Hadronic corrections to muon anomaly (05' status)

A.E. Dorokhov *(JINR, Dubna)*

- ***I.*** *80 years of the problem*
- *Data from BNL (g-2) Collaboration (+/-0.5 ppm)*
- *SM prediction from QED, EW and Strong sectors (2.7σ)*
- ***II.*** *Leading order Hadronic contribution: phenomenology*
  e^+e^- *- annihilation vs* τ *– decay (shape discrepancy,* $a_s(M_Z)$*))*
- ***III.*** *LO and NLO Hadronic contribution from theory (main source of theoretical error)*
- *Conclusions*

Brief preHistory

I. History

Ein Weg zur experimentellen Prüfung der Richtungsquantelung im Magnetfeld.

Von Otto Stern in Frankfurt a. Main.

Mit zwei Abbildungen. — (Eingegangen am 26. August 1921.)

In der Quantentheorie des Magnetismus und des Zeemaneffektes wird angenommen, daß der Vektor des Impulsmomentes eines Atoms nur ganz bestimmte diskrete Winkel mit der Richtung der magnetischen Feldstärke $\mathfrak{H}$ bilden kann, derart, daß die Komponente des Impulsmomentes in Richtung von $\mathfrak{H}$ ein ganzzahliges Vielfaches von $h/2\pi$ ist[1]). Bringen wir also ein Gas aus Atomen, bei denen das

$$\mathfrak{K} = \mathfrak{m}_x \frac{\partial \mathfrak{H}}{\partial x} + \mathfrak{m}_y \frac{\partial \mathfrak{H}}{\partial y} + \mathfrak{m}_z \frac{\partial \mathfrak{H}}{\partial z}.$$

Nun führt das Atom eine gleichförmige Rotation um die Feldrichtung, d. h. um die z-Achse aus[1]), wobei $\mathfrak{m}_z$ konstant bleibt, während der Mittelwert von $\mathfrak{m}_x$ und $\mathfrak{m}_y$ über einen vollen Umlauf Null wird. Mitteln wir also bei konstantem $\frac{\partial \mathfrak{H}}{\partial x}, \frac{\partial \mathfrak{H}}{\partial y}, \frac{\partial \mathfrak{H}}{\partial z}$ über eine gegen die Umlaufdauer (die z. B. für $\mathfrak{H} = 1000$ Gauß 7.10^{-10} sec ist) große Zeit, so wird die mittlere auf das Atom wirkende Kraft:

$$\bar{\mathfrak{K}} = m_z \frac{\partial \mathfrak{H}}{\partial z}.$$

Für die auf das Atom wirkende Kraft ist also beim magnetischen Moment nur die Komponente in Richtung des Feldes selbst maßgebend, also

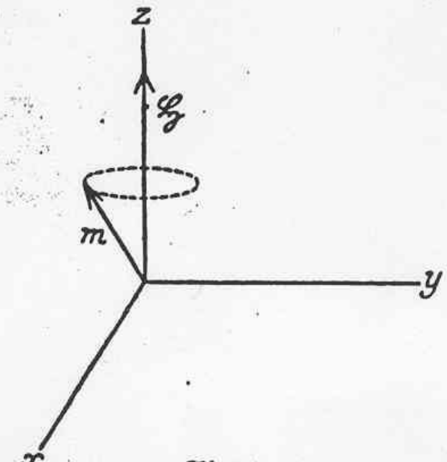


Fig. 1.

Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld.

Von Walther Gerlach in Frankfurt a. M. und Otto Stern in Rostock.

Mit sieben Abbildungen. (Eingegangen am 1. März 1922.)

Vor kurzem[1]) wurde in dieser Zeitschrift eine Möglichkeit angegeben, die Frage der Richtungsquantelung im Magnetfeld experimentell zu entscheiden. In einer zweiten Mitteilung[2]) wurde gezeigt, daß das normale Silberatom ein magnetisches Moment hat. Durch die Fortsetzung dieser Untersuchungen, über die wir uns im folgenden zu berichten erlauben, wurde die **Richtungsquantelung** im **Magnetfeld als Tatsache erwiesen.**

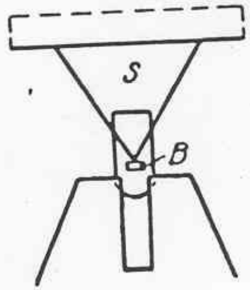


Fig. 1.

1924. № 10.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 74.

1. *Über die Richtungsquantelung im Magnetfeld*[1]; *von Walther Gerlach und Otto Stern.*

(Hierzu Tafel III.)

Nr. der Aufnahme	Entfernung des unabgelenkten Strahles von der Schneide	Mittlere Ablenkung des abgestoßenen Strahles	
		berechnet	beobachtet
15	0,32 mm	$0{,}10_1$ mm	$0{,}10_3$ mm
14	0,21 mm	$0{,}14_8$ mm	0,15 mm

Die Genauigkeit der Messungen schätzen wir auf 10 Proz. Innerhalb dieser Fehlergrenzen zeigen also die Versuche, daß das Silberatom im Normalzustand ein Bohrsches Magneton hat.

$\Rightarrow g = 2$

(in modern language)

§ 9. Ergebnis.

Die im vorstehenden mitgeteilten Versuche erbringen

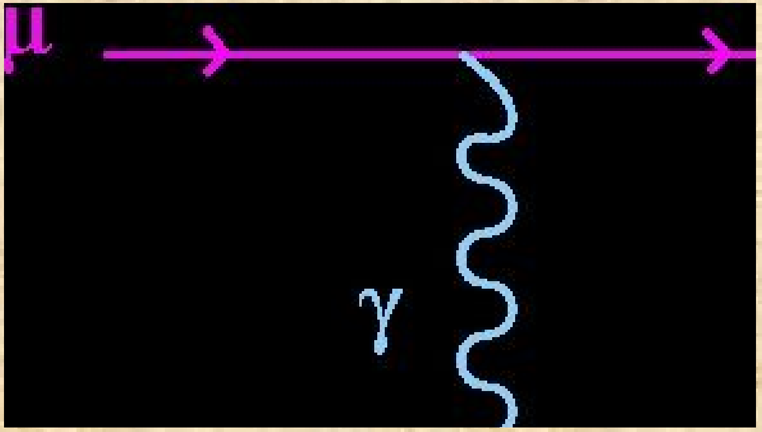
1. den experimentellen Nachweis der Debye-Sommerfeldschen magnetischen Richtungsquantelung
2. die experimentelle Bestimmung des Bohrschen Magnetons.

Schließlich möchten wir dem Institutsmechanikermeister Hrn. Adolf Schmidt für seine unermüdliche und verständnisvolle Hilfe unseren aufrichtigen Dank sagen.

Frankfurt a. M. und Hamburg, 1923.

Dirac Equation Predicts for point-like spin ½ charged particle: g=2

$$i\hbar\frac{\partial\psi}{\partial t} = [\frac{p^2}{2m} - \frac{e}{2m}(\vec{L}+2\vec{S})\cdot\vec{B}]\psi$$



Some Definitions

A charged particle with spin s *has a magnetic moment* μ

$$L_I = \vec{\mu}_S \vec{B},$$

$$\vec{\mu}_S = g_S \left(\frac{e}{2m} \right) \vec{s},$$

Gyromagnetic ratio

PDG

$$a = \frac{g_S - 2}{2}, \quad \mu = (1 + a) \frac{eh}{2m}$$

Anomaly

m_μ =105.6583692(94) MeV,

m_τ = 1776.99 (29) MeV

m_μ/m_e = 206.768 2838(54)

The general form of the ffγ vertex is

$$-ie\bar{u}(p')\left\{\gamma_{\mu}F_1(q^2)+i\sigma_{\mu\nu}\frac{q_\nu}{2m_l}F_2(q^2)-\gamma_5\sigma_{\mu\nu}\frac{q_\nu}{2m_l}F_3(q^2)\right\}u(p)e_\mu(q)$$

- ***F_1* is the electric charge distribution** ***$e_l=eF_1(0)$***

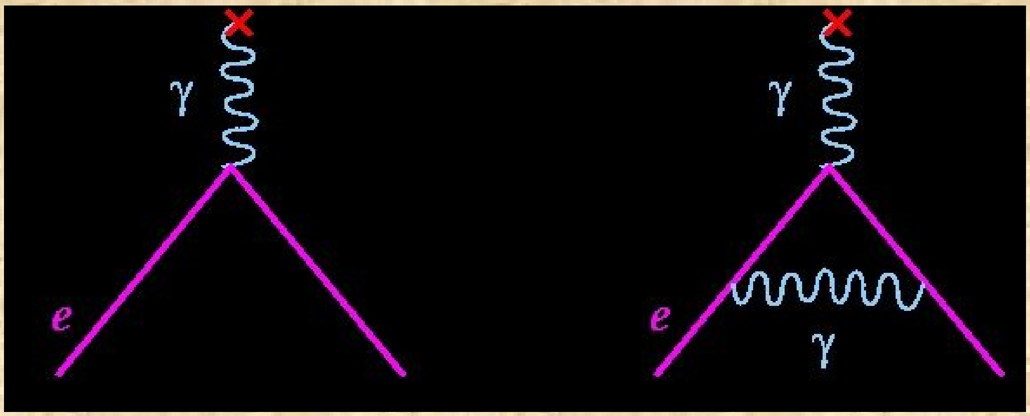
- ***F_2* corresponds to Anomalous Magnetic Moment (AMM)** ***$a_l=(g_l-2)/2=F_2(0)$***

- ***F_3* corresponds to Anomalous Electric Dipole Moment** ***$d_l=-e_l/(2m_l)F_3(0)$***

***$d_l=0$* due to *T*- and *P* symmetries**

However, in SM ***a_l*** is not zero due to ***Radiative Corrections***

The lowest order radiative correction



$$\Gamma_{\mu} = e\gamma_{\mu} + a_{\ell} \frac{ie}{2m}\sigma_{\mu\nu} q_{\nu}$$

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Schwinger, 1948
(Nobel prize 1965)

The Magnetic Moment of the Electron†

P. Kusch and H. M. Foley
Department of Physics, Columbia University, New York, New York

(Received April 19, 1948)

A comparison of the g_J values of Ga in the $^2P_{3/2}$ and $^2P_{1/2}$ states, In in the $^2P_{1/2}$ state, and Na in the $^2S_{1/2}$ state has been made by a measurement of the frequencies of lines in the *hfs* spectra in a constant magnetic field. The ratios of the g_J values depart from the values obtained on the basis of the assumption that the electron spin gyromagnetic ratio is 2 and that the orbital electron gyromagnetic ratio is 1. Except for small residual effects, the results can be described by the statement that $g_L = 1$ and $g_S = 2(1.00119 \pm 0.00005)$. The possibility that the observed effects may be explained by perturbations is precluded by the consistency of the result as obtained by various comparisons and also on the basis of theoretical considerations.

Theory

meets

Experiment

$$a = \frac{\alpha}{2\pi} = 0.00116$$

The CERN Muon (g-2) Experiments

$$a_{\mu^+} = 0.001\ 145(22) \qquad (1966)$$

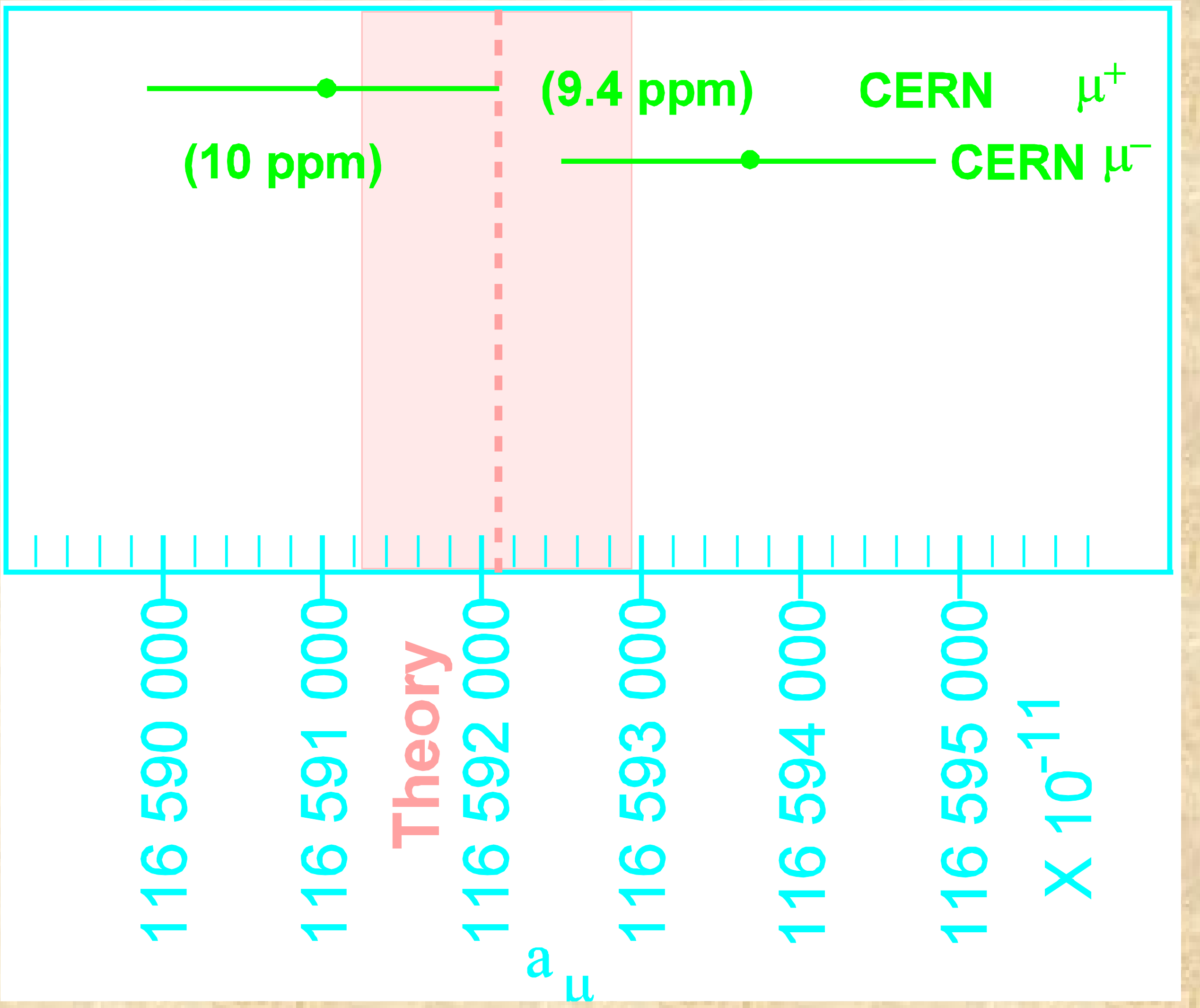
$$\frac{\alpha}{2\pi} = 0.001\ 161\ 410$$

The muon was shown to be a point particle obeying QED

The final CERN precision was 7.3 ppm

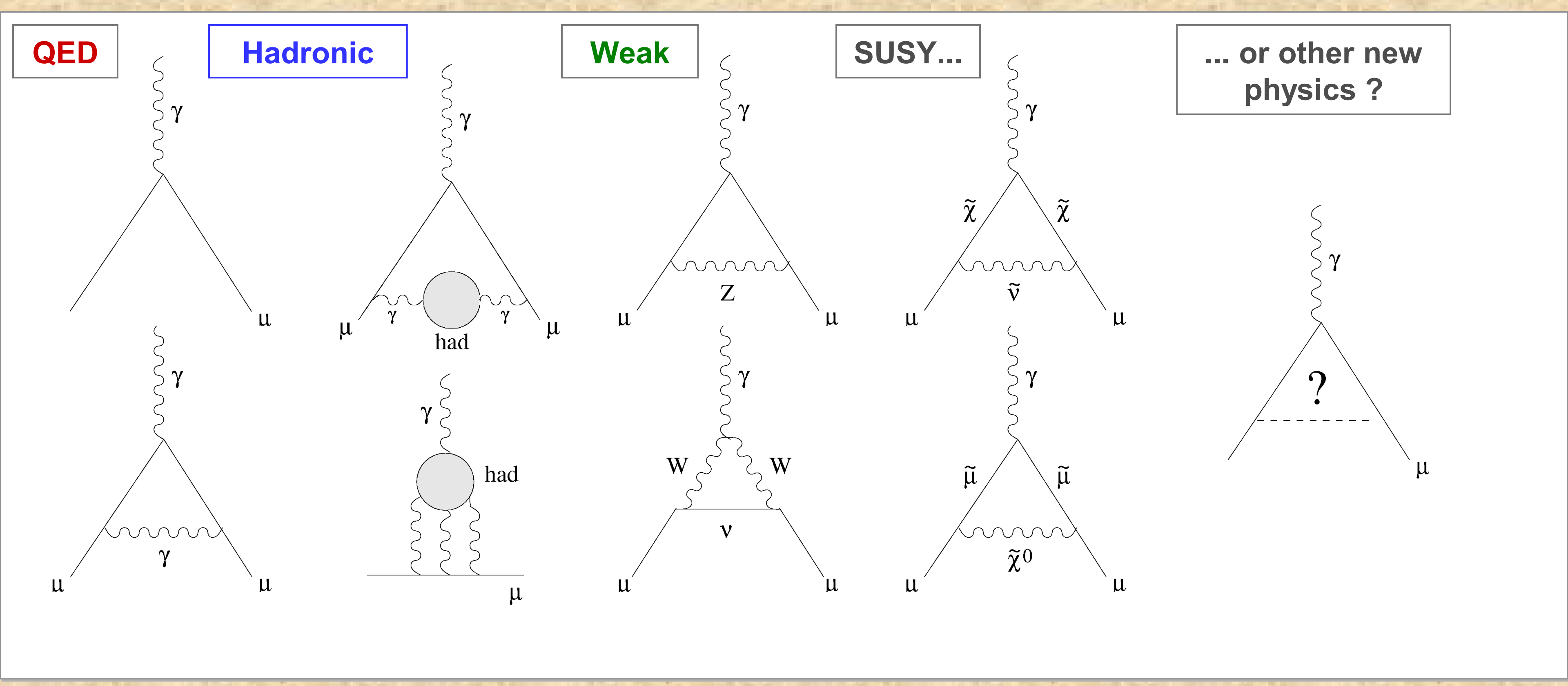
$$a_\mu = 0.001\ 165\ 923(8.5) \qquad (1979)$$

