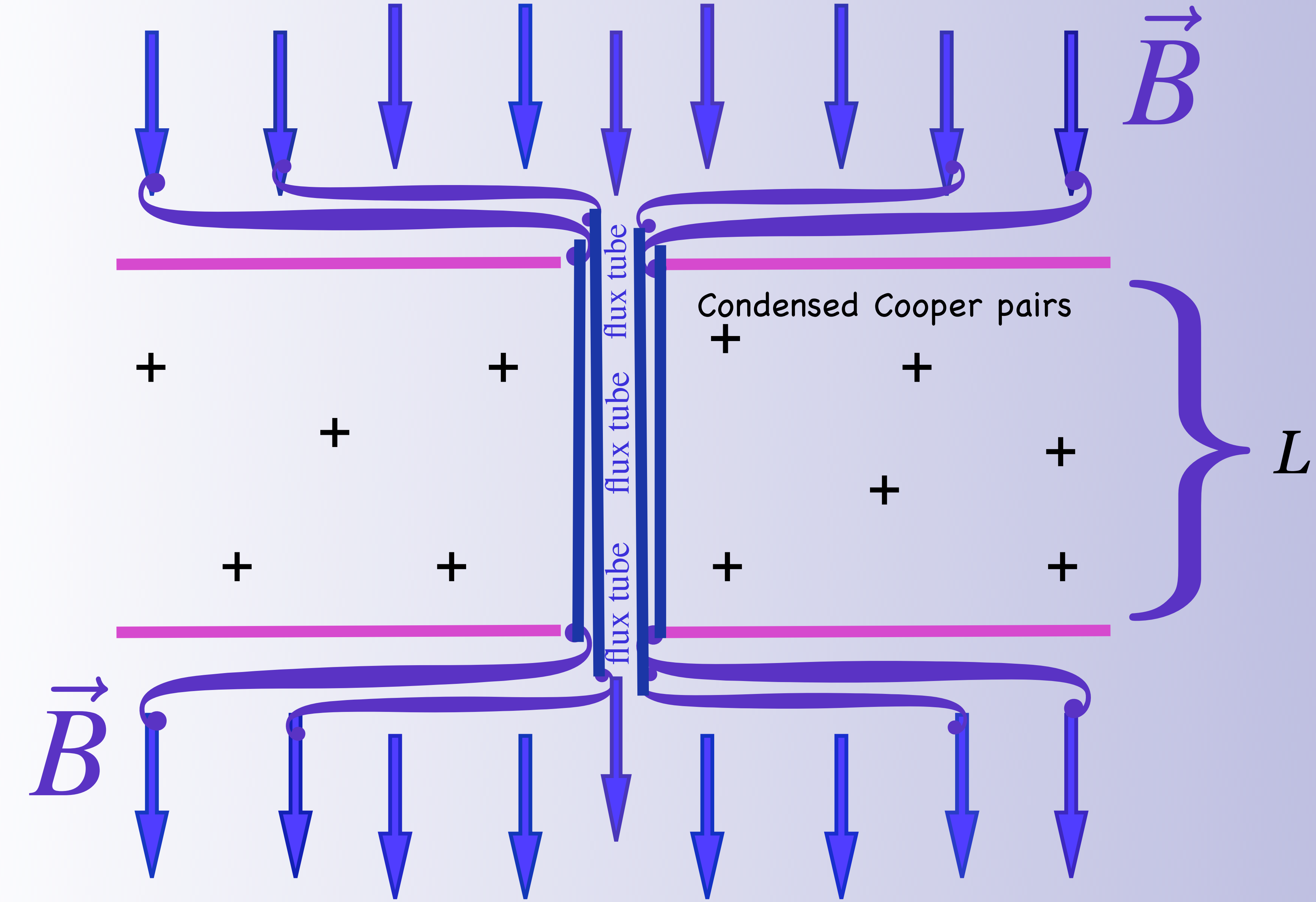


Outline

- Reviewing QCD Strings
- QCD Strings and 1/N expansion
- QCD Strings and SUSY
- Modeling k-Strings at Weak Coupling

