Casimir Effects:
From the Tabletop to the Standard Model

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★ A new tool for computation in renormalizable quantum field theories.

★ Effective action for
- Time-independent field configurations
- One loop (order $\hbar$)
- In renormalizable field theories

★ Exact (at one loop)

★ Unambiguous (as you would expect in a renorm. q.f.t.)

★ Renormalized (in standard schemes)

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Goals

Study QFT extended objects — solitons, domain walls, etc. – in a quantitative, practical way. For example, compute the energy of a static field configuration, to implement variational search for quantum solitons.

Requirements:

- ⋆ Unambiguous treatment of renormalization. Quantum field theory has divergences, which are cancelled by divergent counterterms. Ambiguities are resolved by imposing perturbative renormalization conditions on low-order Green's functions, which must be implemented precisely.
- ⋆ Practical for numerical calculation. Actual calculation must not involve cancellation of large numbers.
- ⋆ Able to handle situations where configuration is not a solution to the equations of motion.
- ⋆ Valid to all orders in the derivative approximation, since often we expect interesting phenomena to occur precisely when the size of the background field configuration is comparable to the Compton wavelength of the dynamical particle. (Derivative approximation is a useful check of our method in the regime where it is valid).

Introduction and Overview

★ An Energy Functional in QFT

For time-independent fields $S_{\text{eff}}[\phi(\vec{x})] \to T E_{\text{eff}}[\phi(\vec{x})]$

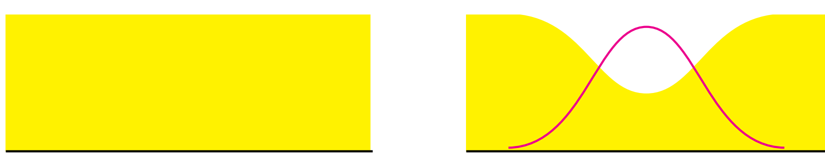
$$E\Big[\{\phi_j(\vec{x})\},\{g\},\{m\}\Big]$$

- Energy functional of renormalized fields, masses and couplings.
- Search for stationary $\{\phi_j(\vec{x})\}$ at fixed g and m,

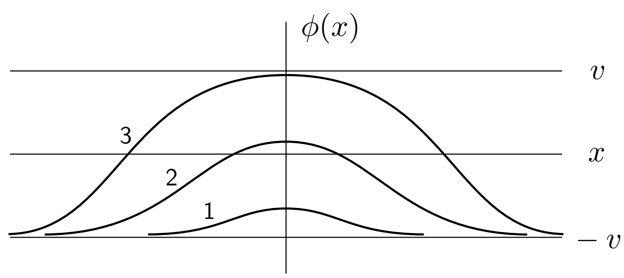
$$\left.\frac{\delta E}{\delta \phi_j(\vec{x})}\right|_{g,m} = 0 \quad \Rightarrow \quad \phi_j = \hat{\phi}_j$$

★ Searching for solitons in renormalizable theories.

- Solitons in the Standard Model ("Top Quark Bags")

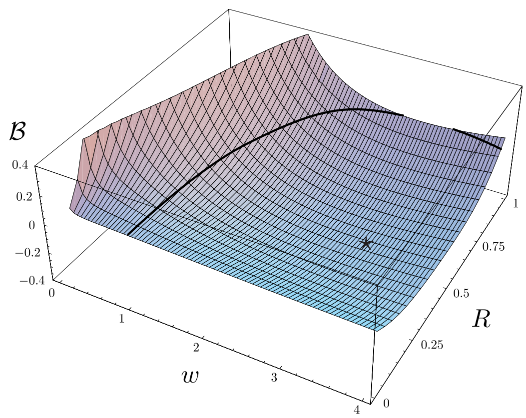


- Unambiguous calculation of mass and central charge for SUSY soliton in $1+1$ dimension...



- Quantum stabilization of solitions in $1+1$ dimensional chiral models

  - $\mathcal{L} = g\bar{\psi}(\phi_1 + i\gamma_5\phi_2)\psi$
  - No classical soliton
  - Robust quantum soliton is a fermion



★ Solitons $\longrightarrow$ "Interfaces"

- Considerable interest in background configurations that are nontrivial in m-dimensions but "trivial" in n-dimensions. . .

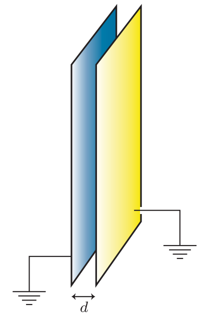
  Cosmic Strings/ Vortices: m=2 n=1

  Branes: m=1 n=4,5,. . .

  Monopoles on interfaces: m=3 n=1,2,. . .

- New computational method takes solitons to interfaces.

★ The Classic Casimir Effect:
Energy Densities and Forces

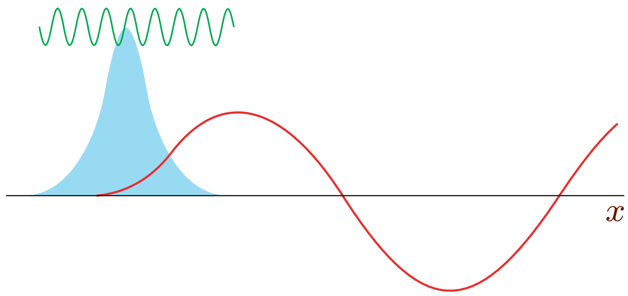


- Quantum zero-point energies ($\equiv$ one-loop effective energy) in the presence of boundaries.

- Boundary conditions are an idealization of interactions with materials.

- Even for the simplest geometries, calculations appear to be fraught with divergences. Are divergences **benign** – ie associated with renormalization of the parameters of the theory? or
  malignant – signatures that the physical effects depend on the cutoffs that characterize the high energy behavior of the material?

- Interpret in light of renormalizable quantum field theory: Replace boundary conditions by renormalizable couplings to background fields. Boundary conditions correspond to singular background fields: "boundary condition limit".



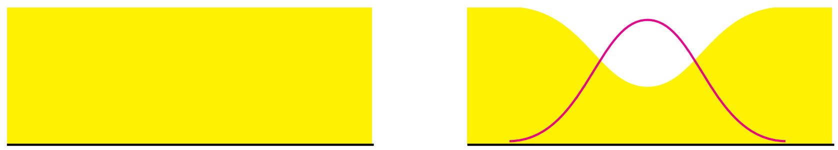
- We can successfully compute and renormalize Casimir energy in the presence of strong, localized, but smooth background fields. Then study what happens as backrgound field approaches boundary condition limit.

- Total **renormalized** Casimir energy always diverges in the boundary condition limit. So there is no meaningful, formal, mathematical "Casimir problem" for the energy in renormalizable QFT.

- However,
  - ⋆ Casimir energy density away from boundaries is finite and calculable even as background fields go to the boundary condition limit.
  - ⋆ The forces between rigid objects also remains finite in the boundary condition limit.
  - ⋆ The Casimir "stress" on a surface cannot be defined in a way that is independent of the details of the dynamics on the surface. Thus, for example, the vacuum pressure on a grounded sphere cannot be defined independent of the detailed treatment of the surface dynamics.