Where we came from:



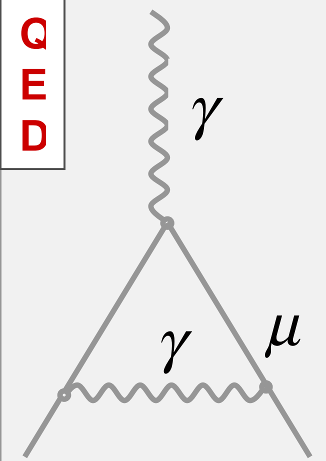
Current Theory Status

Magnetic Anomaly

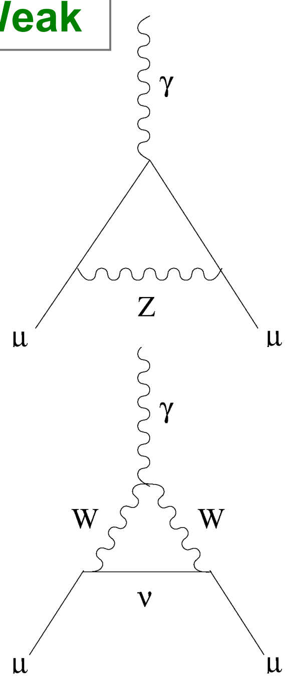


Muon Magnetic Anomaly

QED



Weak



QED Prediction:

Computed up to 4th order [Kinoshita *et al.*]
(5th order estimated [Mohr, Taylor])

$$a_\mu^{QED} = \sum_{n=1} \left(\frac{\alpha}{\pi}\right)^n \approx \left(11614098.1 + 41321.8 + 3014.2 + 38.1 + 0.6\right) \times 10^{-10}$$

EW Prediction:

[Ternov, Rodionov, Studenikin *et al*,…Czarnecki et al]

3 loops is very small 10^{-12}

$$a_\mu^{EW}\left(\text{one} + \text{two loops}\right) = 15.4(3)\cdot 10^{-10}$$

Strong sector prediction:

$$a_\mu^{Had} \approx 700(10)\cdot 10^{-10}$$

BNL (g-2) experiment

Precession Method

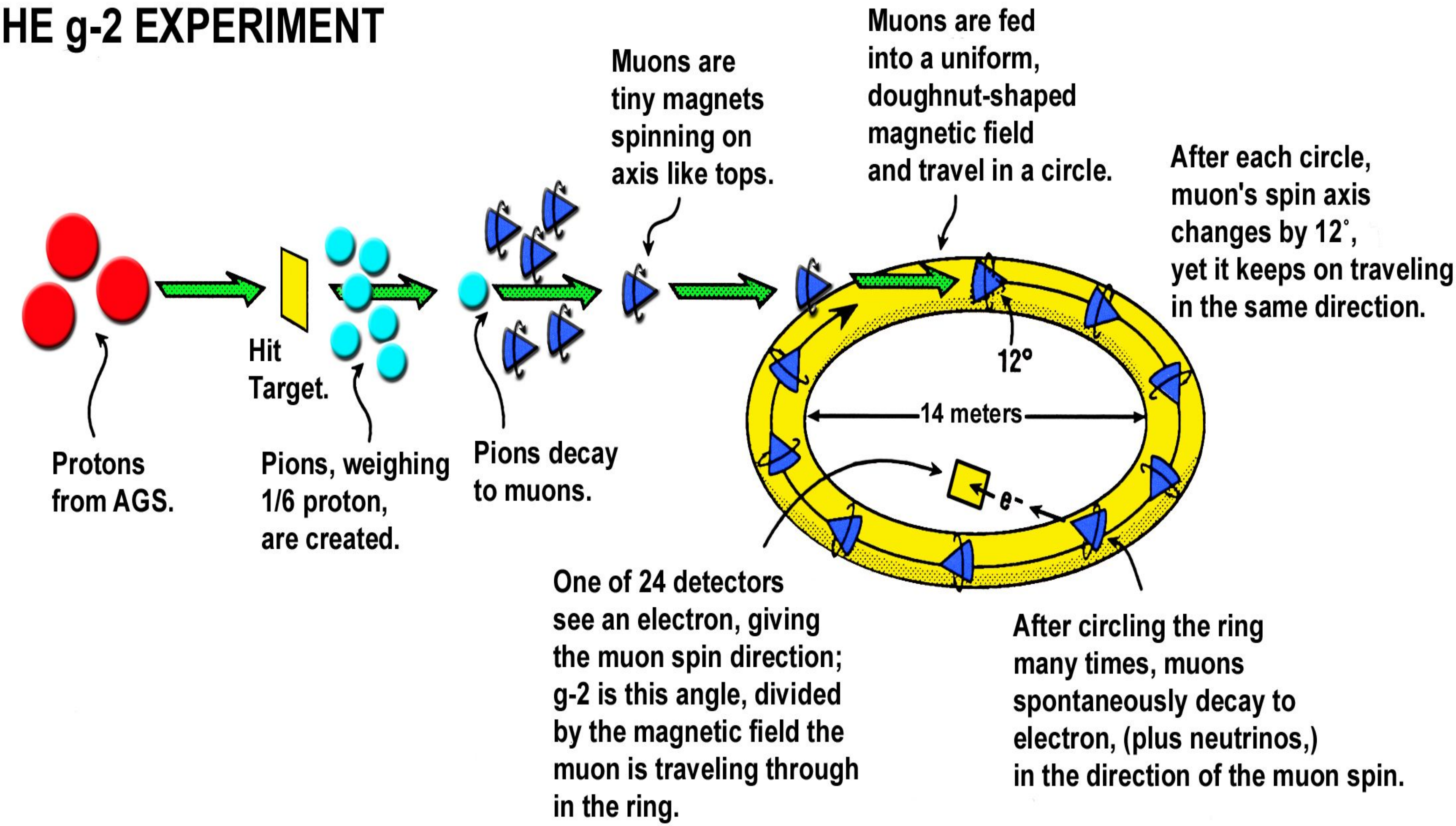
This BNL experiment is based on the fact that for $a_\mu > 0$ the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field.

The difference of the spin frequency (Larmor and Thomas) ω_s and the momentum precession (cyclotron) frequency ω_c is given by

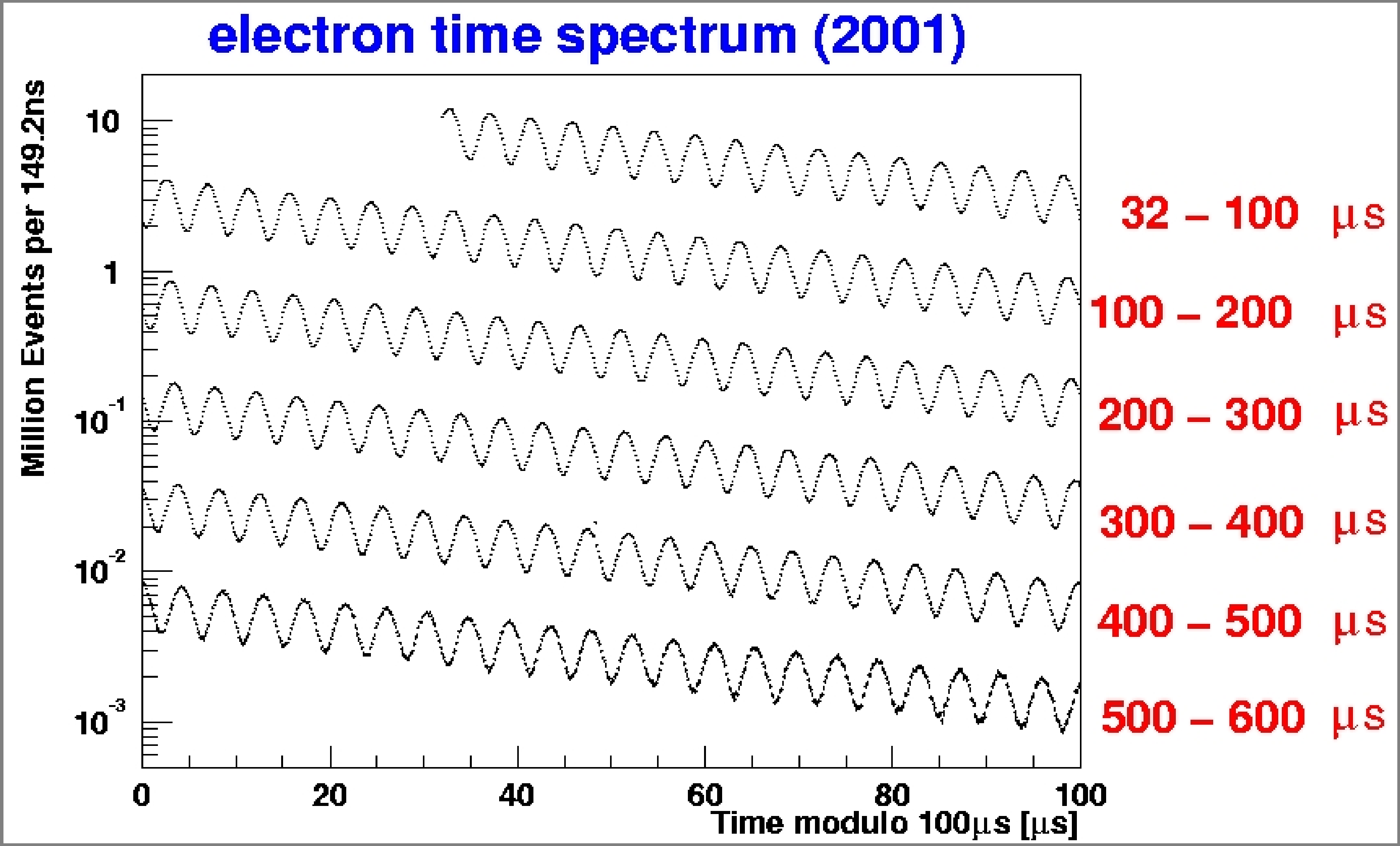
$$\omega_a = \frac{g-2}{2}\frac{eB}{mc}$$

The difference frequency ω_a is the frequency with which the spin precesses relative to the momentum, and is proportional to the anomaly, rather than to g.

LIFE OF A MUON: THE g-2 EXPERIMENT

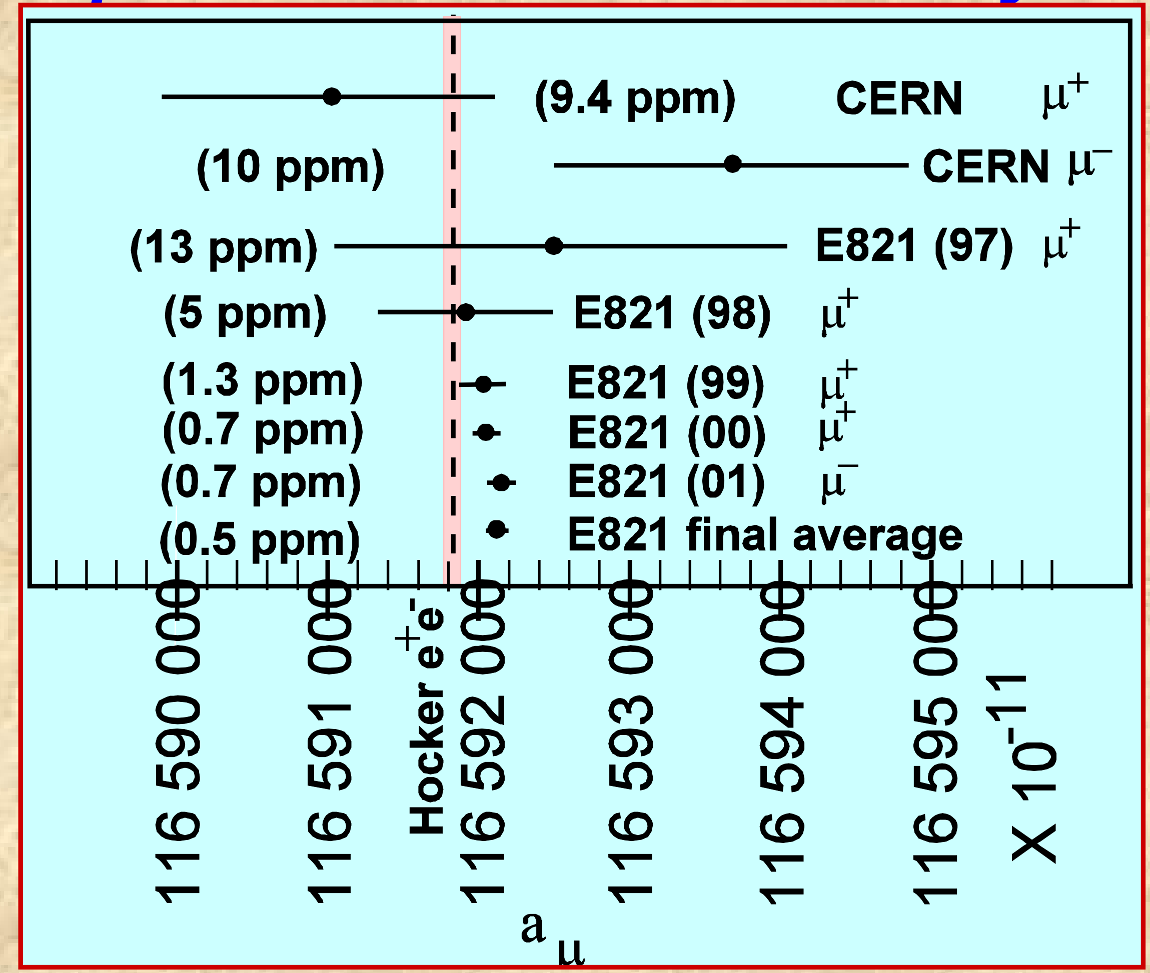


$4 \times 10^9 \;\; e^-, E_{e^-} \geq 1.8$ GeV



$$f(t) = N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

Today: Experiment vs theory



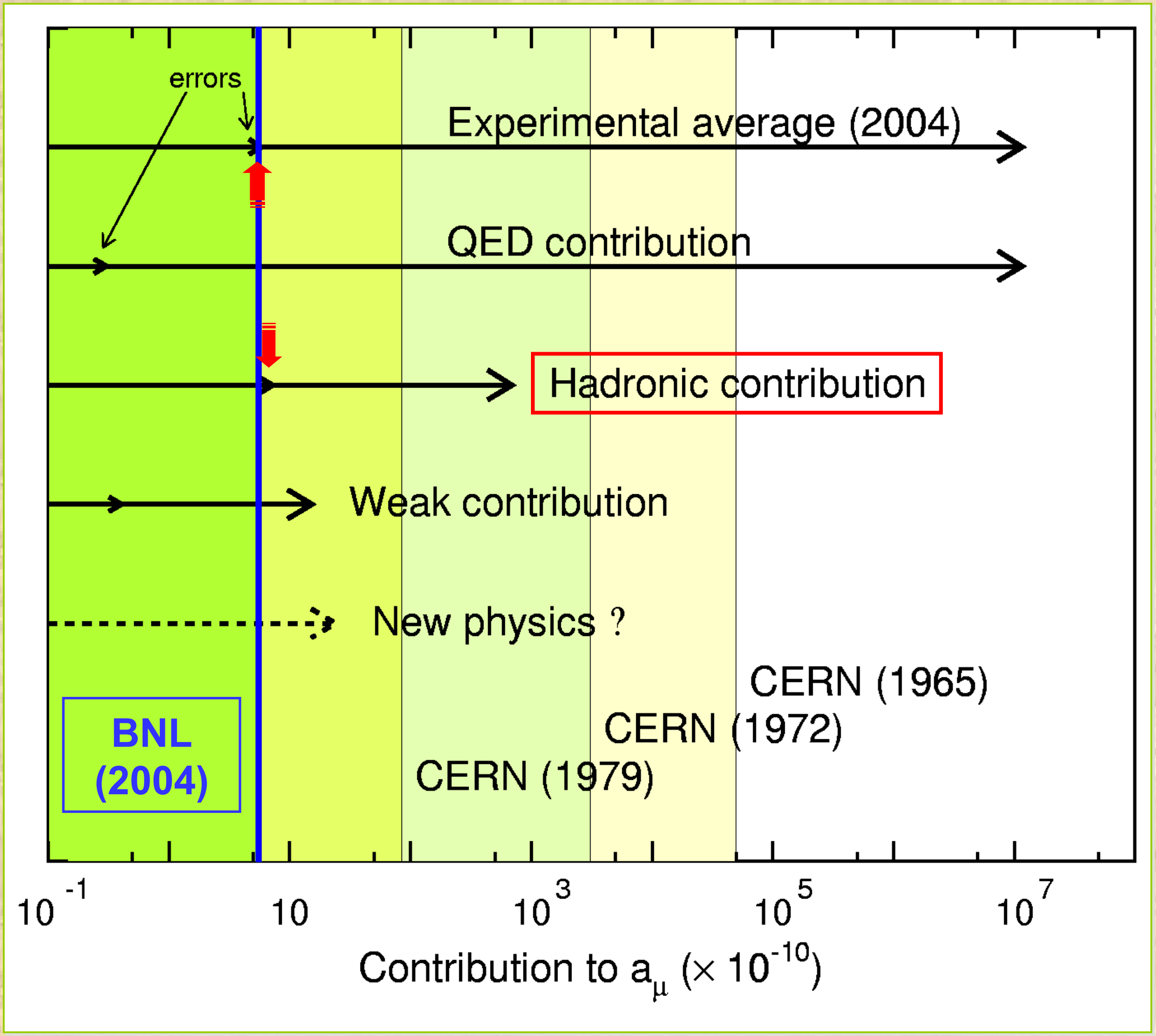
$$a_\mu = \frac{\omega_a}{\frac{e}{mc}B}$$

$$a_\mu = 11\,659\,208(6) \times 10^{-10} \;(0.5\,\mathrm{ppm})$$

2.7σ difference with e^+e^- based SM value

What we expect to find?