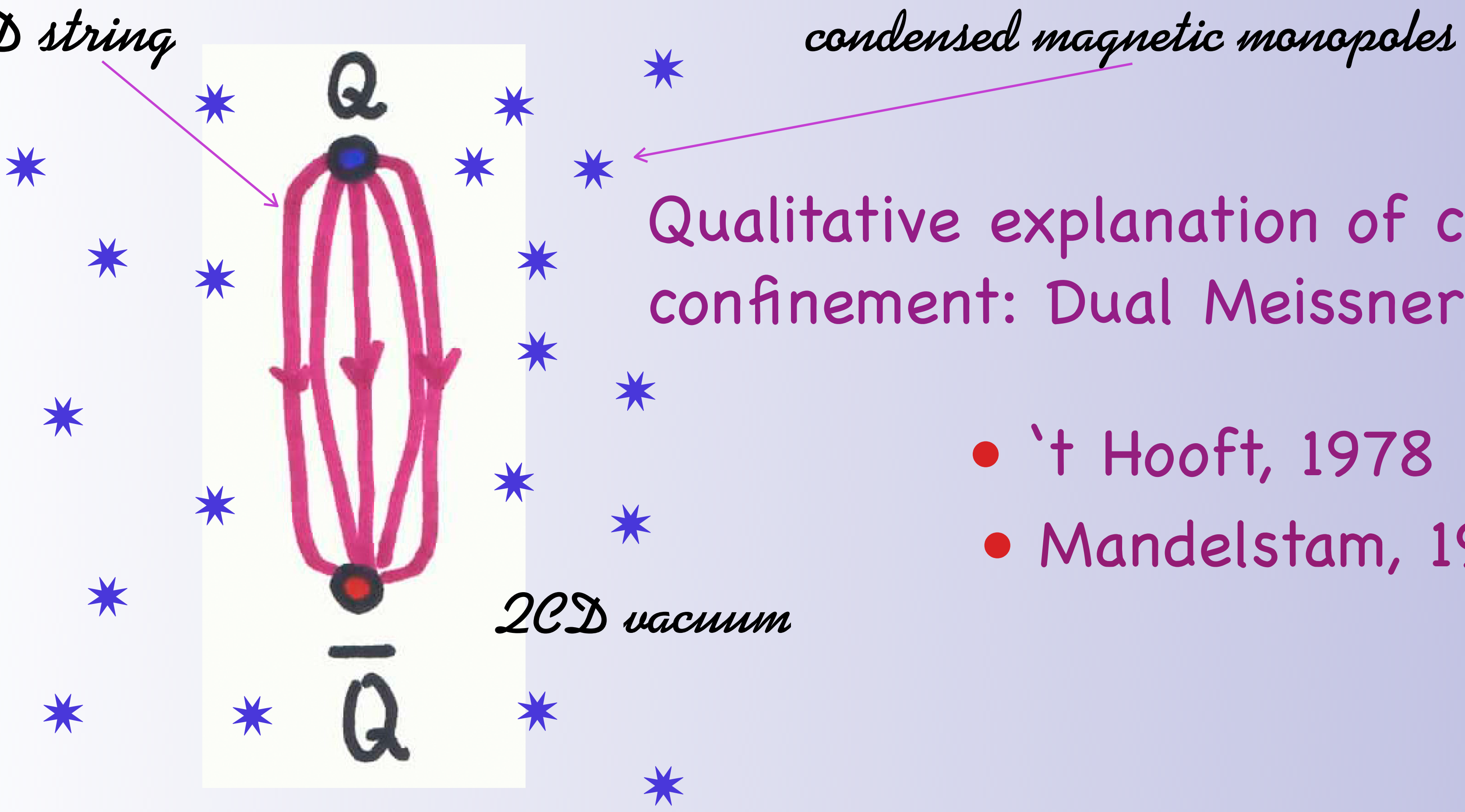
QCD, lattices, ... 1970's-2000
Adi Armoni & MS, 2003
Alyosha Yung & MS, 2004 & 2005
(the last part of the talk)

Well-known strings -- Abrikosov vortices in superconductors



$\vec{B} \times \text{area} = \Phi \qquad \leftarrow$ quantized

$\text{Energy} = \sigma \cdot L$



Qualitative explanation of color confinement: Dual Meissner effect:

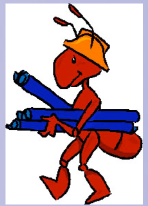
- ‘t Hooft, 1978
- Mandelstam, 1978

- Seiberg & Witten, 1994, + power of N=2 SUSY confirmed monopole condensation and Dual Meissner effect!!!

⋆ At $\mathrm{Tr}\,\phi^2 \gg \Lambda^2$ clear-cut monopoles

✩ Trace their evolution as Tr φ^2 diminishes

⋆ “Former” monopoles condense at $\mathrm{Tr}\,\phi^2 \sim \Lambda^2$

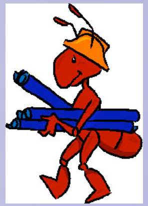


- Seiberg-Witten strings/confinement Abelian!

$\star$ Douglas & Shenker: weird "hadronic" spectrum; $N/2$ Regge trajectories, instead of 1 $\star$

- Qualitatively great! Quantitatively the problem stands ...

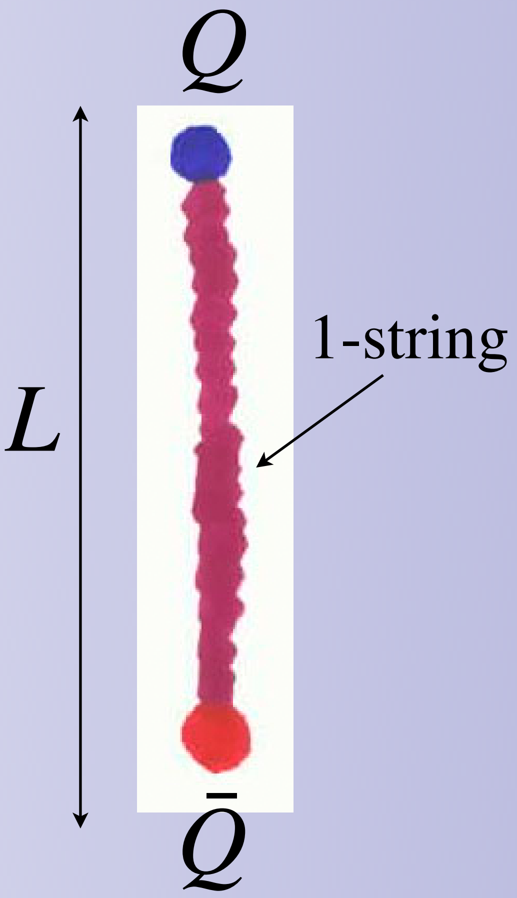
What do we know of QCD strings?