⋆ Heavy fermions $\Longleftrightarrow$ Solitons in the Standard Model

- Naive idea of early 1990's **"Top Quark Bags"**



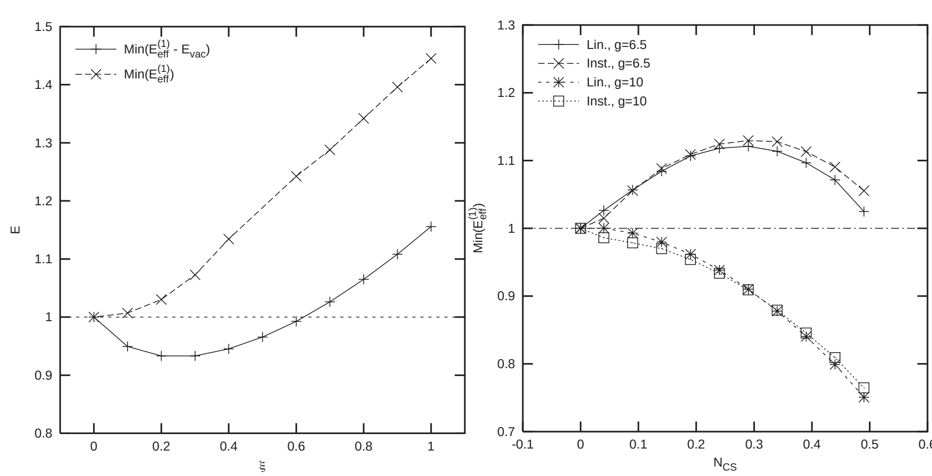
$$E[\phi] = E_{\text{classical}}[\phi] + \hbar\omega_0$$

Favors non-trivial ϕ.

- However, vacuum fluctuation energy cannot be ignored:

$$E[\phi] = E_{\text{classical}}[\phi] + \hbar\omega_0 + \frac{1}{2}\sum_j(\hbar\omega_j - \hbar\omega_0)$$

- Destabilization!



Outline of Remainder of Talk

- ⋆ References
- ⋆ Basic Idea
- ⋆ "Born Renormalization" via dimensional regularization
- ⋆ How it all works...
- ⋆ Application 1: Quantum Soliton Formation in $1+1$ Dimensions.
- ⋆ Application 2: Interfaces
- ⋆ Application 3: True Casimir Energies and Forces
- ⋆ Progress report and future plans

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Typically define, cutoff and calculate Casimir ($\equiv$ one-loop effective) energy (in principle if not in practice), but subtraction of divergences is ad hoc, not connected with perturbative renormalization scheme.

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Basic Idea

Work in $n + 1$ dimensional space-time, where n is chosen so that the entire theory is finite. Typically $0 < n < 1$. Later analytically continue to integer dimensions as appropriate.

[Standard dimensional regularization.]

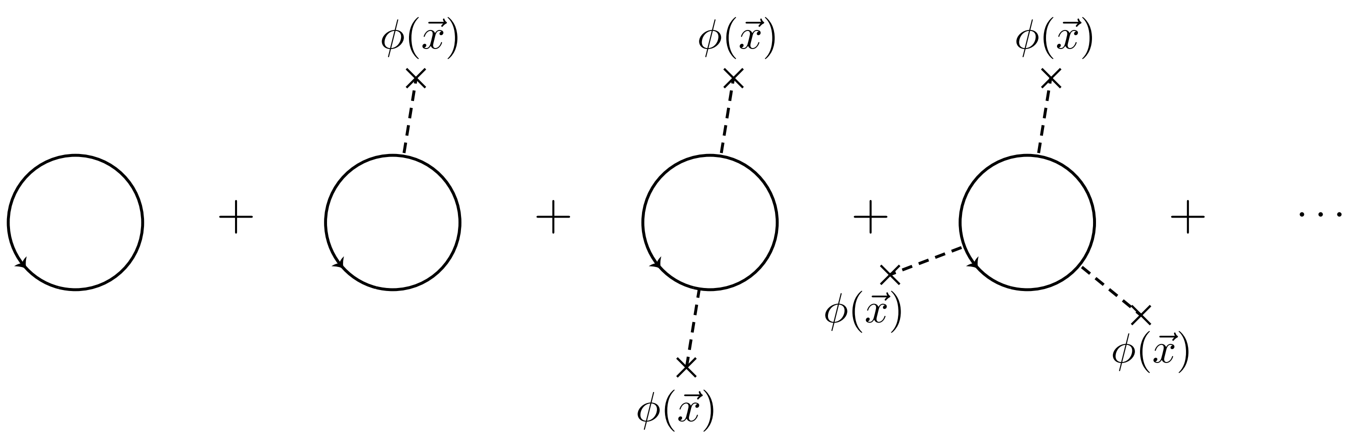
Effective action formalism. For time-independent fields,

$$S_{\text{EFF}}[\phi(\vec{x}, T)] \to T\ E[\phi(\vec{x})]$$

To one-loop order,

$$E[\phi] = E_{\text{classical}} + E_{\text{1-loop}} + E_{\text{counterterm}}$$

$E(1 - \text{loop})$:



$$E_{1-\text{loop}} - E_{\text{vacuum}} = \pm \sum_k \frac{1}{2} \hbar (|\omega_k| - |\omega_k^0|) \equiv E_{\text{Casimir}}[\phi]$$

$$E_{\text{Casimir}}[\phi] = \pm \sum_k \frac{1}{2}\hbar(|\omega_k| - |\omega_k^0|)$$

Work in the continuum: $\sum_k \to \sum_{\text{boundstates}} + \int dk$

$$\sum \frac{1}{2}(|\omega| - |\omega^0|) \Rightarrow \sum_j \frac{1}{2}|\omega_j| + \int_0^\infty \frac{|\omega|}{2}(\rho(k) - \rho^0(k))dk$$

$$= \sum_j \frac{1}{2}(\omega_j - m) + \int_0^\infty \frac{(\omega - m)}{2}(\rho(k) - \rho^0(k))dk$$

★ Levinson's theorem allows subtraction.

★ where ω_j are bound states, $|\omega| = \sqrt{k^2 + m^2}$ on the right hand side, and $\rho(k)$ is density of states.

★ Assume (generalized) spherical symmetry (spherical, grand spin, reduces to symmetric and antisymmetric as $n \to 1$).

$$\rho(k) - \rho^0(k) = \sum_\ell D_\ell \frac{1}{\pi}\frac{d\delta_\ell(k)}{dk}$$

$$\left[\text{General result:} \quad \frac{dn}{dk} = \frac{1}{2\pi i}\frac{d}{dE}\text{Tr}\ln S(E)\right]$$

★ $\delta_\ell(k)$ sums phase shifts for $\pm|\omega(k)|$.

★ n – space dimension – suppressed on degeneracy factor D_ℓ and δ_ℓ.