Why Do We Need to Know it so Precisely?



Experimental progress on precision of $(g-2)_\mu$

outperforms theory precision on hadronic contribution

Lepton Anomalies

- Electron anomaly is measured to a relative precision of about 4 parts in a billion (ppb). ***QED test.***
- Muon anomaly is measured to 0.5 parts in a million (ppm) ***SM test.***
- Tau anomaly is difficult to measure since its fast decay.
- For a lepton L, New Physics contribution to a_L is proportional to $\left(m_L^2 / \Lambda^2\right)$
- Thus muon AMM leads to a $(m_\mu/m_e)^2 \sim 40\,000$ enhancement of the sensitivity to New Physics versus the electron AMM, the muon anomaly is sensitive to ≥ 100 GeV scale physics.
- However Electron AMM one of the best for determining α

Electron anomaly

To measurable level a_e arises entirely from virtual electrons and photons

$$a_e = (115\,965\,218.59 \pm 0.38) \cdot 10^{-11}$$

The theoretical error is dominated by the uncertainty in the input value of the QED coupling $\alpha \equiv e^2/(4\pi)$

$$\alpha^{-1} = 137.035\,999\,11 \pm 0.000\,000\,46$$

Es ist fantastisch!

QED is at the level of the best theory ever built to describe nature

Tau anomaly

- Tau due to its highest mass is the best for searching for New Physics,
- But Tau is short living particle, so the precession method is not perspective
- The best existing limits

$$-0.052 < a_\tau^{Exp} < 0.013$$

are obtained at DELPHI (LEP, CERN) from the high energy process

$$e^+e^- \rightarrow e^+e^- \tau+\tau- ,$$

- While the SM estimate is

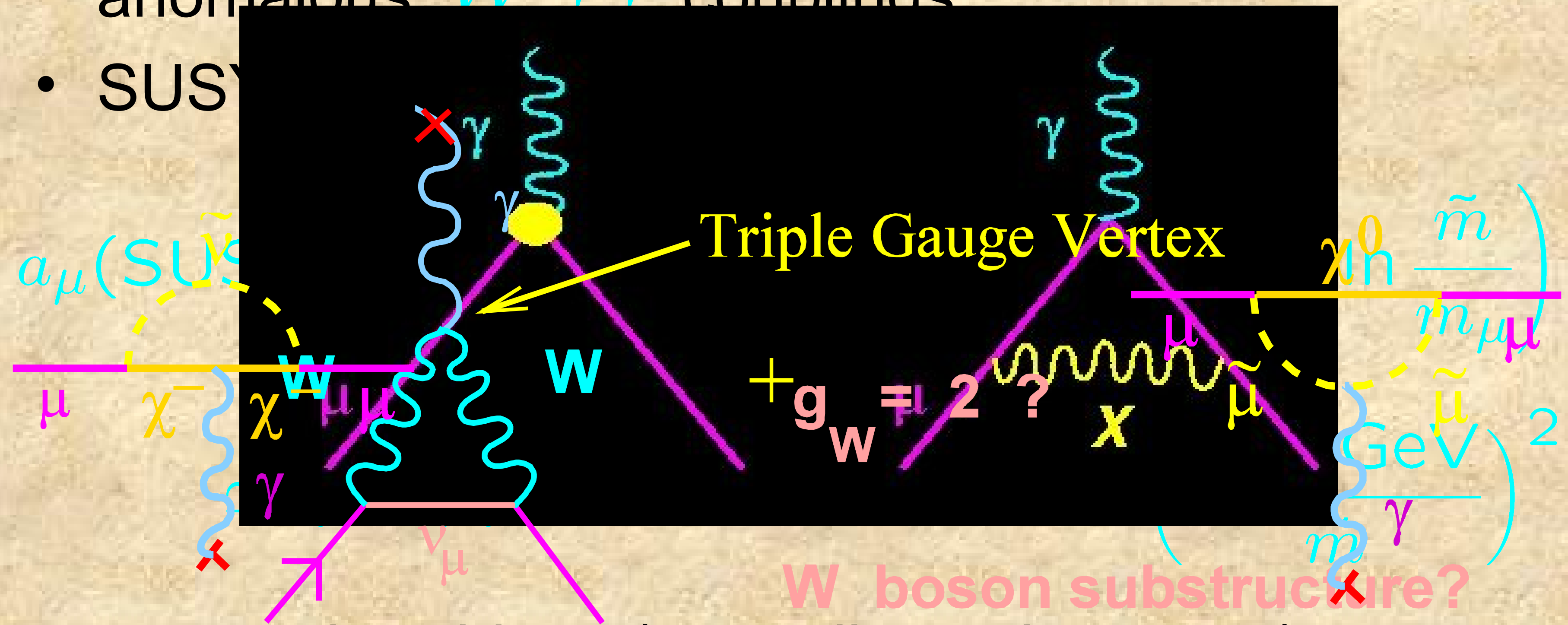
$$a_\tau^{SM} = 1.1773(3)\ 10^{-3}$$

Muon Anomaly is sensitive to a wide range of new physics

- muon substructure

$$\delta a_\mu(\Lambda_\mu) \simeq \frac{m_\mu^2}{\Lambda_\mu^2}$$

- anomalous $W\gamma\gamma$ couplings
- SUSY



- many other things (extra dimensions, etc.)

Standard Model Prediction.

Hadronic Corrections from Phenomenology.

The Muonic $(g-2)_\mu$

Contributions to the Standard Model (SM) Prediction:

$$a_\mu^{SM} \equiv \left(\frac{g-2}{2}\right)_\mu = a_\mu^{QED} + a_\mu^{had} + a_\mu^{weak}$$

The Situation 1995

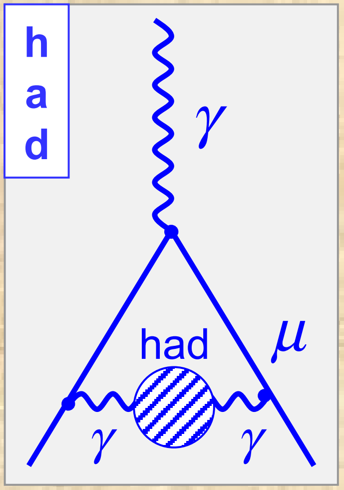
Source	$\sigma(a_\mu)$	Reference
QED	**$\sim 0.3 \times 10^{-10}$**	[Schwinger '48 & others]
Hadrons	**$\sim (15 \oplus 4) \times 10^{-10}$**	[Eidelman-Jegerlehner '95 & others]
***Z, W* exchange**	**$\sim 0.4 \times 10^{-10}$**	[Czarnecki *et al.* '95 & others]

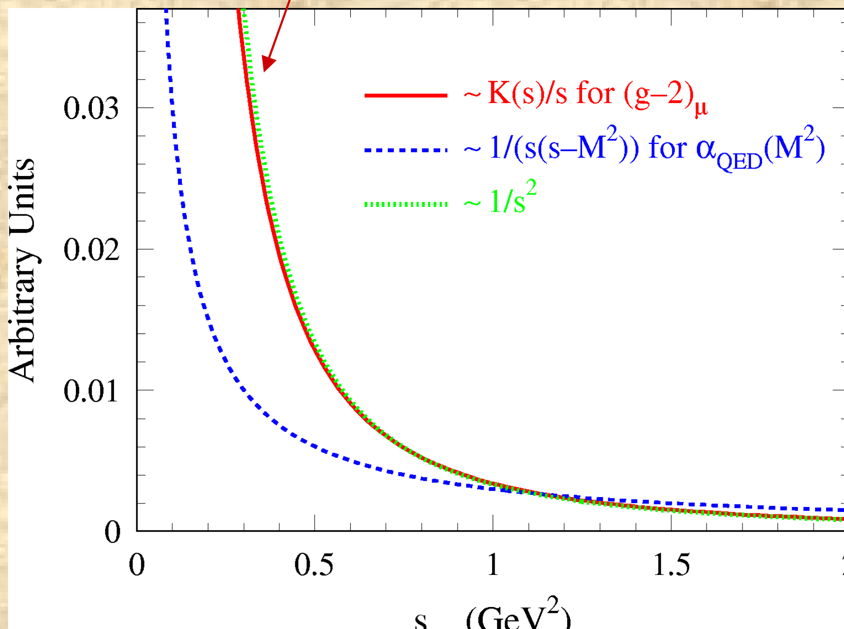
Dominant uncertainty from lowest order hadronic piece. Cannot be calculated from QCD ("first principles") – but: we can use experiment (!)

II. Leading Order Hadronic contributions

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R^{(0)}(s)$$

"Dispersion relation", uses unitarity (optical theorem) and analyticity (Bouchiat and Michel, 1961)

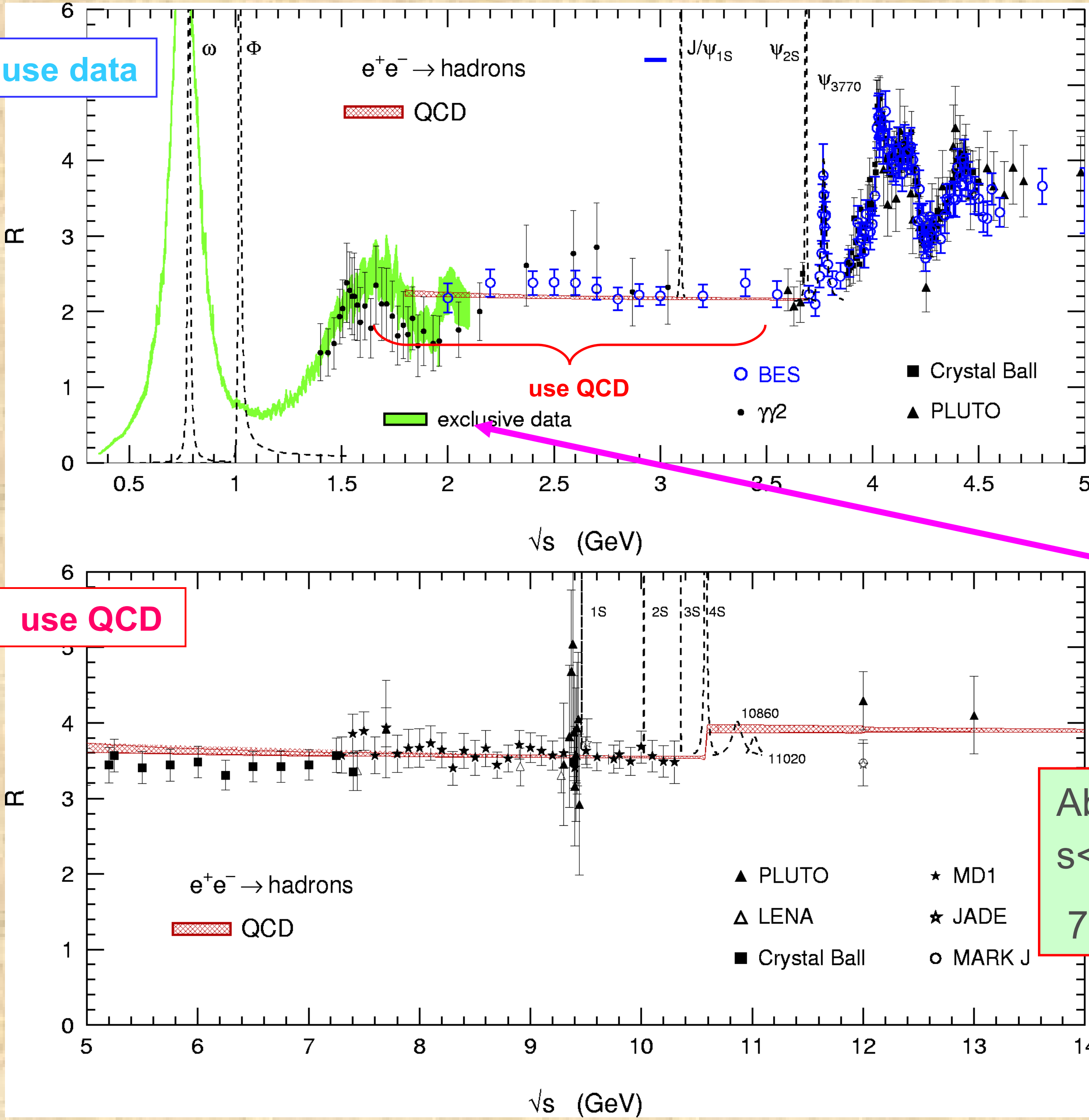




Still it is impossible to calculate LO contribution from Theory with required accuracy

$$R(s) = \frac{\sigma\left[e^+e^- \to \text{hadrons}\right]}{\sigma\left[e^+e^- \to \mu^+\mu^-\right]}$$

Evaluating the Dispersion Integral



Agreement between Data (BES) and pQCD (within *correlated* systematic errors)

Better agreement between exclusive and inclusive ($\gamma\gamma 2$) data

About 91% of a_{ϖ} comes from $s<(1.8\text{GeV})^2$,

73% is covered by final 2π state

Phenomenological Determination of the Hadronic Contribution to $(g-2)_\mu$

Eidelman-Jegerlehner'95, Z.Phys. C67 (1995) 585

Energy [GeV]	Input 1995	Input after 1998
$2m_\pi$ - 1.8	Data	Data (e^+e^- & τ) (+ QCD)
1.8 – $\psi(3770)$	Data	QCD
J/ψ - Υ	Data	Data (+ QCD)
Υ - 40	Data	QCD
40 - ∞	QCD	QCD

Improved determination of the dispersion integral comes from:

- better data
- extended use of QCD

Improvement in 4 Steps:

- Inclusion of precise τ data using SU(2) (CVC)

  Alemany-Davier-Höcker'97, Narison'01, Trocóniz-Ynduráin'01, + later works

- Extended use of (dominantly) perturbative QCD

  Martin-Zeppenfeld'95, Davier-Höcker'97, Kühn-Steinhauser'98, Erler'98, + others

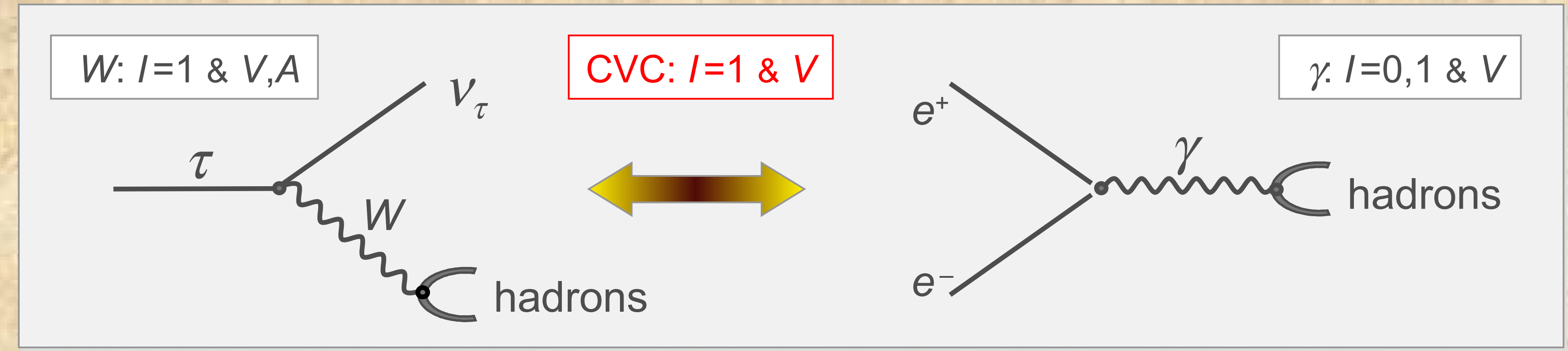
- Theoretical constraints from QCD sum rules and use of Adler function

  Groote-Körner-Schilcher-Nasrallah'98, Davier-Höcker'98, Martin-Outhwaite-Ryskin'00, Cvetič-Lee-Schmidt'01, Jegerlehner et al'00, Dorokhov'04 + others

- Better data for the $e^+e^- \to \pi^+\pi^-$ cross section

  CMD-2'02, KLOE'04 (!)