• First analytic idea: 1/N

$$E = \sigma \cdot L$$

$$\sigma \sim N^0 \Lambda^2$$



🔵 Ψ^i

🔵🔵 Ψ^{ij}

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🔵🔵🔴 Ψ^{ij}_{k}

Ψ_i 🔴

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Ψ_{ijk} 🔴🔴🔴

Ψ_i 🔴

|# of 🔵 − # of 🔴| = "*N*-ality"

Defined mod *N*

triality for SU(3)

- N-ality = 1 ⟶ 1-string, N-ality = 2 ⟶ 2-string, etc.
- 1-string ⟶ fundamental

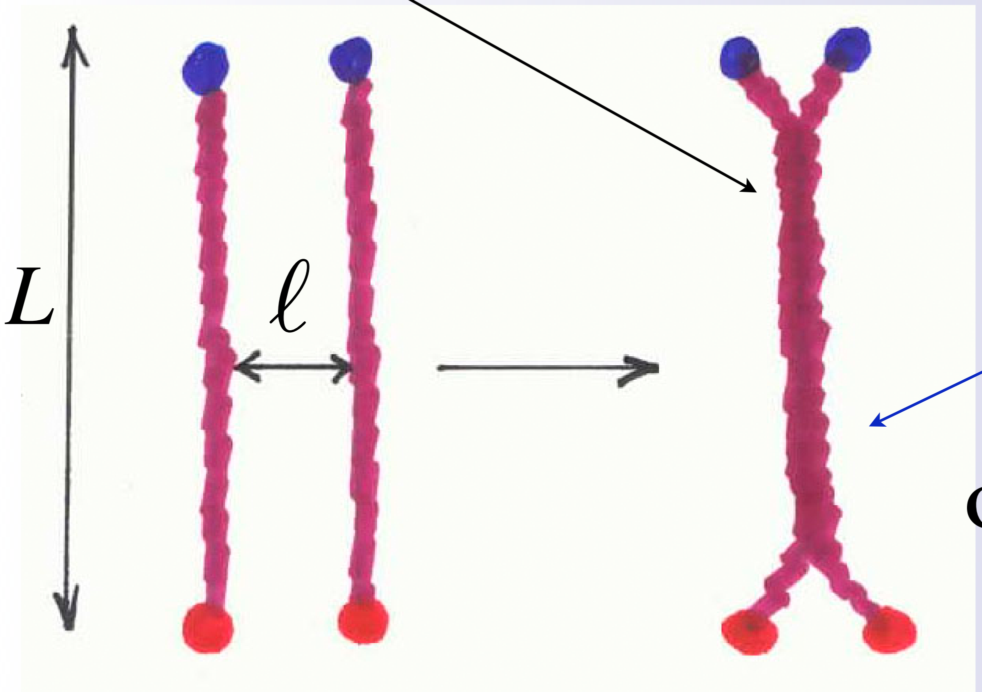
$$\boxed{\sigma_1 \equiv \sigma}$$

SU(N)