Regularization and Renormalization

$$E[\phi] = E_{\text{cl}} + E_{1-\text{loop}} + E_{\text{ct}}$$

To make contact with conventional renormalization theory, must accept a counterterm contribution in some standard perturbative scheme.

$$E_{\text{ct}} = E_{\text{ct}}[\phi, \Lambda] \quad \text{cutoff dependent}$$

★ Implications for Casimir "Sum"

- Integral not sum, $\sum_n E_n \to \int dE$.
- Seek conventional regularization, not
  "energy cutoff" — $\int^\Lambda dE$ or
  "mode number cutoff" — $\int^\Lambda dn(dE/dn)$

★ Cancellation of cutoff dependence

$$\lim_{\Lambda\to\infty} E_{\text{ct}}[\phi, \Lambda] + E_{1-\text{loop}}[\phi, \Lambda] = E_1[\phi]$$

- Numerical difficulties implied by Λ dependence of 1-loop calculation. Imagine (eg) Pauli- Villars scheme —

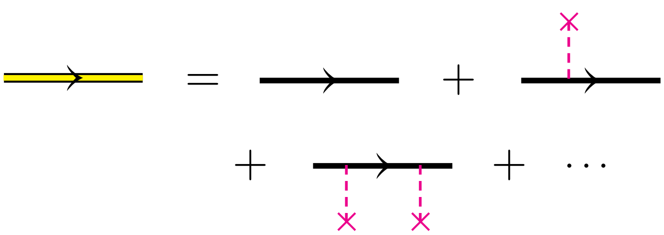
$$E_{1-\text{loop}} \sim \int dk(dn/dk)\left(\sqrt{k^2+m^2} - \sqrt{k^2+\Lambda^2}\right)$$

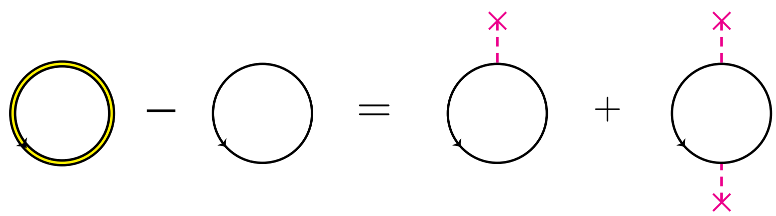
- Must calculate $E_{1-\text{loop}}$ repeatedly to map out, fit, and subtract Λ dependence — including both quadratic and logarithmic.
- A Nightmare

Born Regularization

★ Identify potentially divergent terms and regularize through the Born Approximation.

★ Born expansion (in n dim.) $\delta_\ell(k) = \sum_{i=1}^{\infty} \delta_\ell^{(i)}(k)$





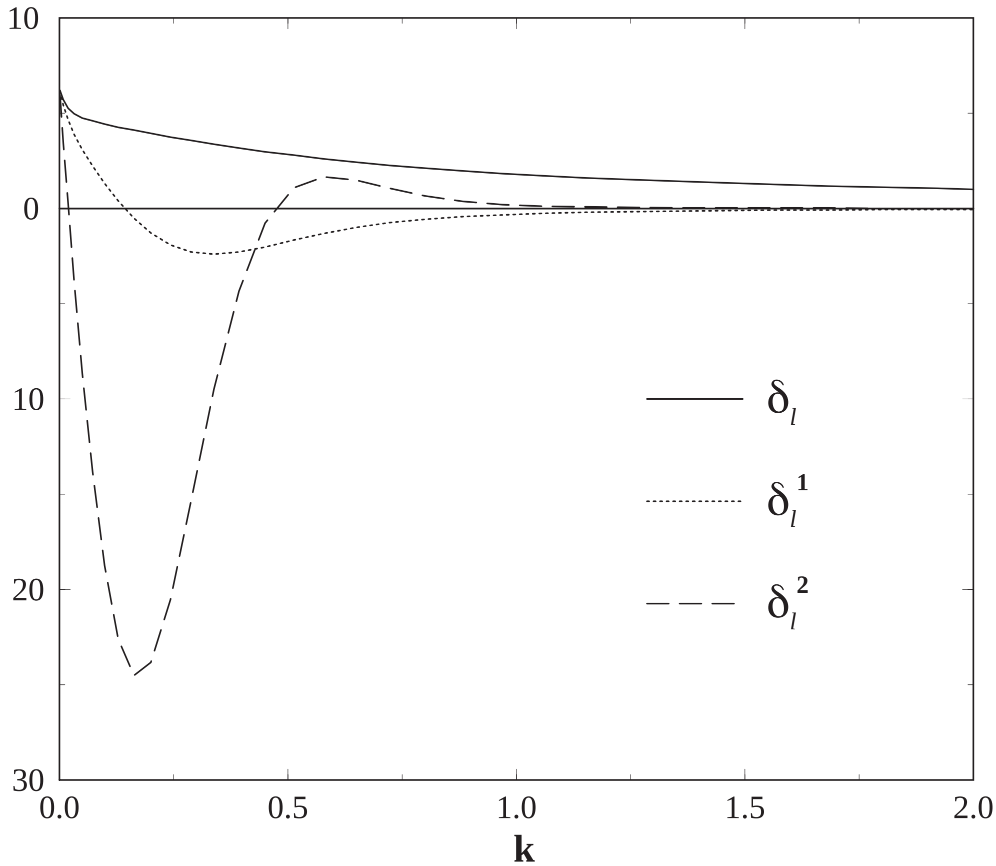
★ One-to-one correspondence between Born contributions to density of states and Feynman diagrams

★ Subtract N Born approximants to regulate

$$\delta_\ell(k) \;\Rightarrow\; \bar{\delta}_\ell(k) \equiv \delta_\ell(k) - \sum_{i=1}^{N} \delta_\ell^{(i)}(k) \quad \text{So} \quad E_{\text{cas}} \Rightarrow \bar{E}_{\text{cas}}$$

Regulated $\bar{E}_{\text{Casimir}}$ is both finite and cutoff independent.

- In theory, because divergent diagrams have been subtracted.
- In practice, because leading large k & large ℓ have been subtracted.



Typical Phase shift in three dimensions before and after subtracting the Born approximation(s)

⋆ Add back in Feynman diagrams

$$\Rightarrow \sum_{n=1}^{N} \Gamma^{(n)}[\phi, \Lambda]$$

Regulate in traditional fashion, combine with counterterms and renormalize.

How Renormalization Works

Formally, both the first Born Approximations and the lowest Feynman diagram are (quadratically) divergent as $n \longrightarrow$ integer. How do we know we are not missing essential finite pieces?

Because we can identify them as analytic functions of n.

To be specific: $\mathcal{L}_I = g\bar{\psi}\phi\psi$

- ⋆ ψ is a $2N_n$ component Dirac field.

- ⋆ $g\langle\phi(r)\rangle = V(r) + m$ with $V(r) \to 0$ as $r \to \infty$.