The Role of τ Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

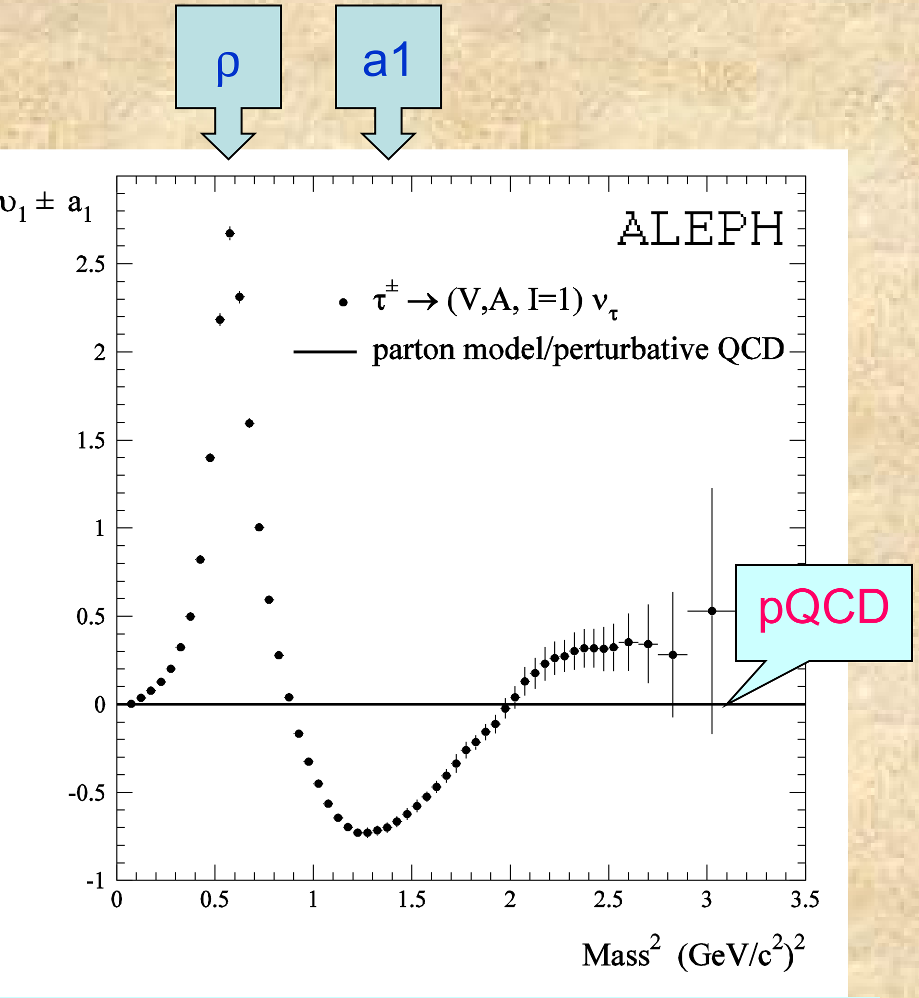
$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\, \upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

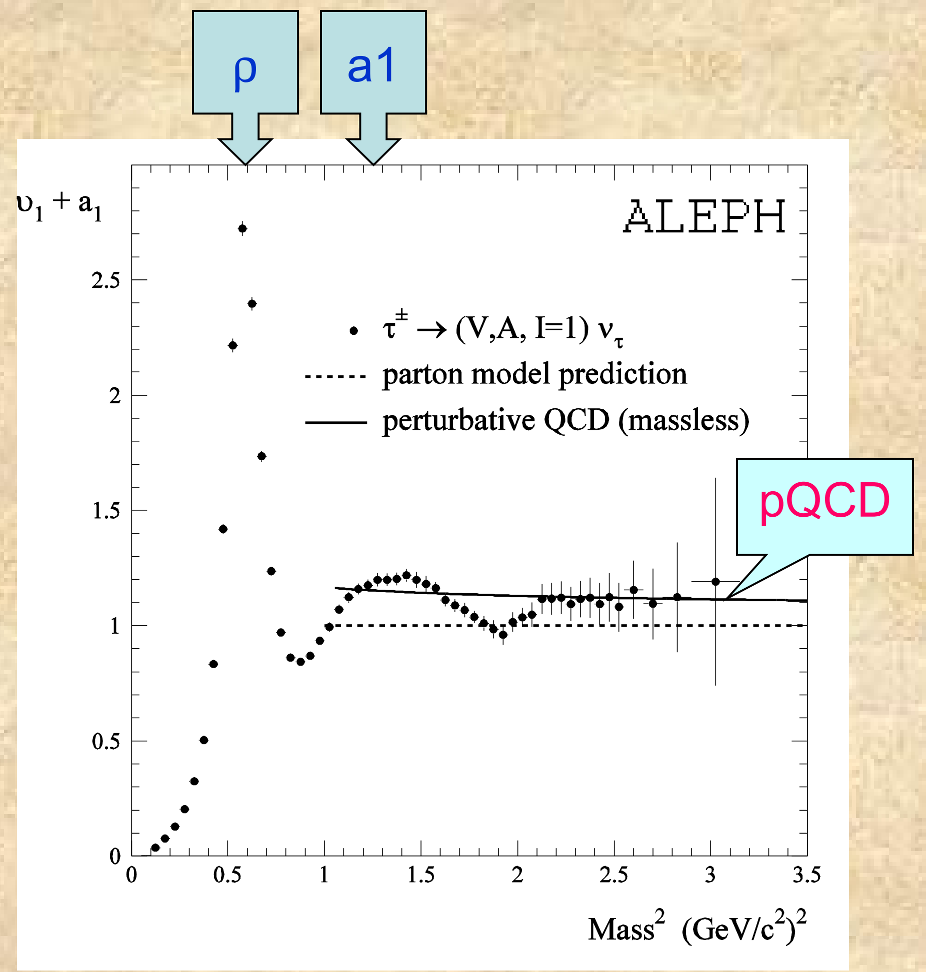
$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{BR\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{BR\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \; \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \; \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

branching fractions — mass spectrum — kinematic factor (PS)

ALEPH data on τ decays



Inclusive v-a spectral function, measured by the ALEPH collaboration



Inclusive v+a spectral function, measured by the ALEPH collaboration

Results: the Compilation (including KLOE)

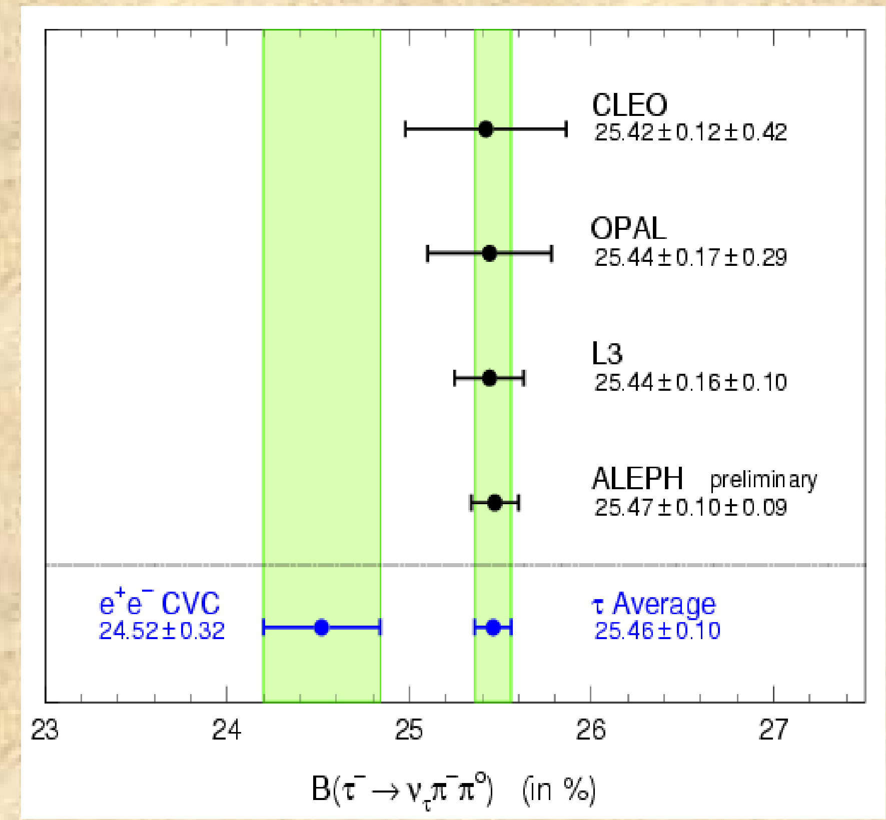
Contributions to a_μ^{had} [in 10^{-10}] from the different energy domains:

Modes	Energy [GeV]	e^+e^-	τ
Low s expansion	$2m_\pi$ – 0.5	$58.0 \pm 1.7 \pm 1.2_{rad}$	$56.0 \pm 1.6 \pm 0.3_{SU(2)}$
[$\pi^+\pi^-$ (DEHZ'03)]	$2m_\pi$ – 1.8	[$450.2 \pm 4.9 \pm 1.6_{rad}$]	$464.0 \pm 3.0 \pm 2.3_{SU(2)}$
$\pi^+\pi^-$ (incl. KLOE)	$2m_\pi$ – 1.8	$448.3 \pm 4.1 \pm 1.6_{rad}$	–
$\pi^+\pi^- 2\pi^0$	$2m_\pi$ – 1.8	$16.8 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.3 \pm 0.6_{SU(2)}$
$2\pi^+ 2\pi^-$	$2m_\pi$ – 1.8	$14.2 \pm 0.9 \pm 0.2_{rad}$	$12.3 \pm 1.0 \pm 0.4_{SU(2)}$
ω(782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{rad}$	–
ϕ(1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{rad}$	–
Other exclusive	$2m_\pi$ – 1.8	$24.0 \pm 1.5 \pm 0.3_{rad}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{rad}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5 \pm 0.0_{rad}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{rad}$	–
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{theo}$	–
Sum (incl. KLOE)	$2m_\pi$ – ∞	$693.4 \pm 5.3 \pm 3.5_{rad}$	$711.0 \pm 5.0 \pm 0.8_{rad} \pm 2.8_{SU(2)}$

Testing CVC

Infer τ branching fractions from e^+e^- data:

$$BR_{CVC}\left(\tau^- \to \pi^-\pi^0\nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\,S_{EW}}{m_\tau^2}\int_0^{m_\tau^2} ds\,\mathrm{kin}(s)\cdot \upsilon^{\text{SU(2)-corrected}}(s)$$



Difference: BR[τ] – BR[e^+e^- (CVC)]:

Mode	$\Delta(\tau - e^+e^-)$	„Sigma“
$\tau^- \to \pi^-\pi^0\nu_\tau$	+ 0.94 ± 0.32	2.9
$\tau^- \to \pi^- 3\pi^0\nu_\tau$	– 0.08 ± 0.11	0.7
$\tau^- \to 2\pi^-\pi^+\pi^0\nu_\tau$	+ 0.91 ± 0.25	3.6

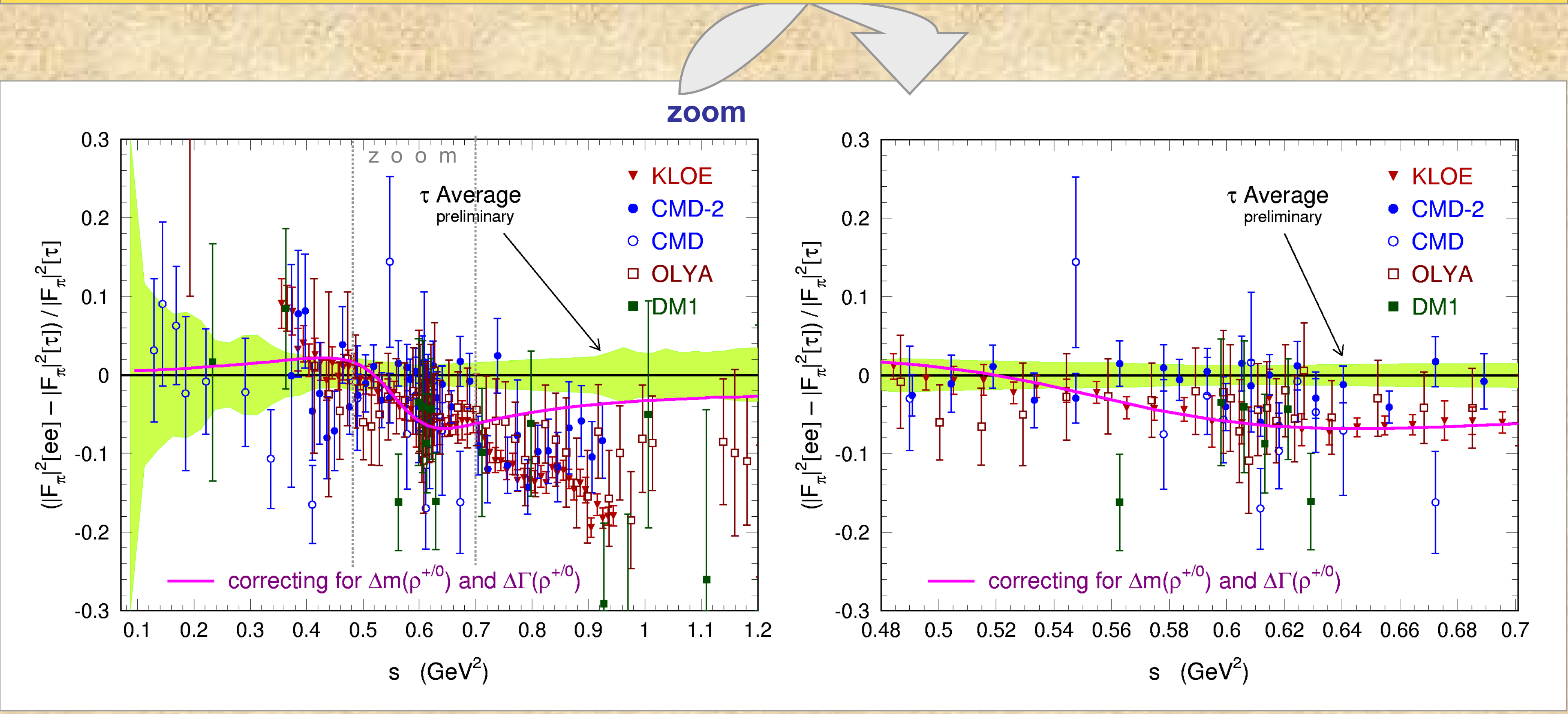
leaving out CMD-2 :

$B_{\pi\pi^0}$ = (23.69 ± 0.68) %

$\Rightarrow$ (7.4 ± 2.9) % relative discrepancy!

Shape of Fπ from e+e- and hadronic τ decay

Relative difference between τ and e^+e^- data:



Correction for $\rho^{\pm}-\rho^{0}$ mass (~2.3 ± 0.8 MeV) and width (~2.9 MeV) splitting applied

Davier, hep-ex/0312064

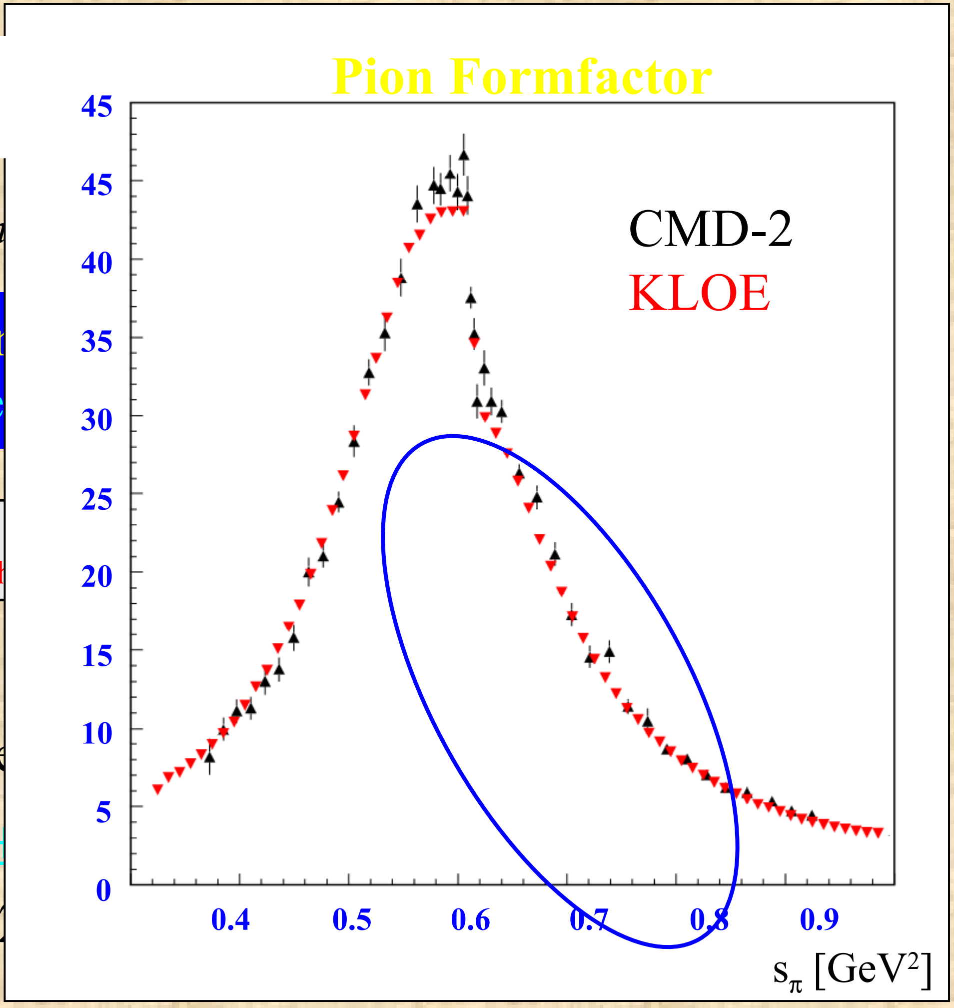
Jegerlehner, hep-ph/0312372

KLOE Data on *R(s)*

2 contribution to a^h

- KLOE has evaluated the Dispersions In
  in the Energy Range 0.35 <s<0.95 Ge

$a_\mu^{\pi\pi} = (388.7 \pm 0.8_{stat} \pm 3.5_{syst} \pm 3.5_{th}$



- Comparison with CMD-2 in the

KLOE (375.6 ± 0.8$_{stat}$ ±

CMD2 (378.6 ± 2.7$_{stat}$ ±

- At large values of s_π ($>m_\rho^2$) KLOE is consistent with CMD and therefore
  They confirm the deviation from -data!