$$\sigma_1,\ \sigma_2,\ \sigma_3, \ldots \sigma_{[N/2]}, \ldots ?,$$

$$\sigma_k/\sigma = f(k) = \ ?$$

2-string

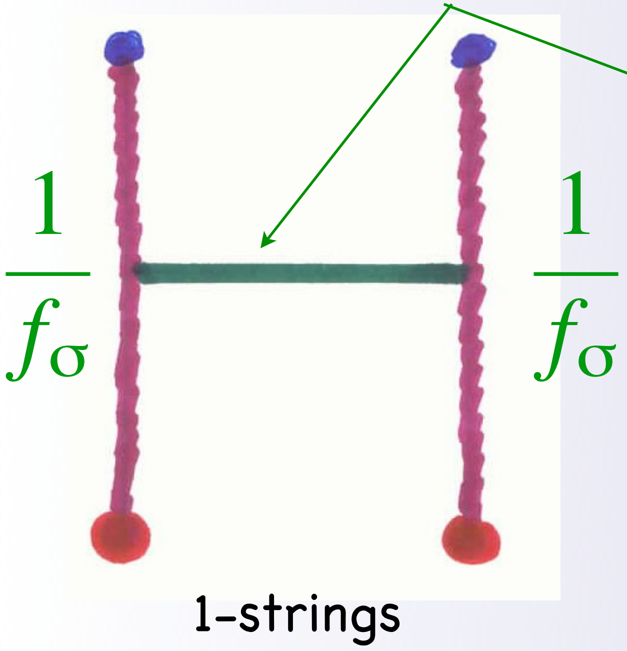


- Start at $\ell \gg \Lambda^{-1}$

$$\sigma_2 = 2\sigma_1 + \text{attraction}$$

At $\ell \sim \Lambda^{-1}$

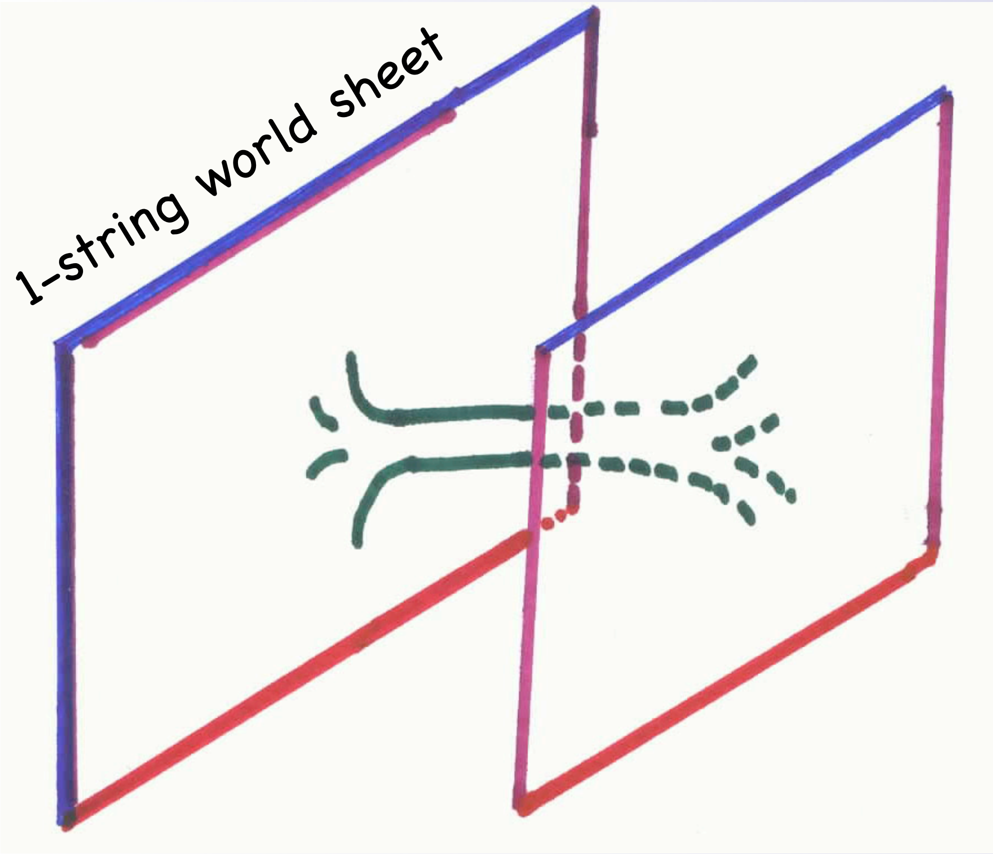
$$\sigma_2 = 2\,\sigma_1 \cdot \begin{cases} 1 - O(1/N) \ \text{or} \\ 1 - O(1/N^2) \end{cases} \quad ?$$



Field theory side:

σ meson ...(the lightest)

$$\text{Interaction} \propto \frac{1}{f_\sigma^2} \propto \frac{1}{N^2}$$

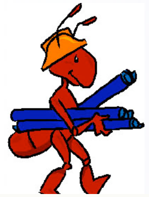


String theory side:

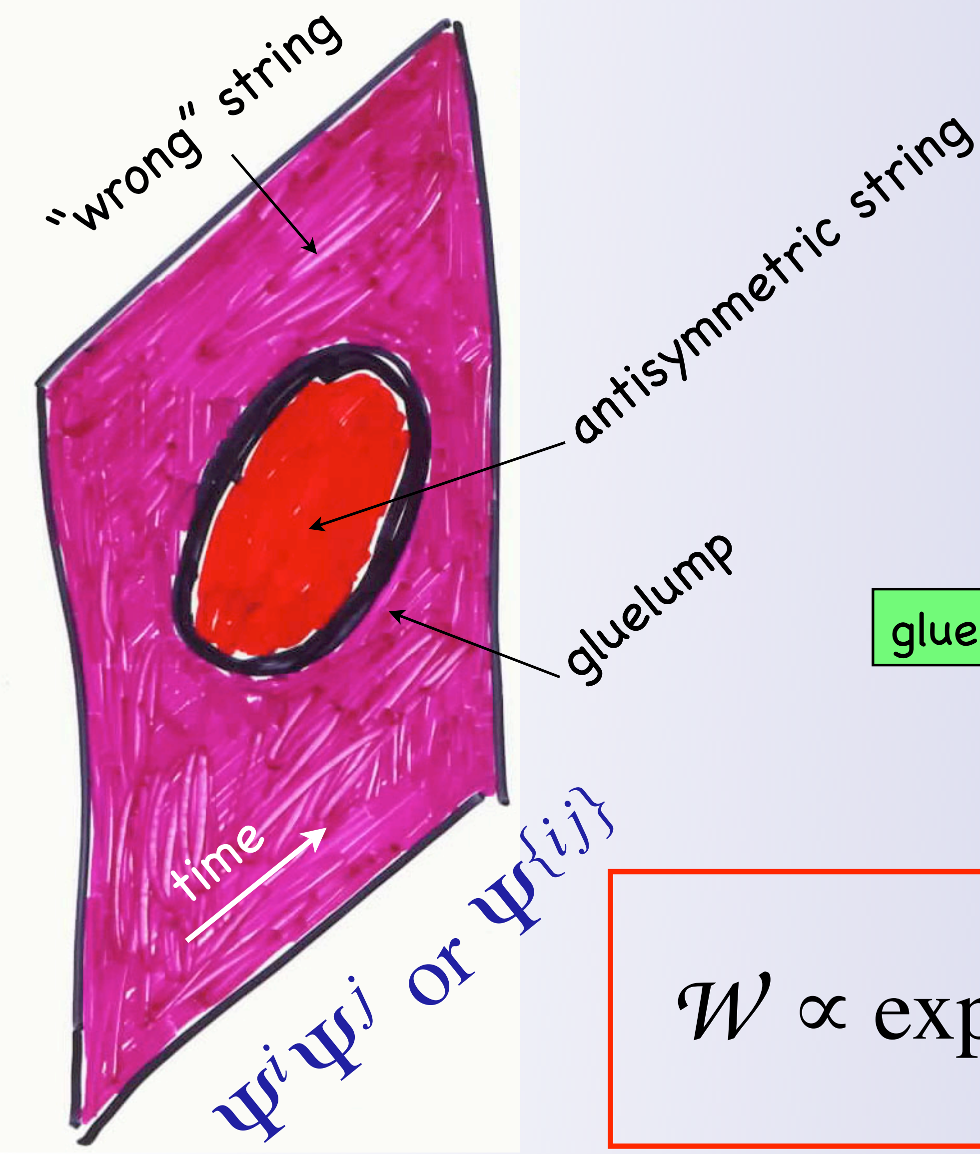
$$\text{Annulus diagram} \propto g_{str}^2 \propto \frac{1}{N^2}$$

Always attraction

Lesson from the above: $\Delta\sigma \sim \frac{1}{N^2}\Lambda^2$ (binding and splittings)



Quasi-Stability



- Consider string world sheet
- False vacuum decay!

$$\Lambda L \sim \Delta\sigma L^2$$

gluelump boundary

inside of the "bubble"

$$L \sim \Lambda^{-1} N^2 \to \infty$$

$$\boxed{\mathcal{W} \propto \exp\left(-\frac{\Lambda^2}{\Delta\sigma}\right) \sim e^{-N^2}}$$

Quasiclassical tunneling

The second lesson from the above:

Casimir scaling $\xcancel{\sigma_k = \sigma k \left(1 - \frac{k-1}{N-1}\right)}$

although popular, has to go!

What can replace it?

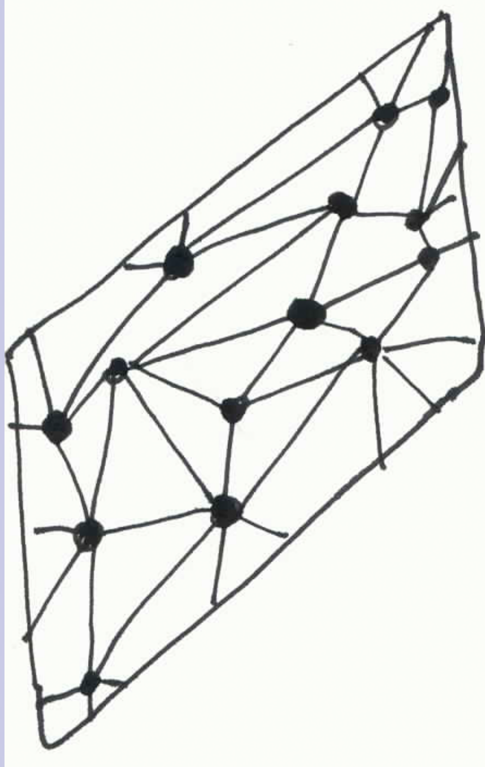
✶ QCD Strings and SUSY →

The sine law: $$\sigma_k = N\Lambda^2 \sin\left(\frac{\pi k}{N}\right)$$

- Douglas-Shenker, 1995 (from SW N=2)
- Hanany-Strassler-Zafaroni, 1997 (from MQCD)
- Herzog-Klebanov, 2002 (from AdS/CFT, Maldacena-Nuñez bckgrd)
- Armoni-MS, 2003 (SUSY gluodynamics domain walls)

If there is one node per Λ^{-2} area of the wall (supported by strings), then the exact wall tension implies

$$T_k = \frac{N}{8\pi^2}\left|\langle\lambda^2\rangle_{n+k} - \langle\lambda^2\rangle_n\right| \rightarrow \text{ Sine formula!}$$