Standard Feynman graph. . .

$$\Gamma^1[\phi, n] = -2N_n \frac{\Gamma(\frac{1-n}{2})m^{n-1}}{(4\pi)^{\frac{n+1}{2}}} \int d^n x\, V(x)$$

Scattering theory in $n+1$ dimensions. . .

$$\delta^{(1)}_{n,j}(k) = -\frac{\pi}{2}\int_0^\infty dr\, rV(r)\left(J^2_{\frac{n}{2}+j-\frac{3}{2}}(kr) + J^2_{\frac{n}{2}+j-\frac{1}{2}}(kr)\right)$$

Bessel function identity

$$\sum_{\ell=0}^{\infty} \frac{(2\ell+2q)\Gamma(2q+\ell)}{\Gamma(\ell+1)} J^2_{\ell+q}(z) = \frac{\Gamma(2q+1)}{\Gamma^2(q+1)}\left(\frac{z}{2}\right)^{2q}$$

Plus a little group theory to work out dimension of Dirac algebra and degeneracy of partial waves as functions of n,

$$E^{(1)}_{\text{cas},n}[\phi, n] = \frac{2N_n(n-2)}{(4\pi)^{n/2}\Gamma(n/2)} \int d^n x\, V(x) \int_0^\infty dk\, k^{n-3}(\omega - m)$$

Which equals Γ^1 as an analytic function of n.

Note: Also confirms Levinson subtraction of m.

$E[\phi(\vec{x}), \{g\}, \{m\}]$

For a case where Feynman 1- and 2-point functions are potentially divergent as $n \longrightarrow$ integer...

$$E[\phi(\vec{x}), \{g\}, \{m\}] = E_{\text{cl}}[\phi(\vec{x}), \{g\}, \{m\}] +$$

$$\left\{\Gamma^1[\phi,\epsilon] + \Gamma^2[\phi,\epsilon] - c_1(\epsilon)\phi - c_2(\epsilon)\phi^2 - c_3(\epsilon)|\vec{\nabla}\phi|^2\right\}$$

$$+\frac{1}{2}\sum_j (E_j - m) - \frac{1}{2\pi}\int_0^\infty dk\,(|\omega(k)| - m)\sum_{\ell=0}^\infty D_\ell \frac{d}{dk}\bar{\delta}_\ell(k)$$

- ⋆ Classical energy.
- ⋆ Potentially divergent Feynman diagrams plus counterterms.
- ⋆ Regulated "Casimir" energy. Finite and smooth as $n \longrightarrow$ integer.
- ⋆ Subtraction of mass protects against infrared divergences and is an identity following from Levinson's theorem.
- ⋆ Renormalization $\bar{\Gamma}^1[\phi] = 0$

$$\left.\frac{d\bar{\Gamma}^2}{dp^2}\right|_{p^2=0} = 1 \qquad \left.\bar{\Gamma}^2[\phi]\right|_{p^2=0} = -m^2$$

  With standard scale and scheme dependence as expected.

- ⋆ Numerical calculations are convergent and quick.

Detailed Example

Charged Scalar Field Coupled to Classical Scalar Background in $3+1$ Dimensions

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\chi)^2 - \frac{\lambda}{4!}\left(\chi^2 - v^2\right)^2 + \partial_\mu\phi^*\partial^\mu\phi - G\phi^*\chi^2\phi$$
$$+ a\,(\partial_\mu\chi)^2 - b\left(\chi^2 - v^2\right) - c\left(\chi^2 - v^2\right)^2$$

⋆ The model

- ϕ appears quadratically and can be integrated out.
- ϕ couples to square of χ so classical potential for χ is positive definite.
- No classical soliton (Derrick's theorem)
- χ potential has minima at $\chi = \pm v$ so define $\chi(x,t) = v + h(x,t)$

⋆ Renormalization

- $\mathcal{L}_{\text{counterterm}} = a(\partial_\mu \chi)^2 - b(\chi^2 - v^2) - c(\chi^2 - v^2)^2$

  coefficients a, b, and c fixed by renormalization conditions.

- **"NO TADPOLE"**

  Tadpole diagram with external $h(x,t)$ vanishes

- **"ON SHELL"**

  Location and residue of pole in h-propagator remain unchanged

⋆ Eigenvalue problem for spherically symmetric $h(r)$

- Small oscillations potential for h is

$$V(r) = G\chi^2(r) - M^2 = G(h^2(r) + 2vh(r))$$

- Eigenvalue problem:

$$-\nabla^2\phi(\vec{r}) + V(r)\phi(\vec{r}) = (\omega^2 - M^2)\phi(\vec{r})$$

  Partial wave expansion...

⋆ Calculating phase shifts and the Born Approximation

- "Variable phase method" (Calegero)

$$
\begin{aligned}
\phi(\vec{r}) &= \frac{1}{r}\sum_{\ell m}\varphi_\ell(k,r)Y_{\ell m}(\Omega) \\
\varphi_\ell(k,r) &\to r h_\ell^{(1)}(kr) \quad \text{as} \quad r\to\infty \\
&\equiv e^{2i\beta_\ell(k,r)} r h_\ell^{(1)}(kr)
\end{aligned}
$$

  $\phi_\ell(k,r)$ is the "Jost solution", asymptotic to a free outgoing wave at infinity.

- The variable phase, β_ℓ obeys (from the wave equation),

$$
-i\beta_\ell'' - 2ikp_\ell(kr)\beta_\ell' + 2(\beta_\ell')^2 + \frac{1}{2}gV(r) = 0
$$

  ⋆ $p_\ell(kr)$ is a rational function,

$$
p_\ell(x) = \frac{d}{dx}\ln\left(h_\ell^{(1)}(x)\right)
$$

  ⋆ $\lim_{r\to\infty}\beta_\ell(k,r) = \beta_\ell'(k,r) = 0$

  ⋆ g is a parameter introduced to count orders in the Born approximation.

  ⋆ Phase shift is $\boxed{\delta_\ell(k) = -2\mathrm{Re}\beta_\ell(k,r)|_{r=0}}$

- Born Approximation

$$\beta_\ell(k,r) \sim \sum_{i=1}^{\infty} g^i \beta_\ell^{(i)}(k,r)$$

⋆ Expand differential equation for β_ℓ in powers of g

$$\begin{aligned}
-i\beta_\ell^{(1)\prime\prime} - 2ikp_\ell(kr)\beta_\ell^{(1)\prime} &= -\frac{1}{2}V(r) \\
-i\beta_\ell^{(2)\prime\prime} - 2ikp_\ell(kr)\beta_\ell^{(2)\prime} &= -2(\beta_\ell^{(1)})^2 \\
-i\beta_\ell^{(3)\prime\prime} - 2ikp_\ell(kr)\beta_\ell^{(3)\prime} &= -4\beta_\ell^{(1)}\beta_\ell^{(2)} \\
&\vdots
\end{aligned}$$

⋆ Simple sequence of linear differential equations with sources known order by order in g.

⋆ Solve together with original equation for the vector

$$\mathbf{B} = \left\{\beta_\ell, \beta_\ell^{(1)}, \beta_\ell^{(2)} \ldots\right\}$$

⋆ Very easy to generate phase shifts and Born approximations.