Tau vs e^+e^-

$a_\mu^{had}[e^+e^-]$	=	$(693.4 \pm 5.3 \pm 3.5) \times 10^{-10}$	
$a_\mu^{had}[\tau]$	=	$(711.0 \pm 5.0 \pm 3.6) \times 10^{-10}$	

The problem of the $\pi^+\pi^-$ contribution :

- Experimental situation:
  - new, precise KLOE results in approximate agreement with latest CMD-2 data
  - τ data without $m(\rho)$ and $\Gamma(\rho)$ corr. in strong disagreement with both data sets
  - ALEPH, CLEO and OPAL τ spectral functions in good agreement within errors
- Concerning the line shape discrepancy:
  - SU(2) corrections: basic contributions identified and stable since long; overall correction applied to τ is (– 2.2 ± 0.5) %, dominated by uncontroversial short distance piece; additional long-distance corrections found to be small
  - ρ lineshape corrections improves, but cannot correct difference above 0.7 GeV^2

The fair agreement between KLOE and CMD-2 devalidates the use of τ data until a better understanding of the discrepancies is achieved

Andreas Hocker (2004)

However, Kim Maltman (2005) found that
the τ decay data are compatible with expectations based on high-scale $\alpha_s(M_Z)$ determinations;
the e+e- data, in contrast, requires significantly lower $\alpha_s(M_Z)$.

The results favor determinations of the Leading Order hadronic contribution to $(g-2)_\mu$ which incorporate hadronic τ decay data over those employing e+e- data only, and
hence suggest a reduced discrepancy between the Standard Model prediction and the current experimental value of $(g - 2)_\mu$.

EM and τ spectral integrals w_k with OPE input

Weight	EM or τ	$\alpha_s(M_Z)$
w_1	EM	$0.1138^{+0.0030}_{-0.0035}$
w_3	EM	$0.1152^{+0.0019}_{-0.0021}$
w_6	EM	$0.1150^{+0.0022}_{-0.0026}$
w_1	τ	$0.1218^{+0.0027}_{-0.0032}$
w_3	τ	$0.1195^{+0.0018}_{-0.0021}$
w_6	τ	$0.1201^{+0.0020}_{-0.0022}$

$$\alpha_s(M_Z) = 0.1200 \pm 0.0020$$

Preliminary Results

$a_\mu^{had}[e^+e^-]$: $(693.4 \pm 5.3 \pm 3.5) \times 10^{-10}$

$a_\mu^{SM}[e^+e^-]$: $(11\,659\,182.8 \pm 6.3_{had} \pm 3.5_{LBL} \pm 0.3_{QED+EW}) \times 10^{-10}$

a_μ^{SM} including:

Weak contribution : $a_\mu^{weak} = +(15.4 \pm 0.3) \times 10^{-10}$

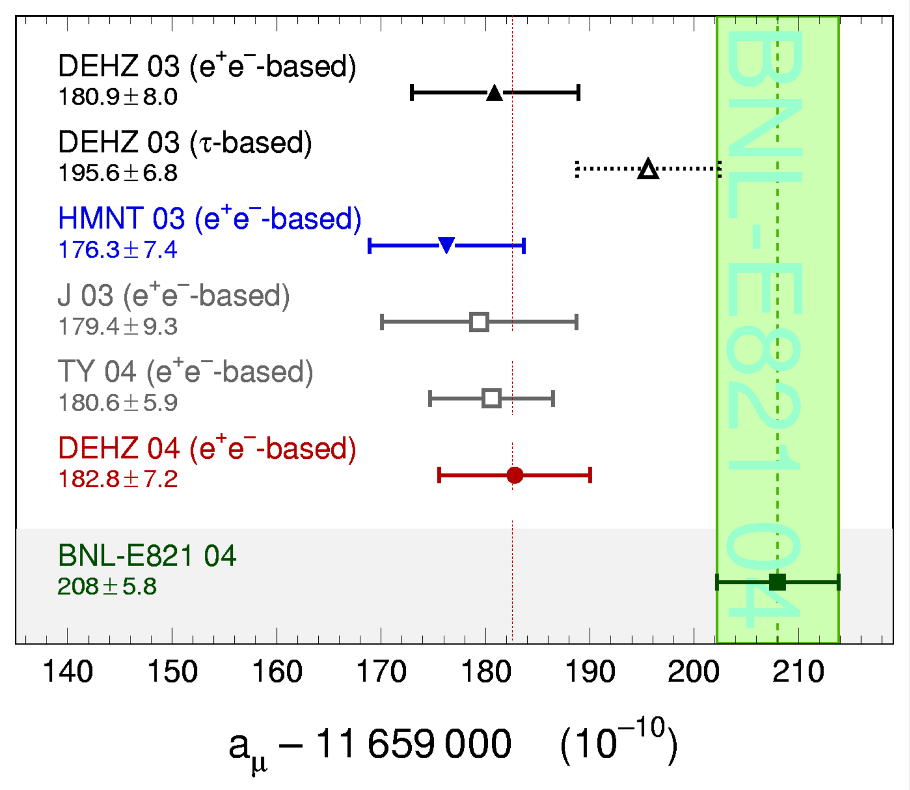
Hadronic contribution from higher order : $a_\mu^{had}[(\alpha/\pi)^3] = -(10.0 \pm 0.6) \times 10^{-10}$

Hadronic contribution from LBL scattering: $a_\mu^{had}[LBL] = +(12.0 \pm 3.5) \times 10^{-10}$

Czarnecki-Marsiano-Vainshtein + others

Knecht-Nyffeler, Phys.Rev.Lett. 88 (2002) 071802

Melnikov-Vainshtein (2003), Dorokhov (2004)



BNL E821 (2004):

$a_\mu^{exp} = (11\,659\,208.0 \pm 5.8)\ 10^{-10}$

Observed Difference with Experiment:

$a_\mu^{exp} - a_\mu^{SM} = (25.2 \pm 9.2) \times 10^{-10}$

➡ 2.7 "standard deviations"

Results and Perspectives

- Hadronic vacuum polarization is dominant systematics for SM prediction of the muon $g-2$
- New data from KLOE in fair agreement with CMD-2 with a (mostly) independent technique
- Discrepancy with τ data (ALEPH & CLEO & OPAL) confirmed
- τ/e^+e^- puzzle has to be solved
- The SM prediction differs by 2.7 σ [e^+e^-] from experiment (BNL 2004) (and by 1.4 σ[τ])

Future experimental input expected from:

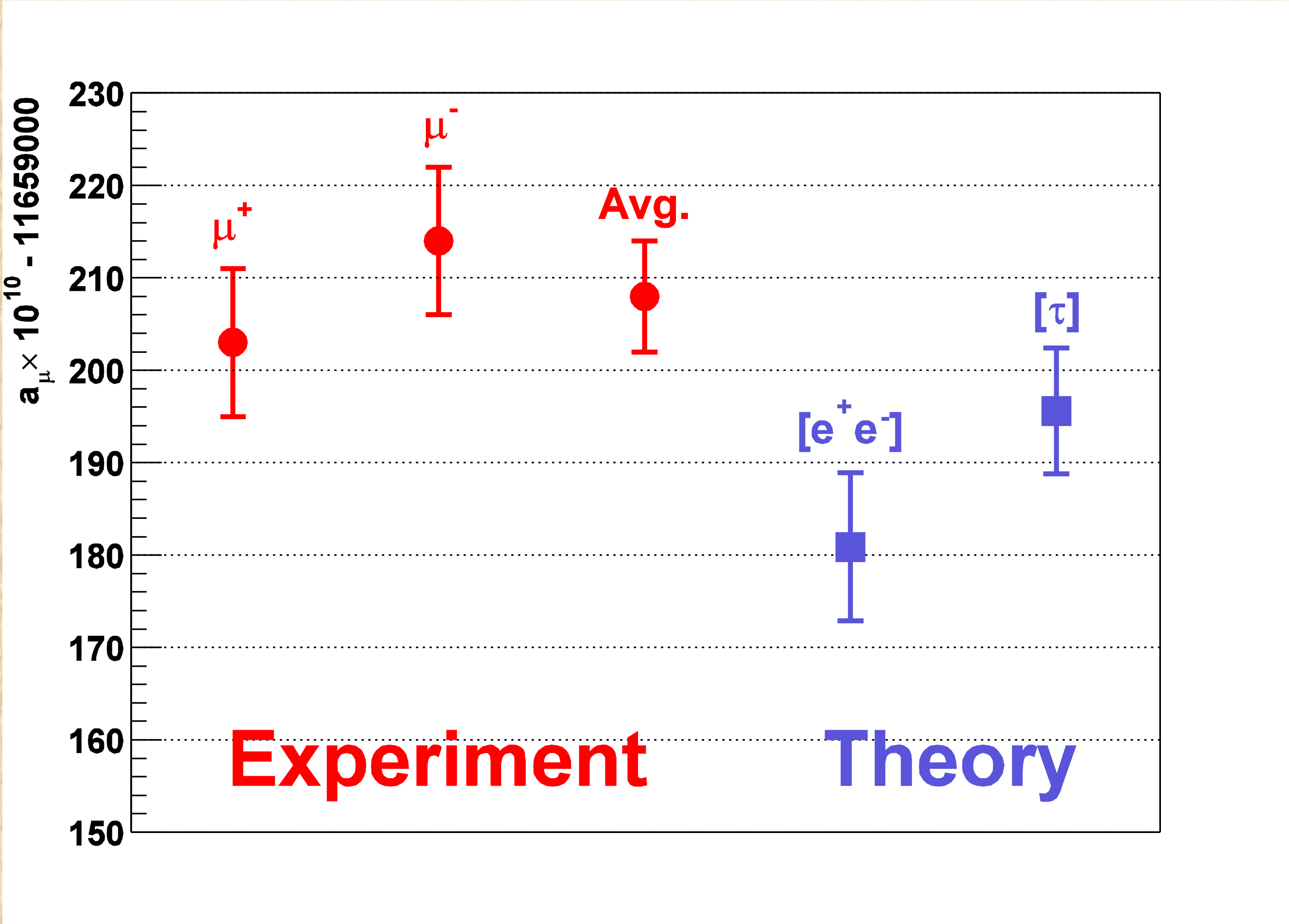
- New CMD-2 results forthcoming, especially at low and large $\pi^+\pi^-$ masses
- BABAR ISR: $\pi^+\pi^-$ spectral function over full mass range, multihadron channels ($2\pi^+2\pi^-$ and $\pi^+\pi^-\pi^0$ already available)

- New proposal submitted by E969 Collaboration aiming at precision of 2.4×10^{-10}
- Ambitious muon $g-2$ project at J-PARC, Japan, aiming at $(0.1 - 0.2) \times \sigma$(BNL-E821)

Hadronic corrections to a_μ within the instanton model of QCD vacuum

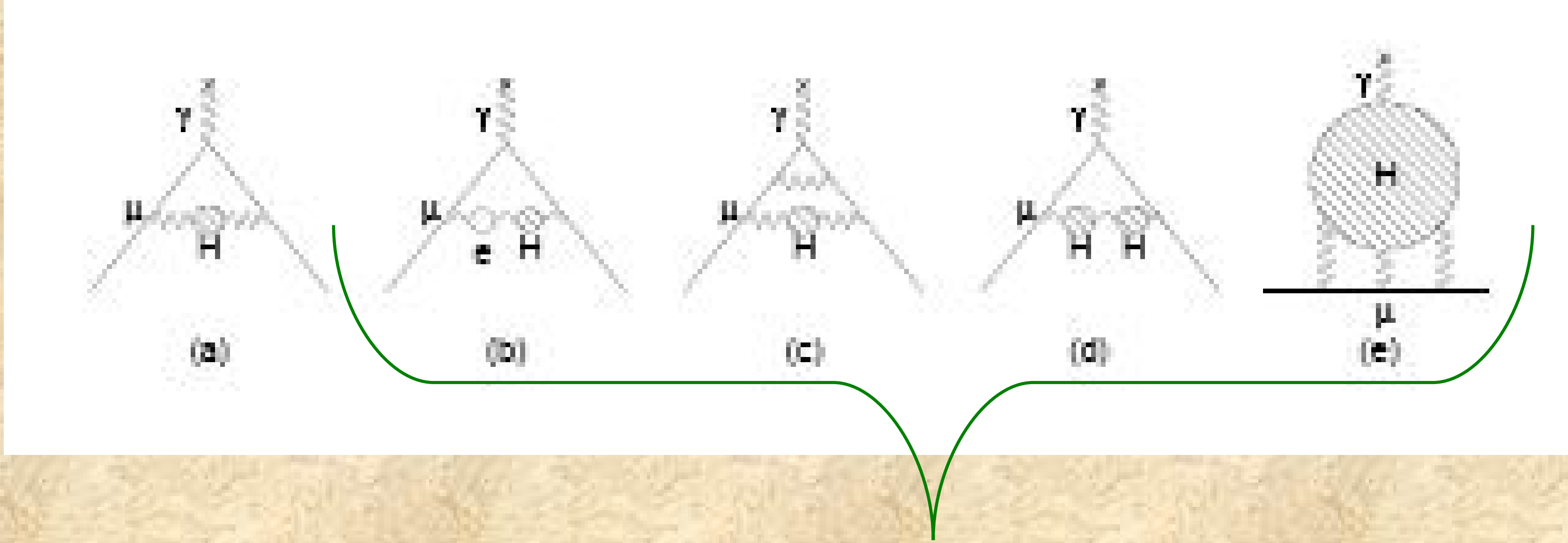
- *Instanton induced effective quark interaction*
- *Conserved V and A currents*
- *Vector Adler function and VAV correlator in the Instanton model*
- *Axial Anomaly and Triangle Dynamics*
- *LO and NLO estimates from Instanton Model*
- *Conclusions*

Muon Anomaly (Current status)



The hadronic contribution to the muon anomaly, where the dominant contribution comes from (a).

The hadronic light-by-light contribution is shown in (e)



α^2

α^3

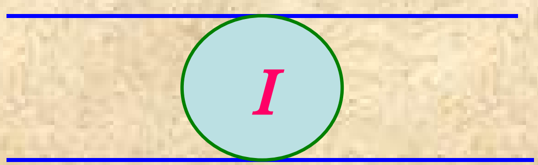
Instanton Liquid Model of QCD Vacuum

Nonlocal Chiral Quark model (χNQM)

SU(2) nonlocal chirally invariant action describing the interaction of soft quarks

$$S = \int d^4x \bar{q}(x)\gamma^{\mu}\left[i\partial_{\mu} - V_{\mu}(x) - \gamma_5 A_{\mu}(x) + m\right]q(x) +$$

$$+\frac{G_i}{2}\int d^4X \int \prod_{n=1}^{4} d^4x_n f_i(x_n)\bar{Q}(X - x_1, X)\Gamma_i Q(X, X + x_3)\bar{Q}(X - x_2, X)\Gamma_i Q(X, X + x_4)$$



Spin-flavor structure of the interaction is given by matrix products $\Gamma_i \otimes \Gamma_i$

$$G(1\otimes 1 + i\gamma_5\tau^a \otimes i\gamma_5\tau^a), \quad G'(\tau^a \otimes \tau^a + i\gamma_5 \otimes i\gamma_5), \quad G_T(\sigma_{\mu\nu} \otimes \sigma_{\mu\nu} + i\sigma_{\mu\nu}\gamma_5 \otimes i\sigma_{\mu\nu}\gamma_5)$$

$$G_{V,A}(\gamma_{\mu}\tau^a \otimes \gamma_{\mu}\tau^a + \gamma_{\mu}\gamma_5 \otimes \gamma_{\mu}\gamma_5), \quad G_{\omega}\gamma_{\mu} \otimes \gamma_{\mu}, \quad G_{f_1}\gamma_{\mu}\gamma_5 \otimes \gamma_{\mu}\gamma_5,$$

(*Instanton interaction: G'=-G*)

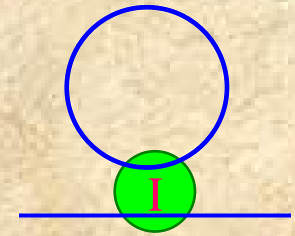
For gauge invariance with respect to external fields V and A the ***delocalized quark fields*** are defined (with straight line path)

$$Q(x,y) = P\exp\left\{ iT^a \int_x^y dz_{\mu}\left[V_{\mu}^a(z) + A_{\mu}^a(z)\gamma_5\right]\right\} q(y)$$

Quark and Meson Propagators

The dressed quark propagator is defined as

$$S^{-1}(p) = \hat{p} - M(p), \quad Z(p) = 1$$

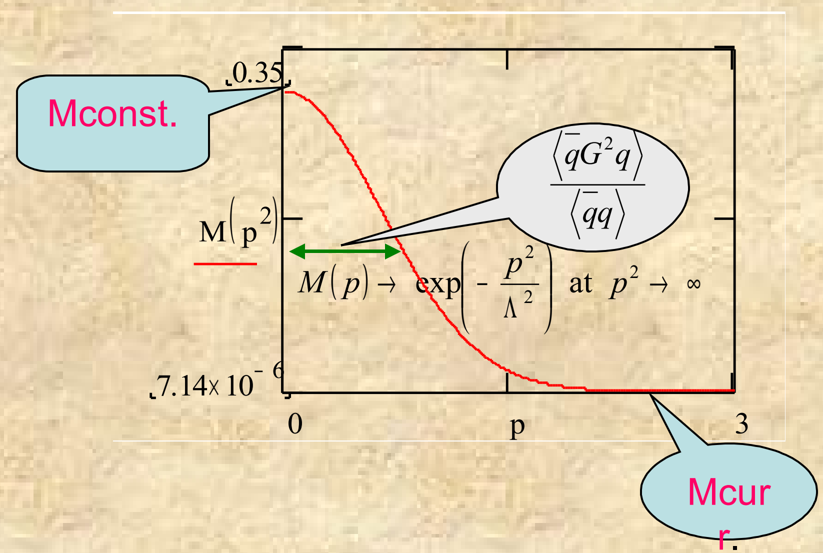


The ***Gap equation***

$$M(p) = m + 4GN_f N_c f^2(p) \int \frac{d^4k}{(2\pi)^4} f^2(k) \frac{M(k)}{k^2 + M^2(k)}$$

has solution

$$M(p) = m + (M_q - m) f^2(p)$$



$$M(p) \to \exp(-p^2/\Lambda^2) \quad \text{at} \quad p^2 \to \infty$$

From existence of derivatives of the quark condensate

$$\langle 0|\bar{q}D^{2n}q|0\rangle = -N_C \int \frac{d^4p}{4\pi^4} p^{2n} \frac{M(p)}{p^2 + M^2(p)}$$

qq *scattering matrix*

$$\hat{T}(q) = G + GJ(q)\hat{T}(q) \Rightarrow \hat{T}(q) = \frac{G}{1 - GJ(q)} \Rightarrow \frac{\overline{V}(q) \times V(q)}{m_M^2 - q^2}$$

with polarization operator $J_{PP}(q^2)\delta^{ab} = -\frac{i}{M_q^2}\int \frac{d^4k}{(2\pi)^4} M(k)M(k+q)Tr\left[S(k)\gamma_5\tau^a S(k+q)\gamma_5\tau^b\right]$

has ***poles*** at positions of meson bound states

$$\det(1 - GJ(q))\big|_{q^2 = m_m^2} = 0, \quad g_{Mqq}^{-2} = (-1)^S \frac{dJ}{dq^2}\bigg|_{q^2 = m_m^2}$$

The pion vertex $\Gamma_\pi^a(k,k') = ig_{\pi qq}\gamma_5\tau^a f(k)f(k')$

with the quark-pion constant $g_{\pi qq}$ satisfying the Goldberger-Treiman relation

$$g_{\pi qq} = \frac{M_q}{f_\pi}$$

Conserved Vector and Axial-Vector currents.