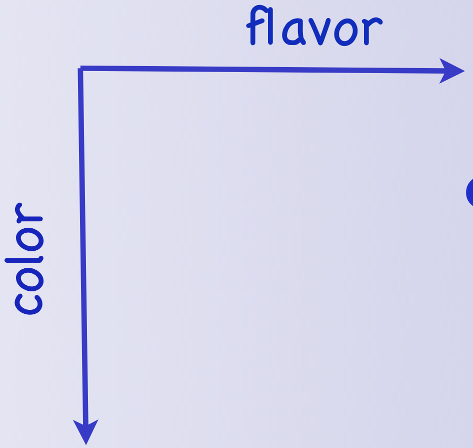


Modeling k-Strings at Weak Coupling: SU(N)xU(1) Yang-Mills

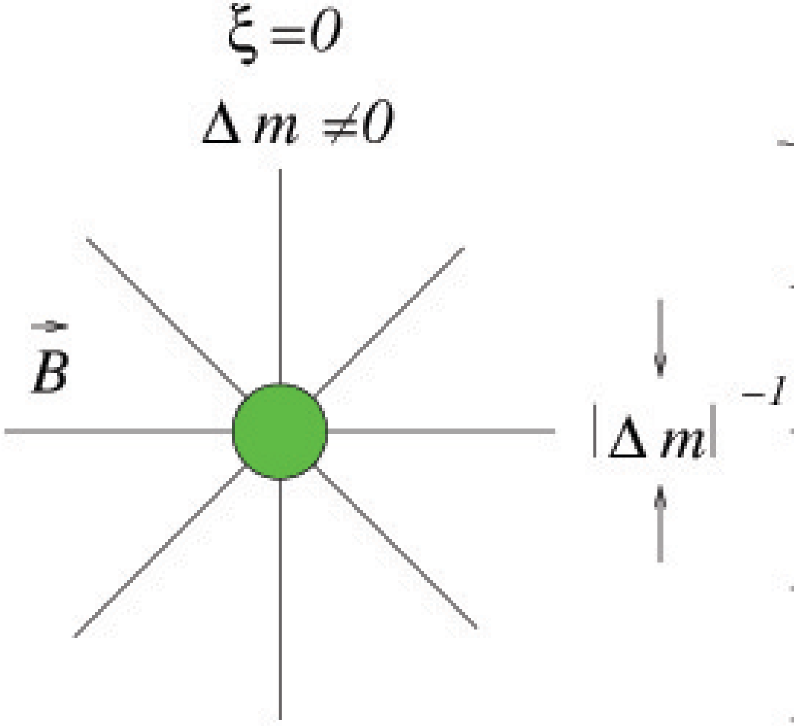
$$S = \int d^4x \left\{ \frac{1}{4g_2^2}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{4g_1^2}\left(F_{\mu\nu}\right)^2 + Tr\left(\nabla_\mu \Phi\right)^\dagger \left(\nabla^\mu \Phi\right) \right.$$

$$\left. + \frac{g_2^2}{2}\left[Tr\left(\Phi^\dagger T^a \Phi\right)\right]^2 + \frac{g_1^2}{8}\left[Tr\left(\Phi^\dagger \Phi\right) - N\xi\right]^2 + \frac{i\theta}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{a\mu\nu} \right\},$$

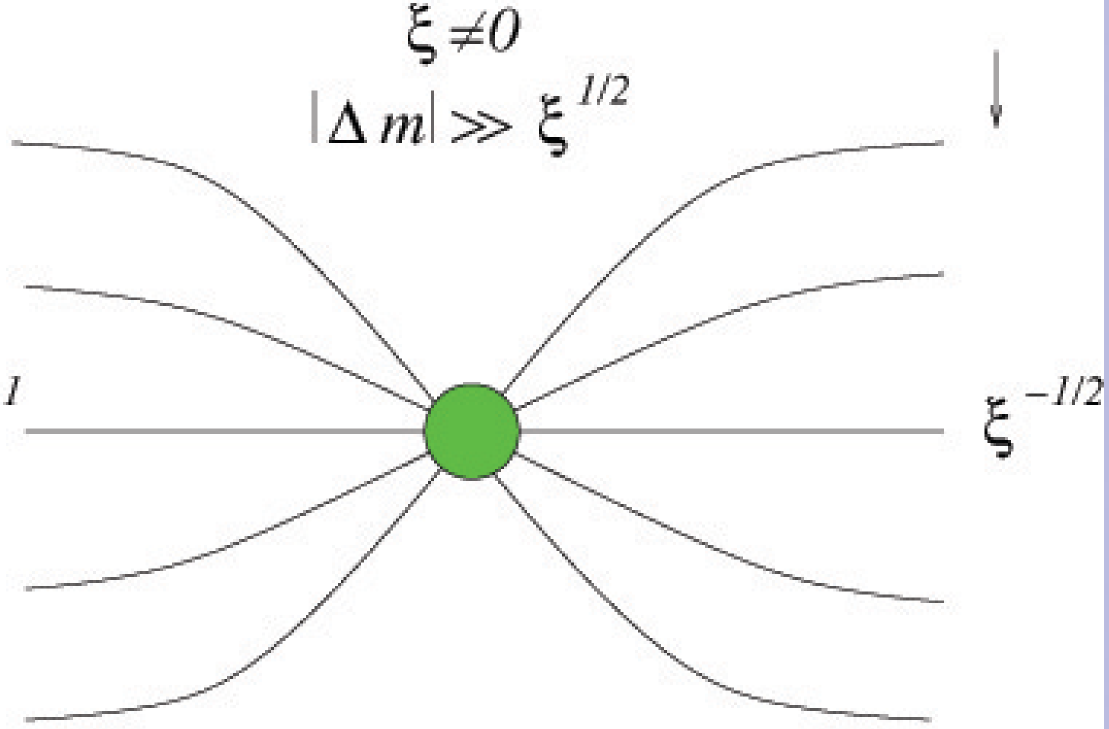
+ adjoints (guiding the evolution)