⋆ Casimir energy and Feynman diagrams

- Three dimensions, complex scalar field, two Born subtractions, two Feynman diagrams,

$$\begin{aligned}\Delta E[h] &= \overline{\Gamma}^{(1)}_{\text{FD}}[\chi] + \overline{\Gamma}^{(2)}_{\text{FD}}[\chi] \\ &+ \sum_{j,\ell}(2\ell+1)(\omega_{j,\ell} - M) - \int_0^\infty \frac{dk}{\pi}\frac{k}{\sqrt{k^2+M^2}} \\ &\times \sum_{\ell}(2\ell+1)\left(\delta_\ell(k) - \delta_\ell^{(1)}(k) - \delta_\ell^{(2)}(k)\right)\end{aligned}$$

- $\overline{\Gamma}^{(1)}_{\text{FD}}[\chi]$ is local and completely cancelled by counterterm.
- $\overline{\Gamma}^{(2)}_{\text{FD}}[\chi]$: Divergent part is cancelled by counterterm b. Diagram also contributes finite wavefunction renormalization, $\propto (\partial_\mu h)^2$, which is renormalized by a.

$$\begin{aligned}\overline{\Gamma}^{(2)}_{\text{FD}}[\chi] &= -\frac{4v^2G^2}{(4\pi)^2}\int_0^\infty \frac{q^2dq}{(2\pi)^2} q^2\tilde{h}^2(q)\int_0^1 dx \frac{x(1-x)}{M^2 - x(1-x)m^2} \\ &+ \frac{G^2}{(4\pi)^2}\int_0^\infty \frac{q^2dq}{(2\pi)^2}\tilde{V}^2(q)\int_0^1 dx\left[\ln\frac{M^2+x(1-x)q^2}{M^2-x(1-x)m^2}\right. \\ &\left. - \frac{x(1-x)m^2}{M^2-x(1-x)m^2}\right]\end{aligned}$$

First line: finite effect of local counterterm a. Second line, standard second order self energy.

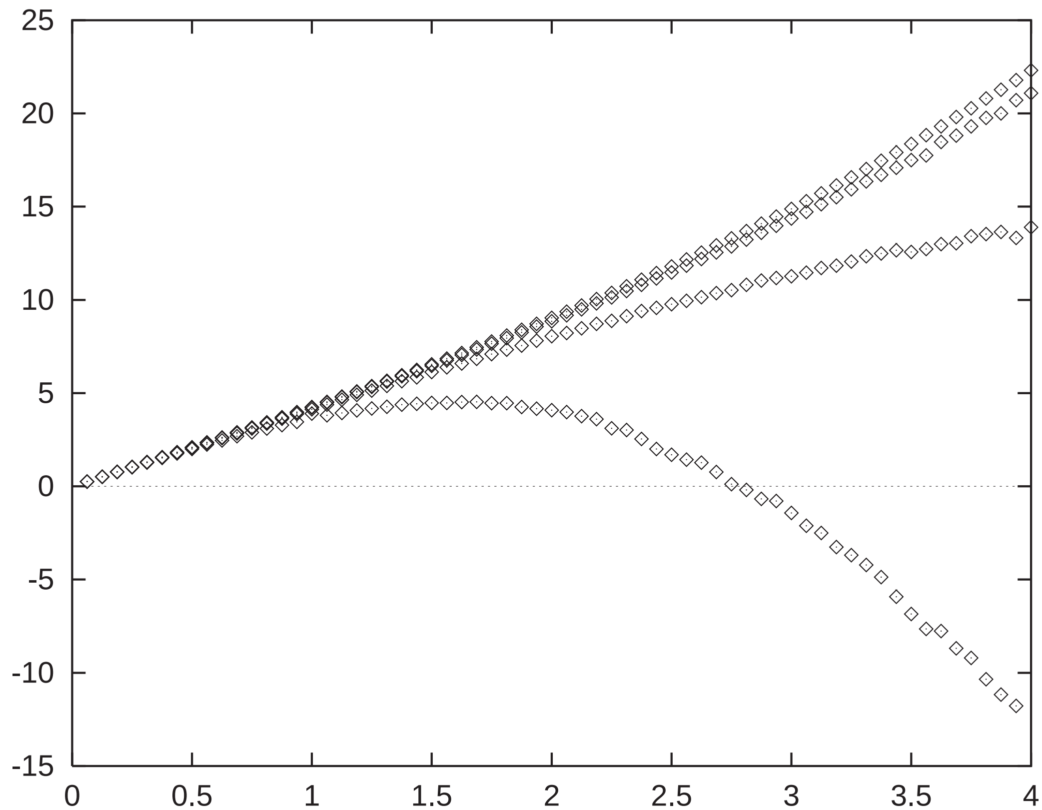
⋆ Results for this (toy) model. . .

- Parametization of ansatz

$$E[h] = E_{\text{cl}}[h] + \Delta E[h]$$

$$h(r) = -dve^{-r^2v^2/2w^2}$$

- Numerical results



$E(d = 1, w)$ in units of v for $G = 1, 2, 4$ and 8 as function of w.

- Small G, no sign of solition
Large G vacuum instability!

Application 1: Quantum Soliton Formation

- ⋆ Original motivation for the whole program was to establish existence (or not) of solitons in the Standard Model at large Yukawa coupling.

- ⋆ In $3+1$ we've studied spherically symmetric and Higg's hedgehog ansätze and find no interesting solitions in internally consistent parameter domains. More on this at the end.

- ⋆ To prove point of principal we studied $1+1$ dimensional chiral model and...

Find a quantum stabilized fermionic soliton

Model – Boson sector:

$$\mathcal{L}_B = \frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} - V(\vec{\phi})\,,$$

$$V(\vec{\phi}) = \frac{\lambda}{8}\left(\vec{\phi}\cdot\vec{\phi} - v^2 + \frac{2\alpha v^2}{\lambda}\right)^2 - \frac{\lambda}{2}\left(\frac{\alpha v^2}{\lambda}\right)^2 - \alpha v^3(\phi_1 - v)\ .$$

If $\alpha = 0$, the $U(1)$ transformation

$$\phi_1 + i\phi_2 \longrightarrow e^{i\varphi}(\phi_1 + i\phi_2)$$

would be a symmetry.

- ⋆ Symmetry breaks at the classical level, but with $\alpha = 0$ radiative corrections always restore symmetry in one-dimension (Coleman, Mermin, Wagner).

- ⋆ So we keep α large enough to suppress restoration of the symmetry and keep

$$\langle\vec{\phi}\rangle = \vec{\phi}_{\text{classical}} = (v, 0).$$

- ⋆ This model has no stable classical solitons. Kink-like configurations with $\phi_1 \to \pm v$ as $x \to \pm\infty$ unravel in ϕ_1, ϕ_2 plane.