The Vector vertex

~αs in pQCD

$q^2 \to \infty$

$$\Gamma_\mu^a(k,q,k'=k+q) = T^a\left[\gamma_\mu - (k+k')_\mu \frac{M(k')-M(k)}{k'^2-k^2}\right] \to T^a\gamma_\mu$$

AF

Nonlocal part

$$q_\mu \Gamma_\mu^a(k,q,k') = S_F^{-1}(k')T^a - T^a S_F^{-1}(k)$$

WTI

The ***iso-triplet*** Axial-Vector vertex has a pole at $q^2 \to 0$

$$\Gamma_\mu^{5a}(k,q,k'=k+q) = T^a\gamma_5\left\{\gamma_\mu - 2\frac{q_\mu}{q^2}f(k)f(k')\left[M - \frac{mG_PJ_P(q^2)}{1-G_PJ_{PP}(q^2)}\right] + (k+k')_\mu\frac{(f(k')-f(k))^2}{k'^2-k^2}\right\}$$

Pion pole

$$m_\pi^2 = -\frac{m\langle\bar{q}q\rangle}{f_\pi^2}$$

$$q_\mu\Gamma_\mu^{5a}(k,q,k') = \gamma_5 S_F^{-1}(k')T^a + T^a S_F^{-1}(k)\gamma_5$$

AWTI

The ***iso-singlet*** Axial-Vector vertex has a pole at $q^2 \to m^2_{\eta'}$

$$\Gamma_\mu^{50}(k,q,k'=k+q) = \left[\gamma_\mu - \frac{G'}{G} 2\sqrt{M(k')M(k)}\,\frac{q_\mu}{q^2}\,\frac{1-GJ_{PP}(q^2)}{1-G'J_{PP}(q^2)} - (k+k')_\mu \frac{\left(\sqrt{M(k')}-\sqrt{M(k)}\right)^2}{k'^2-k^2}\right]\gamma_5$$

1-G'J_{PP}(q²): η' meson pole

$$q_\mu \Gamma_\mu^{50}(k,q,k') = \gamma_5 S_F^{-1}(k') + S_F^{-1}(k)\gamma_5 + \gamma_5 \frac{2\sqrt{M(k')M(k)}}{1-G'J_{PP}(q^2)}\left(1-\frac{G'}{G}\right)$$

anomalous AWTI

Leading Order Hadronic Corrections from Instanton Model

Vector and Axial-Vector correlators.

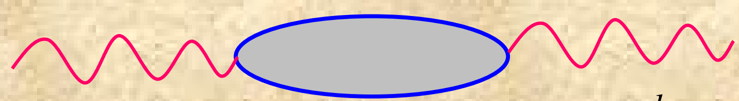
V and A correlators are fundamental quantities of the strong-interaction physics, sensitive to small- and large-distance dynamics. In the limit of exact isospin symmetry they are

$$\Pi_{\mu\nu}^{V,ab}(q) = i\int d^4x e^{iqx}\Pi_{\mu\nu}^{V,ab}(x) = \delta^{ab}\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi_T^V\left(Q^2\right)$$

$$\Pi_{\mu\nu}^{A,ab}(q) = i\int d^4x e^{iqx}\Pi_{\mu\nu}^{A,ab}(x) = \delta^{ab}\left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi_T^A\left(Q^2\right) +$$

$$+ \delta^{ab}q_\mu q_\nu \Pi_L^A\left(Q^2\right)$$

$$\Pi_{\mu\nu}^{J,ab}(x) = \left\langle 0\left|T\left\{J_\mu^a(x)J_\nu^b(0)\right\}\right|0\right\rangle$$



J_μ^a J_ν^b

where the QCD currents are

$$J_\mu^a = \bar{q}\gamma_\mu T^a q, \qquad J_\mu^{5a} = \bar{q}\gamma_\mu\gamma_5 T^a q,$$

Current-current correlators

Current-current correlators are sum of ***dispersive*** and ***contact*** terms

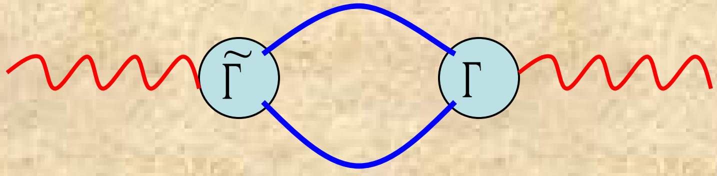
$$-Q^2\Pi^{J}_{\mu\nu}\left(Q^2\right)=K^{J}_{\mu\nu}\left(Q^2\right)+S^{J}_{\mu\nu}\left(Q^2\right),$$

$$K^{J}_{\mu\nu}\left(Q^2\right)=\int\frac{d^4k}{(2\pi)^4}Tr\left[\Gamma^{J}_{\mu}(k,Q,k')S(k')\overline{\Gamma}^{J}_{\nu}(k',-Q,k)S(k)\right],$$

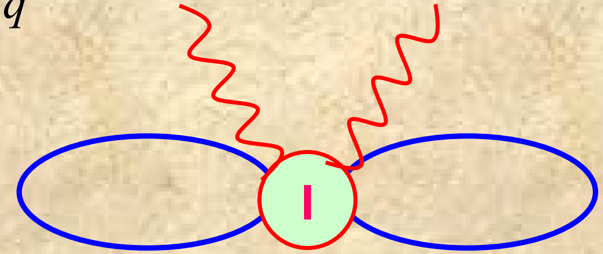
$$S^{J}_{\mu\nu}\left(Q^2\right)=2M_q\int\frac{d^4k}{(2\pi)^4}Tr\left[\Gamma^{J}_{\mu}(k,Q,-Q,k')S(k)\right]$$

The transverse and longitudinal part of the correlators are extracted by projectors

$$P^{T}_{\mu\nu}=\frac{1}{3}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right),\quad P^{L}_{\mu\nu}=\frac{q_\mu q_\nu}{q^2}$$



Dispersive term



Contact term

Current-current correlators in χNQM

V correlator

$$-Q^2\Pi^V(Q^2) = 2N_C\int\frac{d^4k}{(2\pi)^4}\frac{1}{D_+D_-}\left\{M_+M_- + k_+k_- - \frac{2}{3}k_\perp^2 + \frac{4}{3}k_\perp^2\left[\left(\Delta^{(1)}M(k)\right)^2(k_+k_- - M_+M_-) - \Delta^{(1)}M^2(k)\right]\right\} +$$

$$+ 4N_C\int\frac{d^4k}{(2\pi)^4}\frac{M(k)}{D(k)}\left\{M'(k) + \frac{4}{3}k_\perp^2\Delta^{(2)}M(k)\right\}$$

and the difference of the V and A correlators

$$-Q^2\Pi^{V-A}(Q^2) = 4N_C\int\frac{d^4k}{(2\pi)^4}\frac{1}{D_+D_-}\left\{M_+M_- + \frac{4}{3}k_\perp^2\left[-\sqrt{M_+M_-}\,\Delta^{(1)}M(k) + \left(\Delta^{(1)}\sqrt{M(k)}\right)^2\left(\sqrt{M_+}\,k_+ + \sqrt{M_-}\,k_-\right)^2\right]\right\}$$

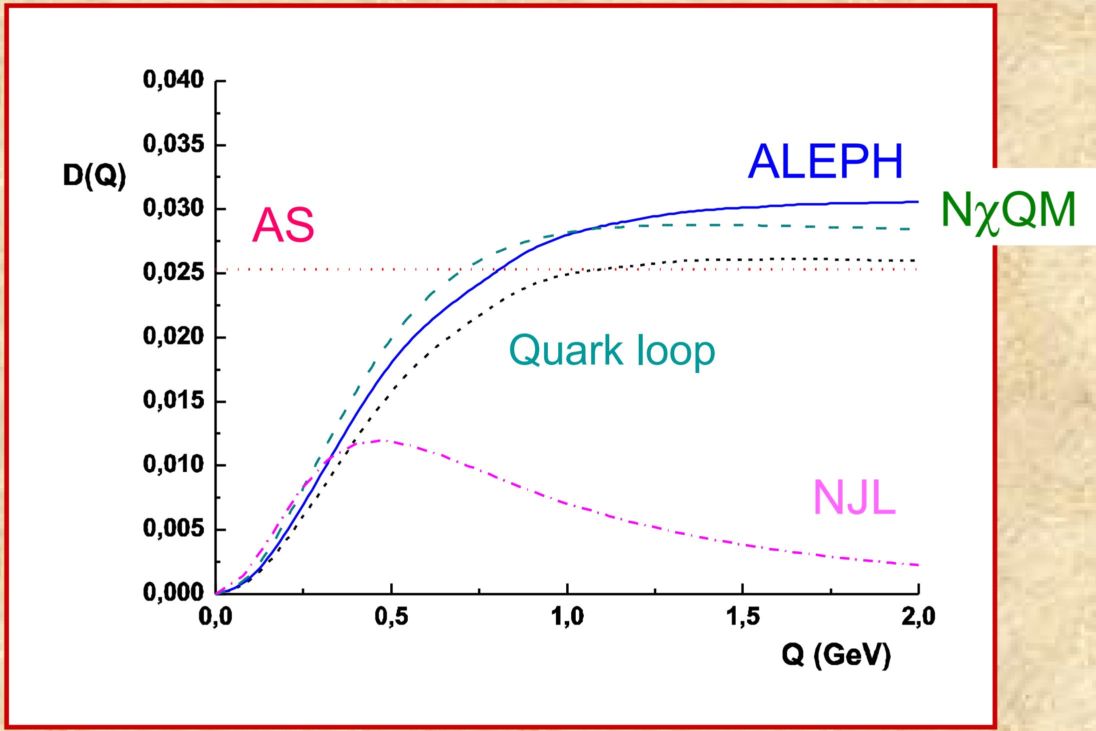
One may explicitly verify that the Witten inequality is fulfilled
and that at Q^2=0 one gets the results consistent with the first Weinberg sum rule

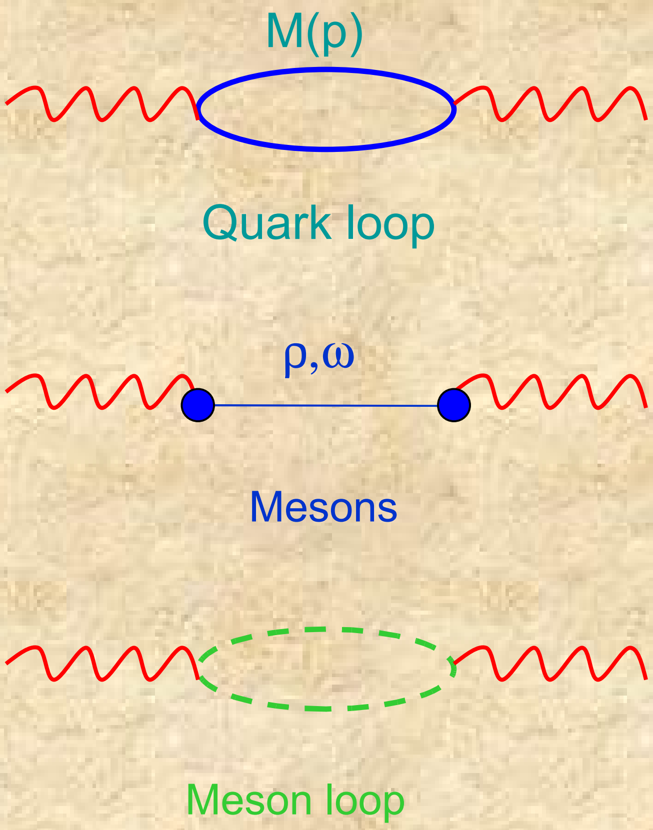
$$-Q^2\Pi_T^{V-A}(Q^2=0) = f_\pi^2, \quad \Pi_L^{V,A}(Q^2) = 0$$

Above we used definitions

$$\Delta^{(1)}M(k) = \frac{M_+ - M_-}{k_+^2 - k_-^2}, \quad \Delta^{(2)}M(k) = \frac{1}{k_+^2 - k_-^2}\left(\frac{M_+ - M_-}{k_+^2 - k_-^2} - M'(k_-)\right)$$

NχQM Adler function and ALEPH data

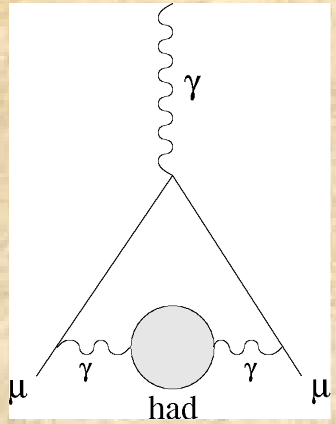




Adler function is defined as

$$D_V(Q^2) = -Q^2 \frac{d\Pi_V(Q^2)}{dQ^2} = \int_0^\infty dt \frac{Q^2}{(t+Q^2)^2} \rho_V(t)$$

III. LO Hadronic contribution to g_μ–2



The calculations are based on the ***spectral representation***

$$a_\mu^{(2)\mathrm{hvp}} = \frac{8}{3}\alpha^2 \int_{4m_\pi^2}^{\infty} dt \frac{K(t)}{t} \frac{1}{\pi} \mathrm{Im}\Pi(t), \quad K(t) = \int_0^1 dx \frac{x^2 m_\mu^2}{x^2 m_\mu^2/(1-x) + t}$$

which is rewritten via Adler function as

$$a_\mu^{(2)\mathrm{hvp}} = \frac{8}{3}\alpha^2 \int_0^1 dx \frac{(1-x)(1-x/2)}{x} D\left(\frac{x^2}{1-x} m_\mu^2\right)$$

Phenomenological estimates give

$$a_\mu^{(2)\mathrm{hvp}} = \begin{cases} (6,849 \pm 0.077) \cdot 10^{-8}, & e^+e^-, e\pi \\ (6,932 \pm 0.076) \cdot 10^{-8}, & e^+e^-, e\pi, \tau \end{cases}$$

and from NχQM one gets

$$a_{\mu,\mathrm{N}\chi\mathrm{QM}}^{(2)\mathrm{hvp}} = (6,23 \pm 0.5) \cdot 10^{-8},$$

$$a_{\mu,\mathrm{Qloop}}^{(2)\mathrm{hvp}} = 5.33 \cdot 10^{-8}, \quad a_{\mu,\rho\omega\text{-mesons}}^{(2)\mathrm{hvp}} = 0.13 \cdot 10^{-8}, \quad a_{\mu,\mathrm{Mloop}}^{(2)\mathrm{hvp}} = 0.77 \cdot 10^{-8},$$

Other model approaches

Phenomenological estimate: $a_\mu^{(2)\mathrm{hvp}} = (6{,}849 \pm 0.077)\cdot 10^{-8}, \quad e^+e^-, e\pi$

NχQM: $a_{\mu,\mathrm{N}\chi\mathrm{QM}}^{(2)\mathrm{hvp}} = (6{,}53 \pm 0.3)\cdot 10^{-8}$

Extended Nambu-Iona-Lasinio:
(Bijnens, de Rafael, Zheng)

$a_{\mu,\mathrm{ENJL}}^{(2)\mathrm{hvp}} \approx 7.5\cdot 10^{-8}$

Minimal hadronic approximation
(Local duality):
(Peris, Perrottet, de Rafael)

$a_{\mu,\mathrm{MHA}}^{(2)\mathrm{hvp}} \approx (4.7 \pm 1.7)\cdot 10^{-8}$

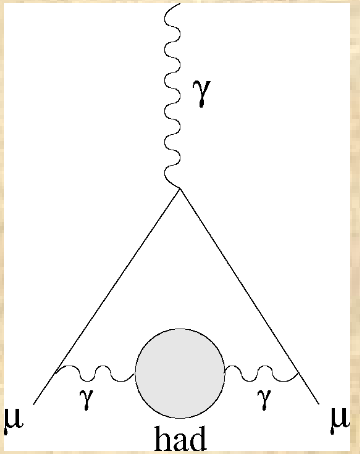
Lattice simulations:
(Blum;
Goeckler et.al. QCDSF Coll.)