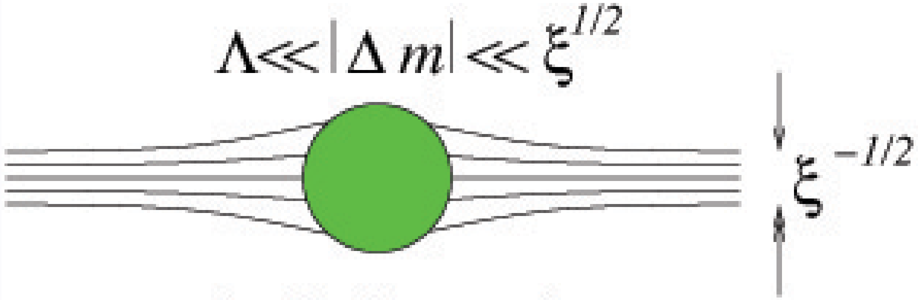
$$\Phi = \begin{pmatrix} \varphi^{11} & \varphi^{12} & \ldots & \varphi^{1N} \\ \varphi^{21} & \varphi^{22} & \ldots & \varphi^{2N} \\ \ldots & \ldots & \ldots & \ldots \\ \varphi^{N1} & \varphi^{N2} & \ldots & \varphi^{NN} \end{pmatrix}$$



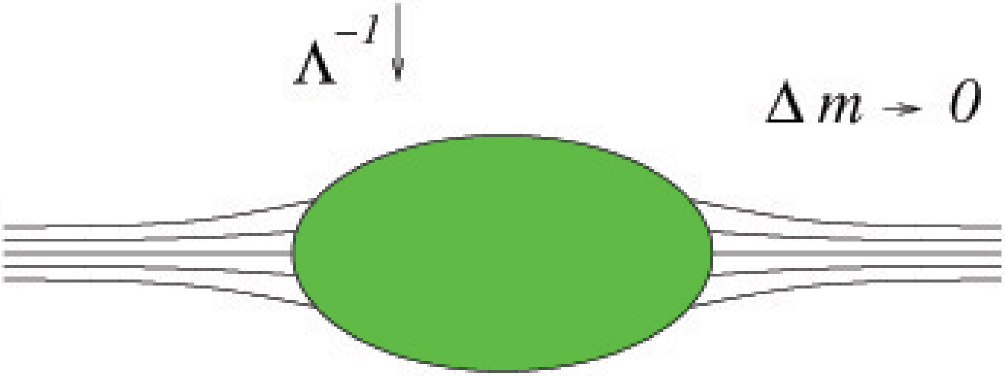
The 't Hooft–Polyakov monopole



Almost free monopole

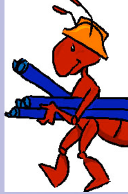


Confined monopole, quasiclassical regime



Confined monopole, highly quantum regime

$U(N)_{gauge} \times SU(N)_{flavor} \to SU(N)_{global}$

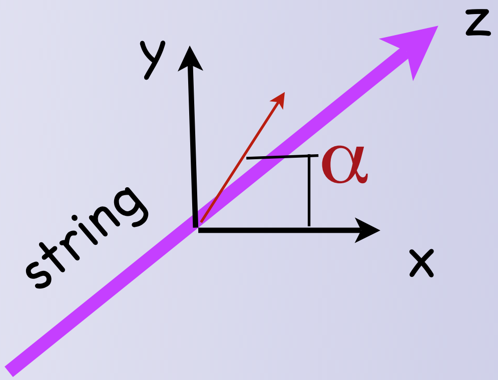


Color-flavor locked vacuum

- ✷ Weak coupling in the bulk ! $(\text{If} \quad \xi \gg \Lambda^2)$
- ✷ Non-Abelian Strings:

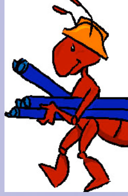
$$\boxed{\pi_1 \left[SU(N) \times U(1)/Z_N \right] \neq 0}$$

$$\Phi_{string} = \sqrt{\xi} \begin{pmatrix} 1 \; 0 \; \ldots \; 0 \\ 0 \; 1 \; \ldots \; 0 \\ \ldots\ldots\ldots \; \ldots \\ 0 \; 0 \; \ldots \; e^{i\alpha} \end{pmatrix}$$