Model – Fermion sector:

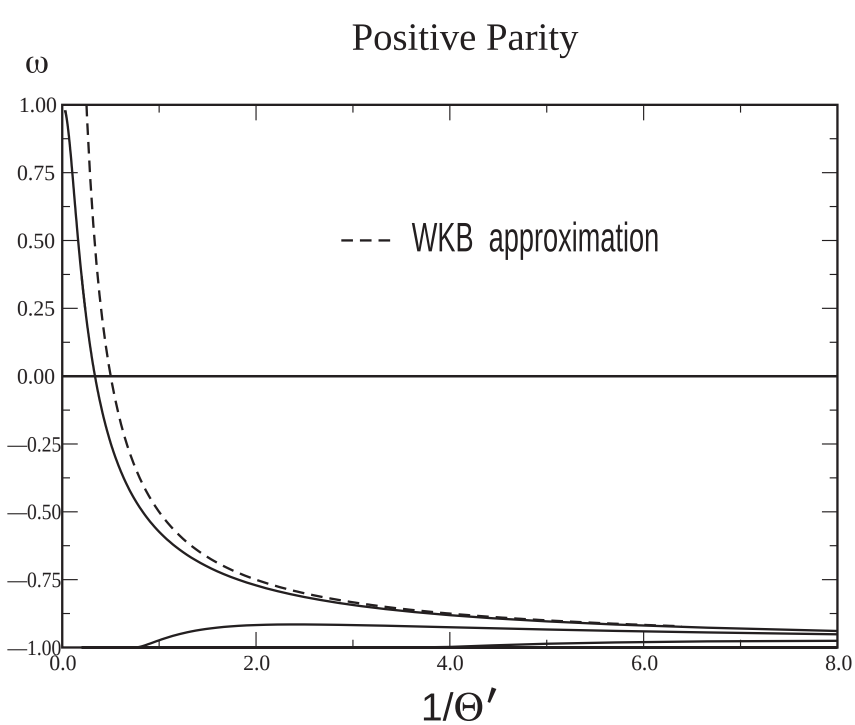
$$\mathcal{L}_F = \frac{i}{2}[\bar{\Psi}, \not{\partial}\,\Psi] - \frac{G}{2}\left([\bar{\Psi}, \Psi]\,\phi_1 + i\,[\bar{\Psi}, \gamma_5\Psi]\,\phi_2\right) .$$

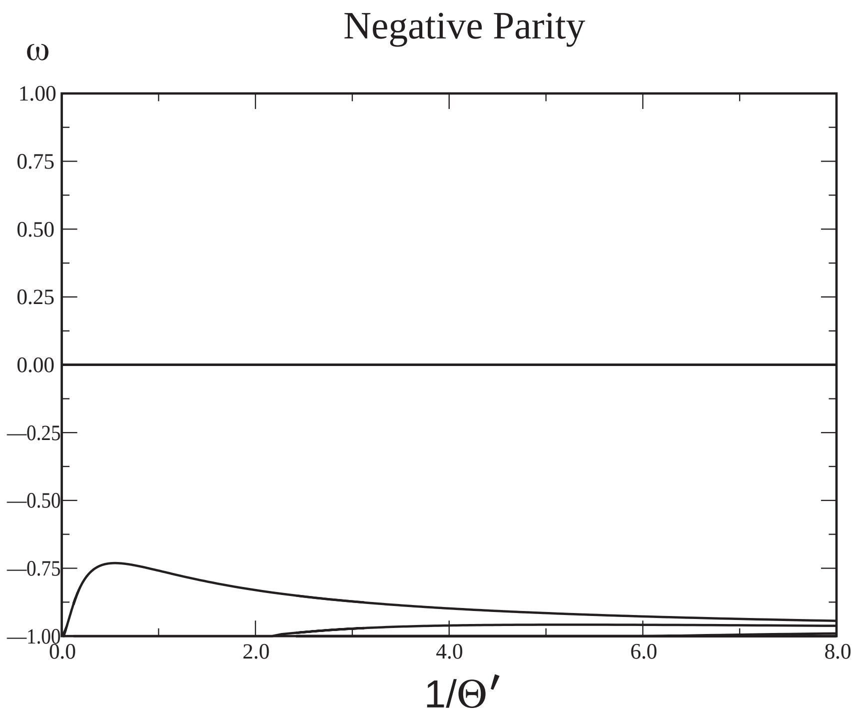
- ⋆ Note careful treatment of charge conjugation.

- ⋆ Take $N_f \to \infty$ so one-fermion loop dominates.

- ⋆ Vacuum is non-degenerate: $\vec{\phi} = v(1, 0)$, but

- ⋆ Domain near "chiral circle", $\vec{\phi} = v(\cos\Theta, \sin\Theta)$ has low energy, and binds a fermion mode tightly. Θ' measures width of soliton.





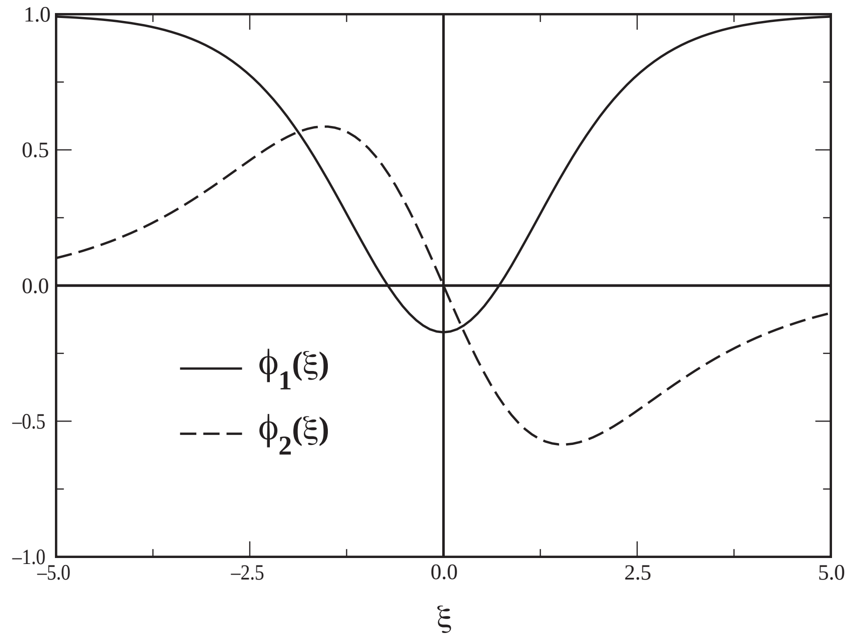
- ⋆ Dynamics will be balance of excursion of $\vec{\phi}$ from its minimum, classical "kinetic energy" $|\vec{\phi}'|^2$, tightly bound fermion level, and Casimir energy from deformation of the fermion continuum.