$a_{\mu,\mathrm{Lattice}}^{(2)\mathrm{hvp}} \approx (4.46 \pm 0.23)\cdot 10^{-8}$

Next-to-Leading Order Hadronic Corrections from Instanton Model

LO vs NLO_Hadron corrections to Muon Anomalous magnetic moment
(Theory)

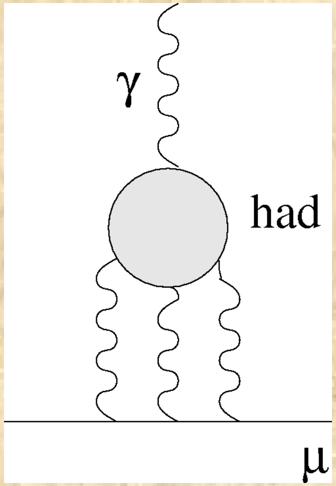
Vacuum Polarization



Lowest Order
(EM and Tau data;
Adler function)

1% from phenomenology,
10% from the model

Light-by-Light scattering

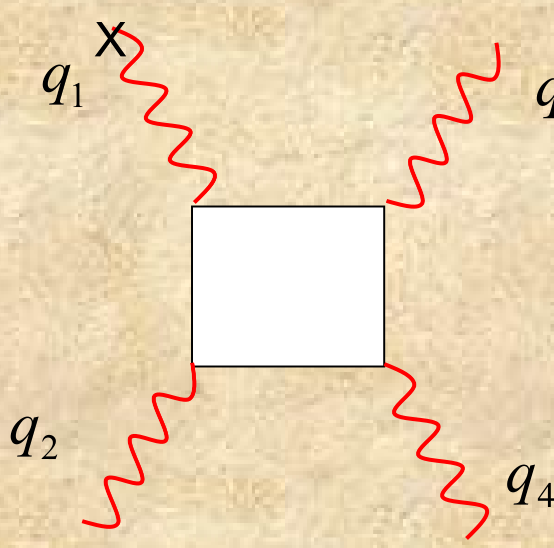


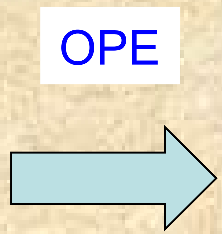
Higher Order
(OPE and Triangle diagram)

NO phenomenology,
50% from the existing model,
The aim to get 10% accuracy

The hadronic light-by-light contribution is likely to provide the ultimate limit to the precision of the standard-model value of a_μ.

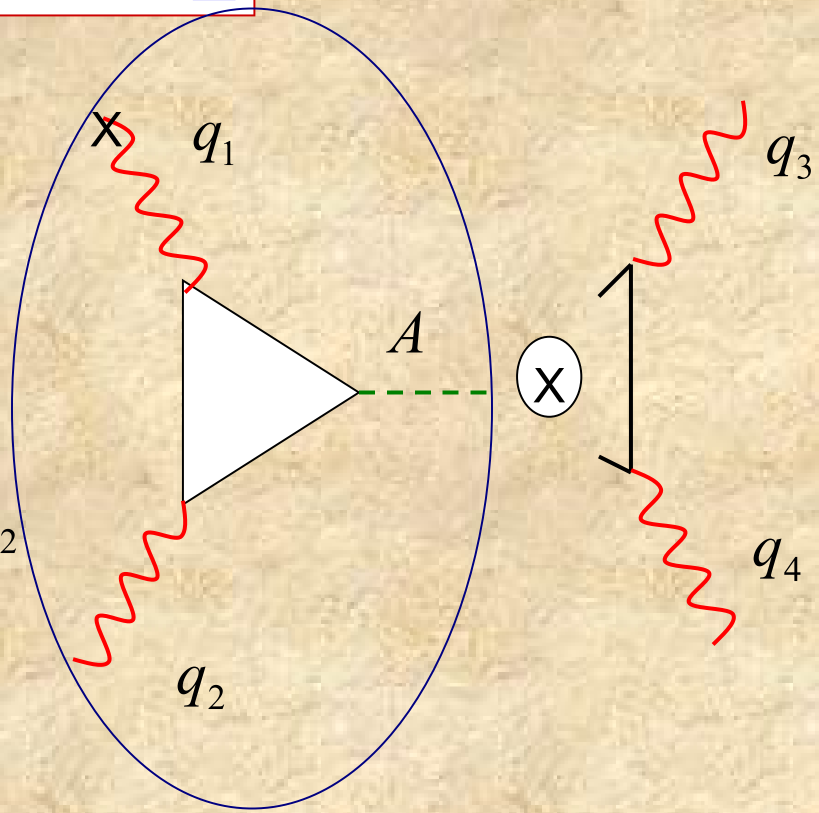
LBL, OPE and Triangle diagram VAV





$$q_1^2 = 0,$$

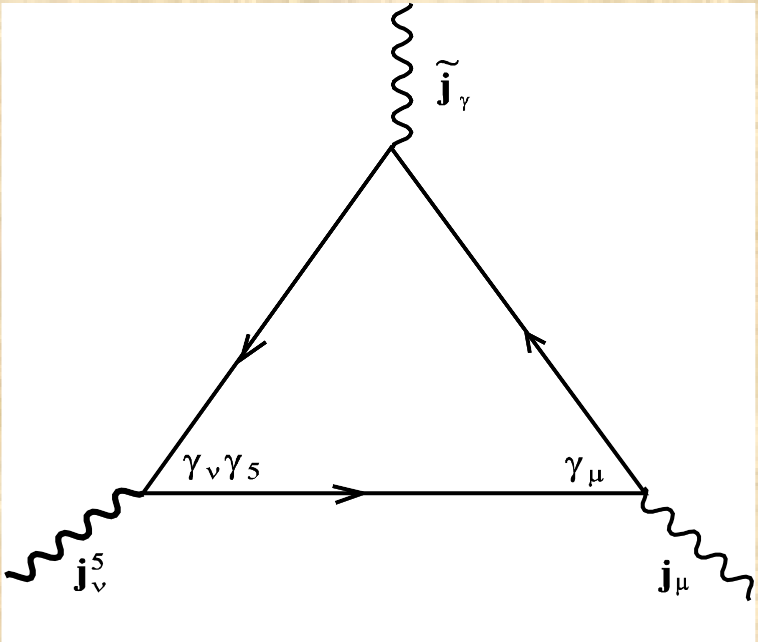
$$q_3^2 \approx q_4^2 >> q_2^2$$



$$i\int d^4x d^4y e^{-iq_3x-iq_4y} T\left\{ j_{\mu_1}(x), j_{\mu_2}(y) \right\} = \int d^4z e^{-i(q_3+q_4)z} \frac{2i}{\hat{q}^2} \varepsilon_{\mu_1\mu_2\delta\rho} \hat{q}^\delta j_5^\rho(z) + \ldots$$

(Similar to $\pi^0\gamma^*\gamma^*$ amplitude at large photon virtualities)

Triangle diagram



The structure of V^*AV amplitude

For specific kinematics $q_2 \equiv q$ - arbitrary, q_1 is real photon with $q_1 \to 0$

Only 2 structures survives in the triangle amplitude

$$T_{\mu\nu\lambda}(q_1,q_2) = w_T\left(q_2^2 q_1^\rho \varepsilon_{\rho\mu\nu\lambda} + q_2^\nu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\mu\lambda} + q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu}\right) - w_L q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu},$$

where

$$w_T(q) = A_4(q_1 = 0, q_2 = q) + A_6(q_1 = 0, q_2 = q), \quad w_L(q) = A_4(q_1 = 0, q_2 = q)$$

The amplitude is transversal with respect to vector current

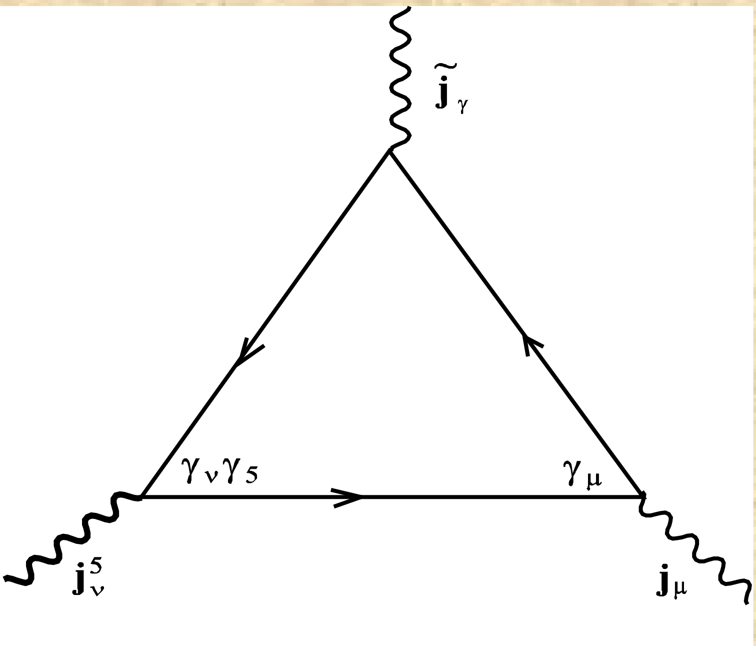
$$q_1^\mu T_{\mu\nu\lambda} = q_2^\nu T_{\mu\nu\lambda} = 0$$

but longitudinal with respect to axial-vector current

$$q_2^\lambda T_{\mu\nu\lambda} = -\left(w_L q_2^2\right) \cdot q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu}$$

Thus Adler-Bell-Jackiw anomaly

Operator Product Expansion and V^*AV amplitude



In local theory for massive quarks one gets

$$w_L = 2w_T = \frac{2N_C}{3}\int_0^1 dx \frac{x(1-x)}{x(1-x)q^2 + m^2}$$

Which in chiral limit (m=0) becomes

$$w_L = 2w_T = \frac{2N_C}{3}\frac{1}{q^2}$$

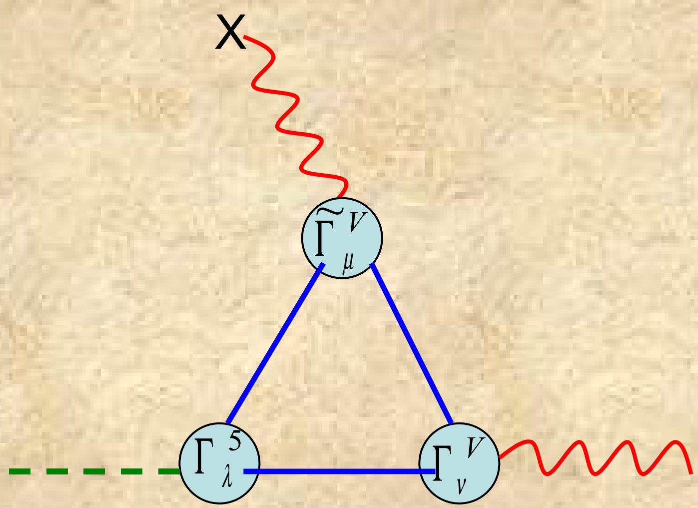
- Perturbative nonrenormalization of wL (Adler-Bardeen theorem, 1969)
- Nonperturbative nonrenormalization of wL (‘t Hooft duality condition, 1980)
- Perturbative nonrenormalization of wT (Vainshtein theorem, 2003)
- Nonperturbative corrections to wT at large q are $O(1/q^6)$ (De Rafael et.al., 2002)
- Absence of Power corrections at large q in Instanton model (this work)

VAV* correlator

$$T_{\mu\nu\lambda}(q_1,q_2) = -2N_C \int \frac{d^4k}{(2\pi)^4} Tr\left[\Gamma_\mu^V(k+q_1,k)S(k+q_1)\Gamma_\lambda^{A5}(k+q_1,k-q_2)S(k-q_2)\Gamma_\nu^V(k,k-q_2)S(k)\right]$$

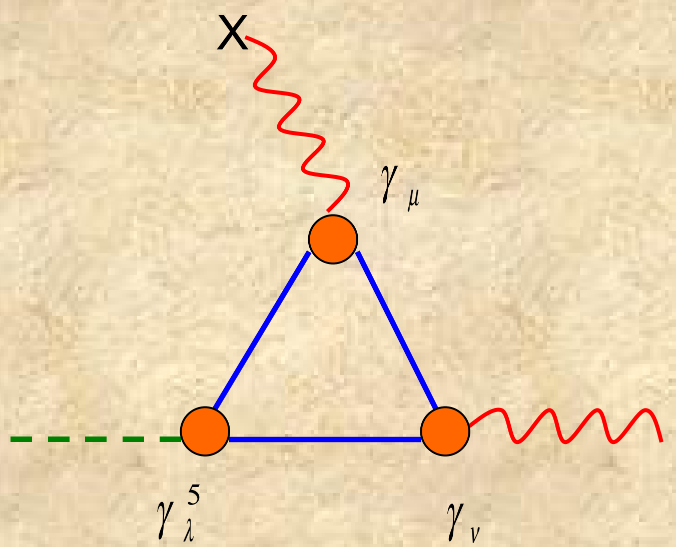
It is transverse with respect to vector currents

$$q_1^\mu T_{\mu\nu\lambda}(q_1,q_2) = q_2^\nu T_{\mu\nu\lambda}(q_1,q_2) = 0$$

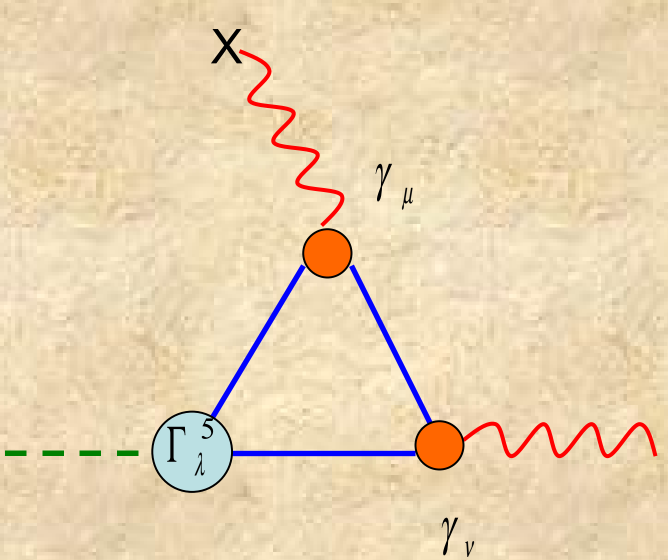


Triangle diagram

Anomalous wL structure (NonSinglet)



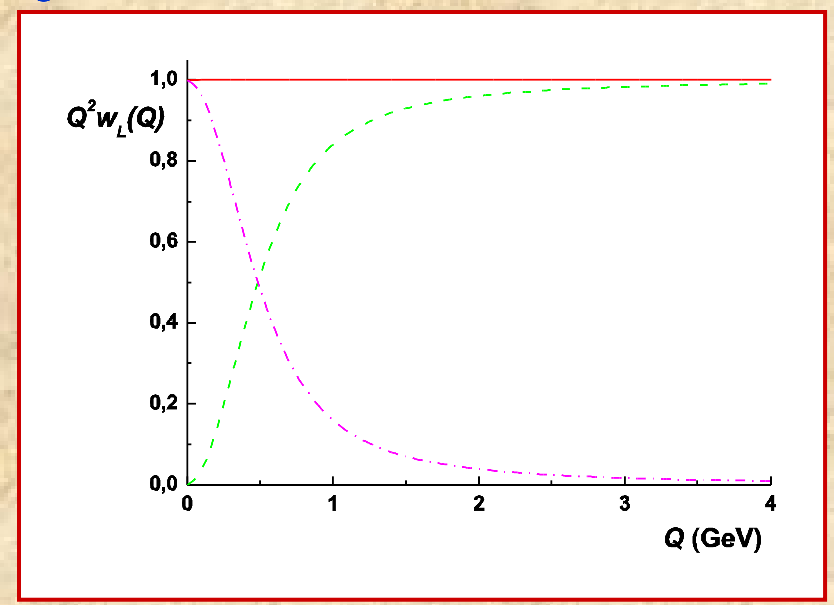
+



+ rest

Diagram with Local vertices

Diagram with NonLocal Axial vertices



$$w_L^{(3)} = \frac{2N_C}{3}\frac{1}{q^2}$$

With accordance with Anomaly and 't Hooft

Anomalous wL structure (Singlet)

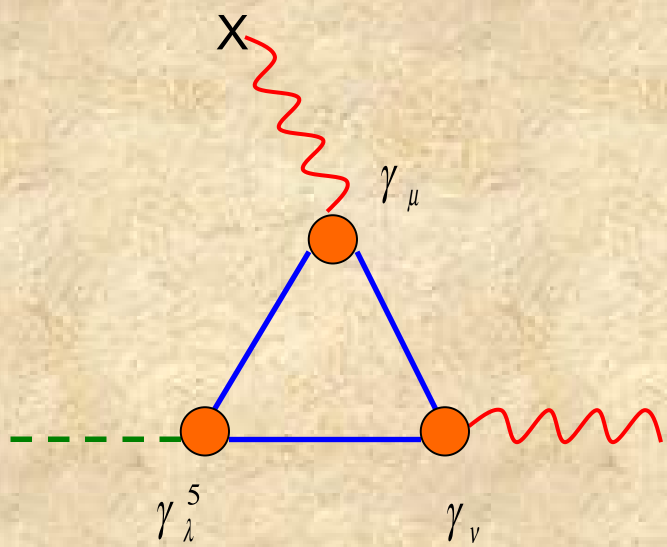


Diagram with Local vertices

+

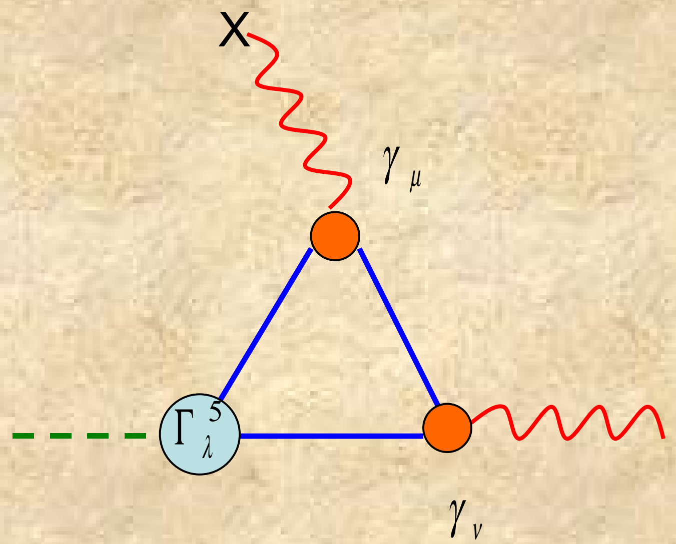
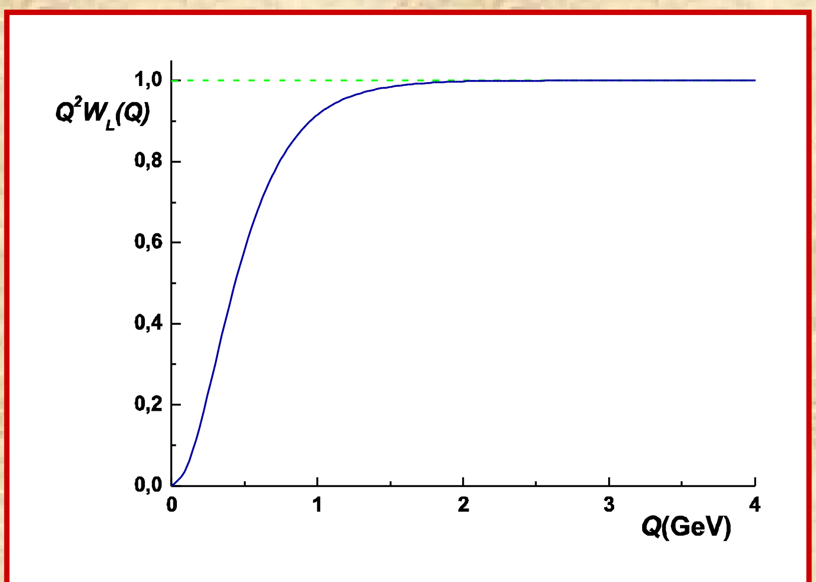