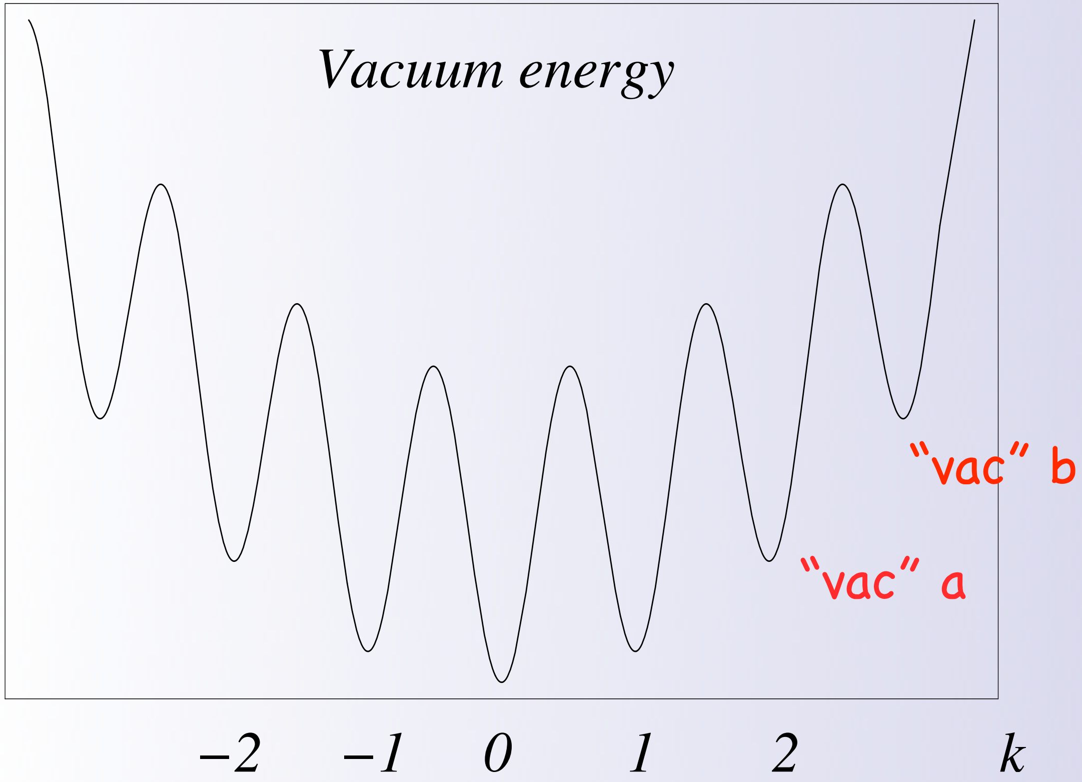
Flux =1/N Abrikosov

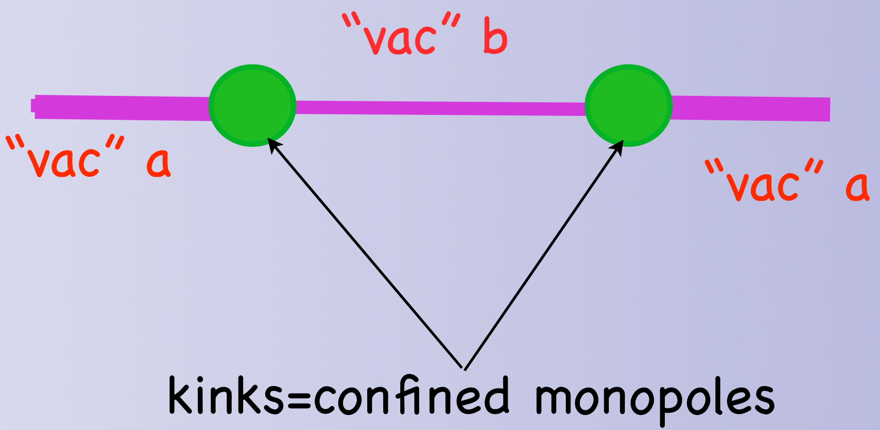
Tension =1/N Abrikosov



✶ String breaks SU(N) down to SU(N-1)xU(1) !

Orientational moduli on the string worldsheet appear. They are governed by CP(N-1) model !



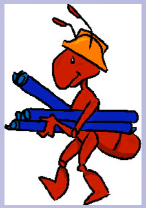


kinks=confined monopoles

Sine law for k-string tensions can be obtained!

Conclusions:

- String excitations: splittings $1/N^2$
- Casimir scaling gone! Sine law ?
- Relaxation time $\exp(-N^2)$
- SUSY and toy models support Sine law



Valya Zakharov was the first to introduce me to QCD, even before QCD was officially born. In the summer of 1972 he was sent, as an ITEP Ambassador, to the Rochester conference which took place at Fermilab that year. After his return, he told us of his impressions.

Apparently, Gell-Mann's talk produced the strongest impression on Valya since he kept saying that Gell-Mann had been preaching octet gluons as mediators of the inter-quark force, and we ought to do something. I was in the very beginning of my PhD work at that time, and knew very little as to how to orient myself in the sea of literature, and whom to trust. Valya repeated, more than once, that Gell-Mann had a direct line to God; hence, Gell-Mann's revelations ought to be taken seriously.

It would be fair to say that Valya's persistence and foresight shaped my career to a large extent. It is gratifying to note that now, 33 years later, he continues a noble mission of analytic thinking, deep insight and promotion of innovative ideas in the lattice QCD community. Thank you, Valya, and Happy Birthday ...