- ⋆ Parameterization of ansatz for $\vec{\phi}$:

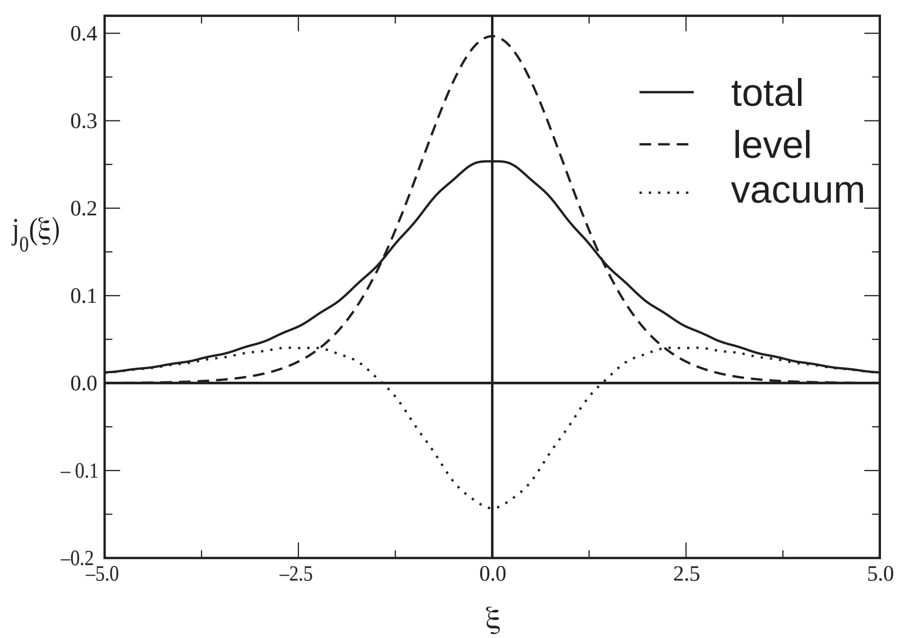
$$\begin{aligned} \vec{\phi}_I(\xi, R, w) &= \Big(1 - R + R\cos\Theta_I(\xi, w), R\sin\Theta_I(\xi, w)\Big) \\ \Theta_I(\xi, w) &= \pi\Big(1 + \tanh(\xi/w)\Big). \end{aligned}$$

- ⋆ Parameters R and w: radius of circle in chiral boson plane and width of soliton.

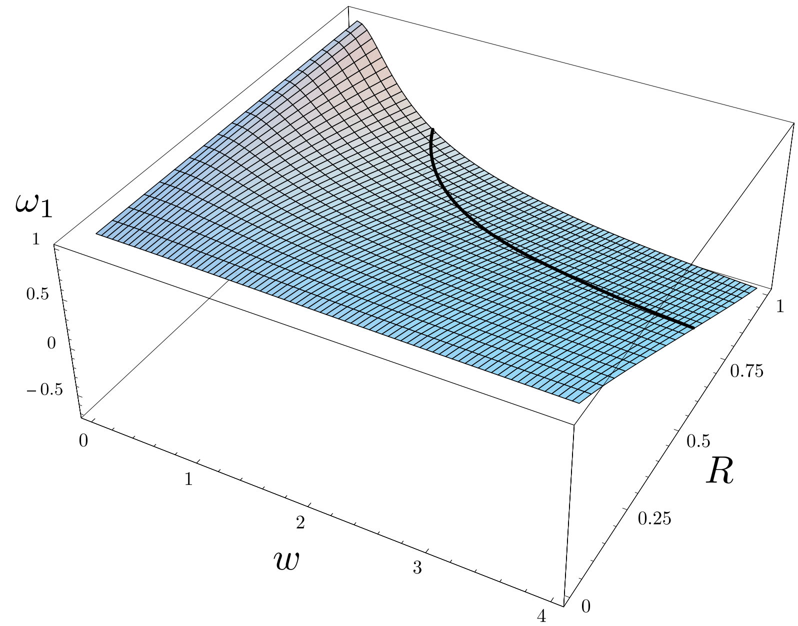
Results



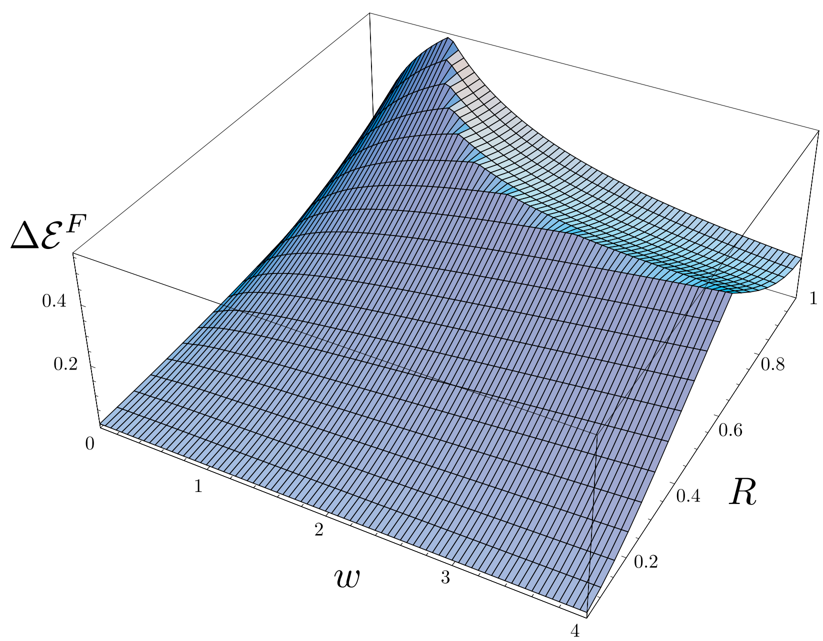
ϕ_1, ϕ_2 at the variational minimum for $\tilde{\alpha} = 0.5$, $\tilde{\lambda} = 1.0$, and $v/\sqrt{N_F} = 0.375$, which is at $R = 0.586$, $w = 2.808$.



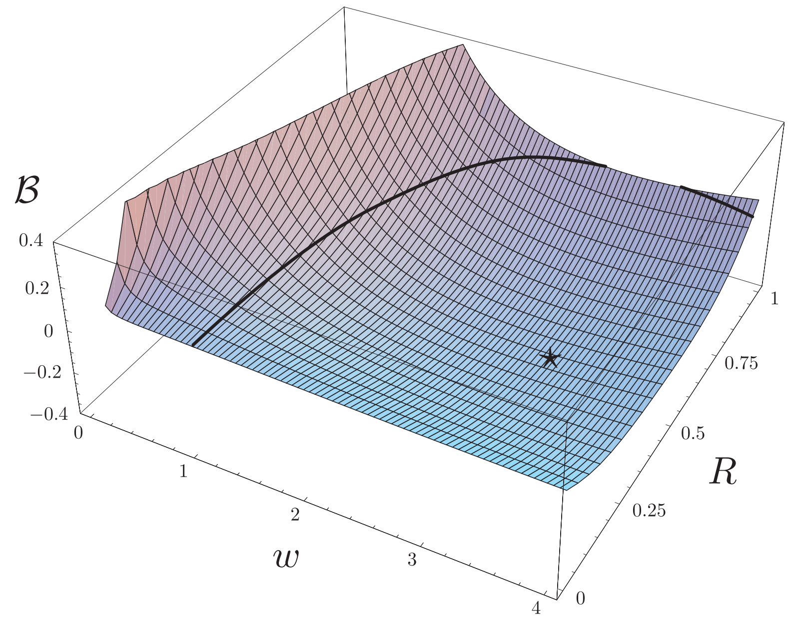
the fermion number density j_0 at the variational minimum



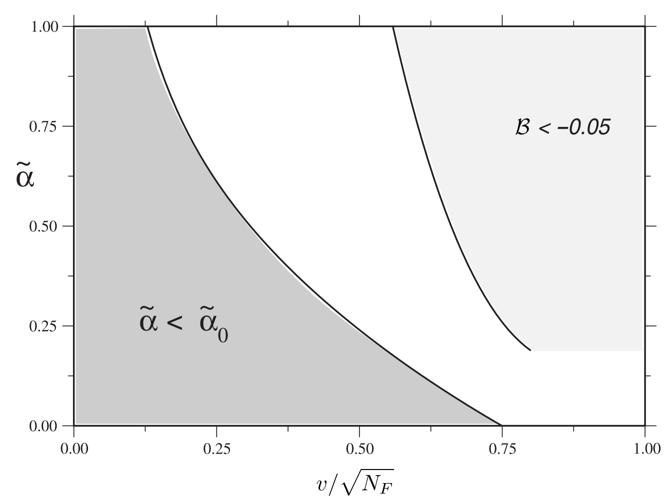
The lowest quark eigenenergy, ω_1, as a function of R and w. Note that for large R and w, ω_1 is negative. A solid curve marks the contour $\omega_1 = 0$.