Diagram with NonLocal Axial vertices

+ rest



$$q^2 w_L^{(0)}(q^2)\Big|_{q^2 \to 0} = 0$$

With accordance with Anomaly and 't Hooft duality principle (no massless states in singlet)

wLT structure

$$w_{LT} = w_L - 2w_T \qquad w^{PT}{}_{LT} = 0$$

$$w_{LT}\left(q^2\right) = \frac{4N_C}{3q^2}\int \frac{d^4k}{\pi^2}\frac{\sqrt{M_-}}{D^2{}_+ D_-}\left\{\sqrt{M_-}\left[M_+ - 2M'_+\left(k^2 - 2\frac{(kq)}{q^2}\right)\right] - \right.$$

$$\left. - \frac{4}{3}k_\perp^2\left[-\sqrt{M_+}\,\Delta^{(1)}M(k) + 2(kq)M'_+\,\Delta^{(1)}\sqrt{M(k)}\right]\right\}$$

$$w_{LT}\left(q^2 \to \infty\right) \cong f\left(q^2\right)$$

Exponentially suppressed, no power corrections!

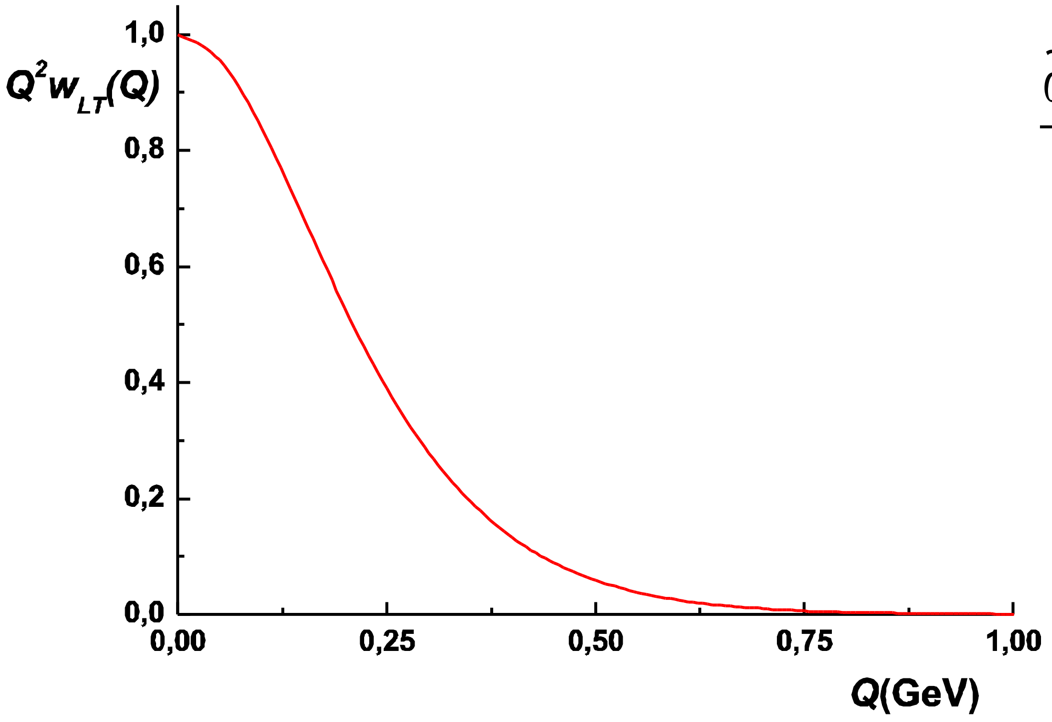
Above we used definitions

$$k_+ = k,\; k_- = k - q,\; M_\pm = M\left(k_\pm\right),$$

$$\Delta^{(1)}M(k) = \frac{M_+ - M_-}{k_+^2 - k_-^2},$$

wLT in the Instanton Model (NonSinglet)

$$w_{LT} \equiv w_L - 2w_T$$

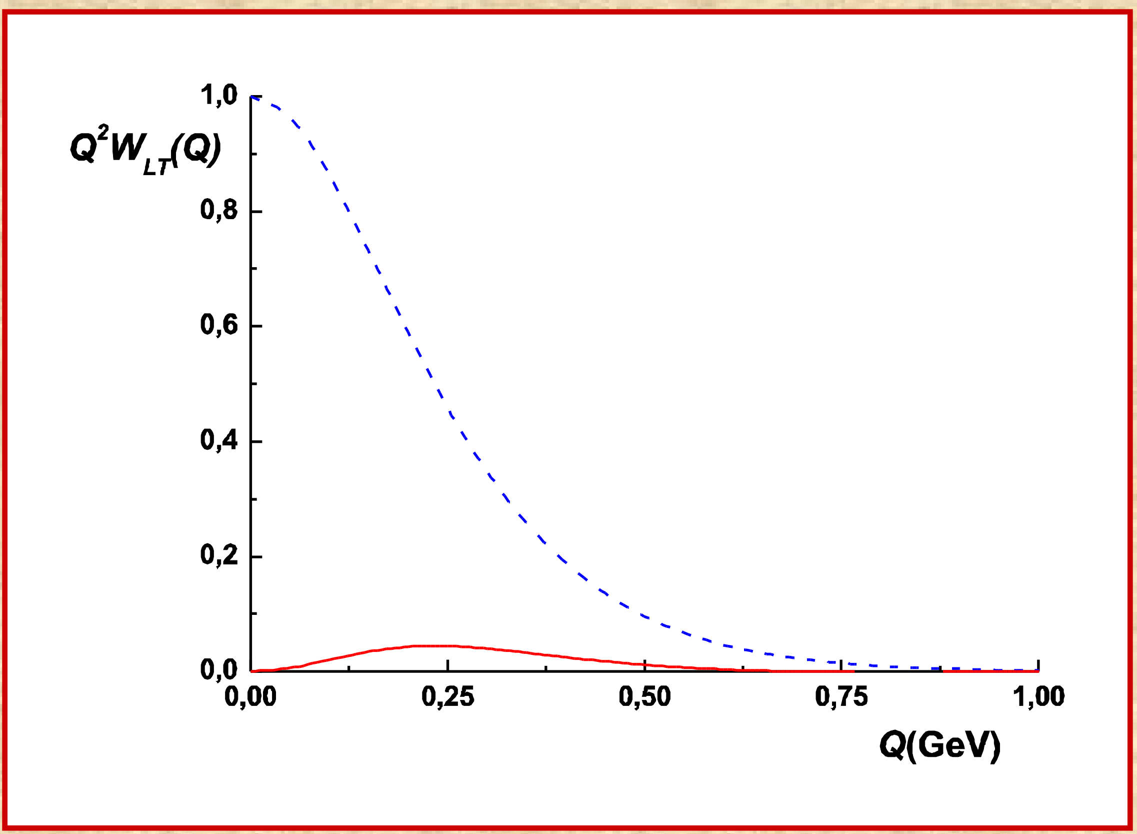


$$\left.\frac{\partial w_{LT}^{(3)}(q^2)}{\partial q^2}\right|_{q^2 \to 0} = -4.6\,\mathrm{GeV}^2$$

- Perturbative nonrenormalization of wL (Adler-Bardeen theorem, 1969)
- Nonperturbative nonrenormalization of wL ('t Hooft duality condition, 1980)
- Perturbative nonrenormalization of wT (Vainshtein theorem, 2003)
- Nonperturbative corrections to wT at large q $O(1/q^6)$ (De Rafael et.al., 2002)
- Absence of Power corrections at large q in Instanton model (this work)

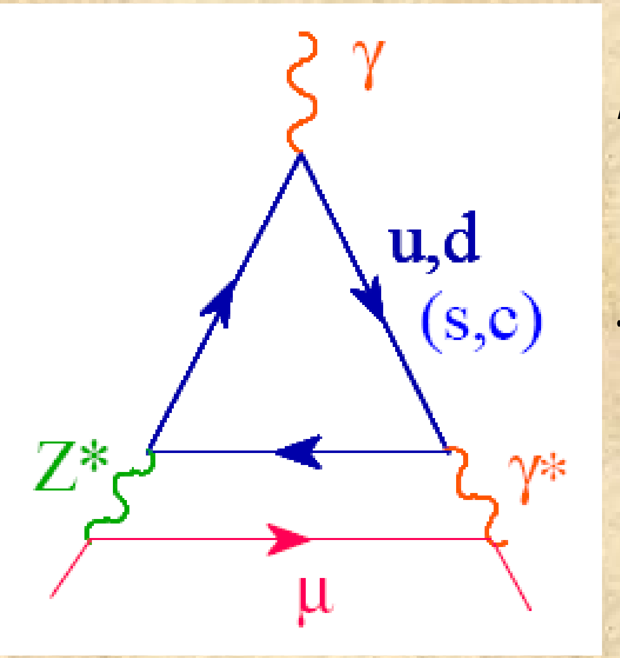
wLT in the Instanton Model (Singlet)

$$w_{LT} \equiv w_L - 2w_T$$



$$w^{(0)}{}_{LT}\left(q^2 = 0\right) = 0.6\,\mathrm{GeV}^2$$

Z*γγ* contribution to a_μ



$$\Delta a_\mu^{EW} = 2\sqrt{2}\frac{\alpha}{\pi} G_\mu m_\mu^2 i \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + 2kp} \cdot$$

$$\cdot \left[\frac{1}{3}\left(1 + \frac{2(kp)^2}{k^2 m_\mu^2}\right)\left(w_L - \frac{m_Z^2}{m_Z^2 - k^2} w_T\right) + \frac{m_Z^2}{m_Z^2 - k^2} w_T \right]$$

Perturbative QCD
(Anomaly cancelayion)

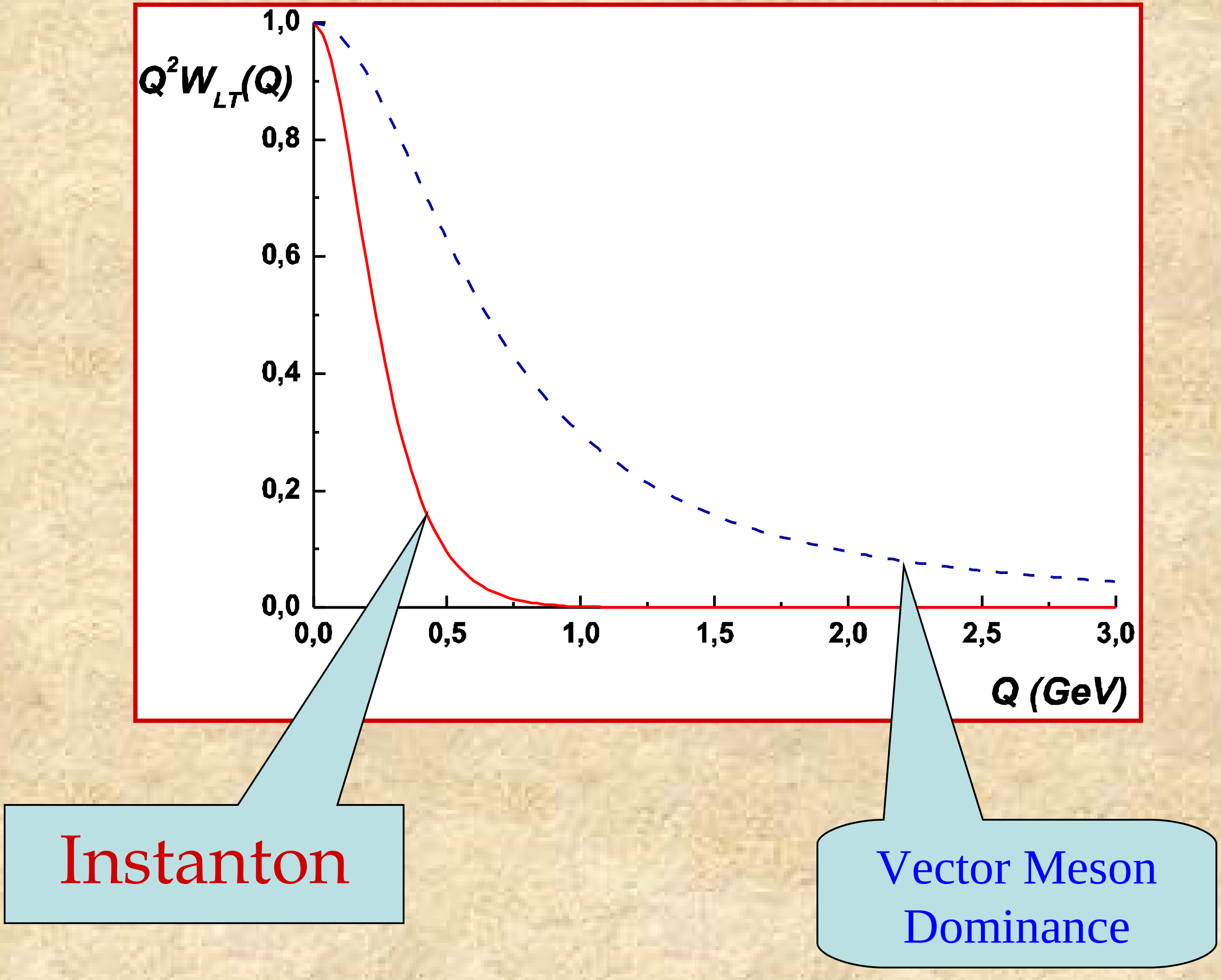
$$\Delta a_\mu^{\mathrm{EW}} = 0$$

VMD + OPE
(Melnikov, Vainshtein, 2003)

$$\Delta a_\mu^{\mathrm{EW}} \approx -2.02 \cdot 10^{-11}$$

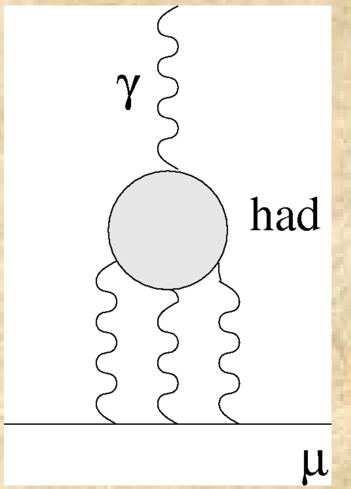
Instanton model:
(Dorokhov, 2005)

$$\Delta a_\mu^{\mathrm{EW}} = -1.48 \cdot 10^{-11}$$



1,0
$Q^2W_{LT}(Q)$
0,8
0,6
0,4
0,2
0,0
0,0
0,5
1,0
1,5
2,0
2,5
3,0
Q (GeV)
Instanton
Vector Meson Dominance

Light-by-Light contribution to muon AMM



- This contribution must be determined by calculation.
- the knowledge of this contribution limits knowledge of theoretical value.

Vector Meson Dominance like model:
(Knecht, Nyffeler 2002)

$$a_\mu^{(3)\mathrm{LbL}}\left(\pi^0\right) = 0.058 \cdot 10^{-8}$$

VMD + OPE
(Melnikov, Vainshtein 2003)

$$a_\mu^{(3)\mathrm{LbL}}\left(\mathrm{PS},\mathrm{PV}\right) = \left(0.136 \pm 0.07\right) \cdot 10^{-8}$$

Instanton model:
(Dorokhov 2005)

$$a_\mu^{(3)\mathrm{LbL}}\left(PS, PV\right) = \left(0.105 \pm 0.01\right) \cdot 10^{-8}$$

Conclusions for the second part

- Instanton model is appropriate for the study of vacuum and light meson internal structure.
- The longitudinal structure of triangle diagram is norenomalized by nonperturbative corrections in agreement with ‘t Hooft arguments
- Transverse structure is calculated for arbitrary *q.*
- At large *q* the transversal amplitude has exponentially decreasing corrections, that reflects nonlocal structure of QCD vacuum in terms of **instantons**
- **Instanton model is a way to extrapolate the results of OPE and χPT to the regions not achievable by these methods.**

Summary

- g-2 continues to be at the center of interest in particle physics.
- E821 reached 0.5 ppm precision with a 2.7 σ discrepancy with SM
- The e+e- & τ puzzle remains
- LO hadronic error could be reduced by a factor about 2 over the next few years.
- Hadronic LBL piece could be realistically estimated within Instanton model
- New BNL experiment is proposed to improve precision of $(g-2)_{\mu}$ by a factor 2.5.