The vacuum contribution to the one-loop fermion energy as a function of R and w. Note the discontinuity in gradient when the negative energy level is filled.



$\mathcal{B}$ as a function of the *ansatz* parameters for $\tilde{\alpha} = 0.5$, $\tilde{\lambda} = 1.0$, and $v/\sqrt{N_F} = 0.375$. A solid curve marks the contour $\mathcal{B} = 0$, and a star indicates the minimum at $w = 2.808$ and $R = 0.586$.



The regions of soliton stability in the plane of $v/\sqrt{N_F}$ and $\tilde{\alpha}$. In the shaded area on the left, a growing width indicates potential infrared instabilities. In the shaded area on the right, the soliton is bound by less than 5 percent. In between, we have a stable, tightly bound soliton.

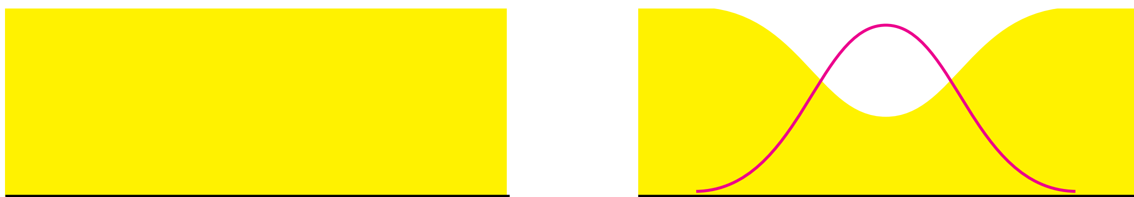
Extension to Standard Model

⋆ A Very Heavy Quark in the Standard Model

$$m \sim gv \qquad \langle\phi\rangle = v$$

Would seem to favor "evacuation" of Higgs VeV near quark

t-quark bag a la Friedberg-Lee



$$\Delta\phi \sim v$$
$$\Delta p \sim m \sim gv$$

Derivative expansion
unlikely to be useful

- Classical

  - F. Wilczek, IASSNS/90-20
  - G. Anderson, L. Hall, S. Hsu, Phys. Lett. **B249** (1990)
  - S. Dimopoulos, B. Lynn, S. Selipsky, N. Tetradis, Phys. Lett. **B253** (1991) 237.

- Derivative Expansion

  - J. Bagger, S. Naculich, Phys. Rev. Lett. 76 (1991) 2252; hep-ph/9209283

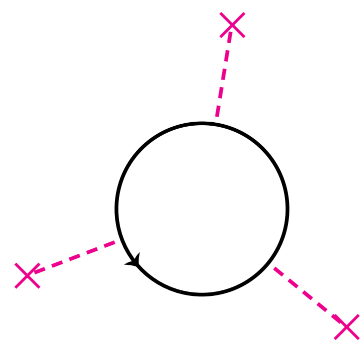
⋆ Decoupling a fermion in the Standard Model

- Decoupling is non-trivial because Higgs-fermion coupling $\longrightarrow \infty$ as $m_f \longrightarrow \infty$.

- If one succeeded in decoupling a fermion doublet in an $SU(2)_{\mathrm{L}}$ gauge one would have a conceptual problem: Residual gauge theory would be anomalous (Witten anomaly)

- Imagine originally two doublets. As $m_f \longrightarrow \infty$ for one doublet, what cancels Witten anomaly at the level of the states?

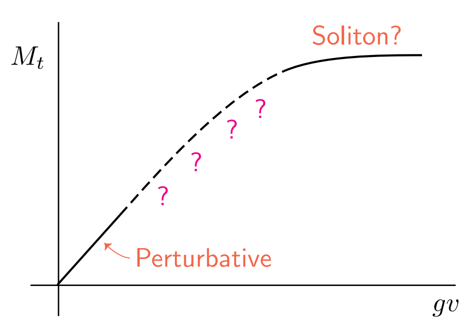
- Decoupling induces a Wess-Zumino-Witten term via heavy fermion loop



  So Higgs field carries heavy fermion number

- Suspect that a hedgehog-soliton in the Higgs sector carries heavy fermion number.

- Something must give as m_f exceeds $M_{\text{Sphaleron}}$



Complete study of hedgehog ansatz

E. Farhi, N. Graham, RLJ, V. Khemani, H. Weigel
hep-th/0303159

- ⋆ $SU(2)_L$ gauge theory with a single (degenerate) fermion doublet.

- ⋆ Field parametrization:

$$\Phi = \begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}$$

$$\Phi(x) = v\left(s(x) + ip^a(x)\tau^a\right)$$

$$V(A,\Phi) = -g\gamma^\mu A_\mu(x)\frac{1-\gamma_5}{2} + f\left(h(x) + ivp^a(x)\tau^a\gamma_5\right)$$

$$h(x) \equiv v(s(x) - 1)$$

$$\mathcal{L}_F = \overline{\Psi}\left(i\gamma^\mu\partial_\mu - fv\right)\Psi - \overline{\Psi}V\Psi$$

- ⋆ Spherical ansatz (in $A^0 = 0$ gauge)

$$A_i(\vec{x}) = \frac{1}{2g}\Big[a_1(r)\tau_j\hat{x}_j\hat{x}_i + \frac{\alpha(r)}{r}(\tau_i - \tau_j\hat{x}_j\hat{x}_i) + \frac{\gamma(r)}{r}\epsilon_{ijk}\hat{x}_j\tau_k\Big]$$

$$\Phi(\vec{x}) = v\left[s(r) + ip(r)\tau_j\hat{x}_j\right]$$

- ⋆ Moduli and phase:

$$-i\rho e^{i\theta} \equiv \alpha + i(\gamma - 1) \quad \text{and} \quad \Sigma e^{i\eta} \equiv s + ip$$

One example: Twisted Higgs Ansatz

★ Starting point: $\eta = -\pi e^{-r/w} \quad \Sigma = \rho = 1$

Variations:

$$\eta = -\pi e^{-r/w} + p_0 \frac{r/w}{1+(r/w)^2} e^{-r/w}$$

$$\Sigma = 1 + p_1 \frac{1}{1+(r/w)} e^{-r/w}$$

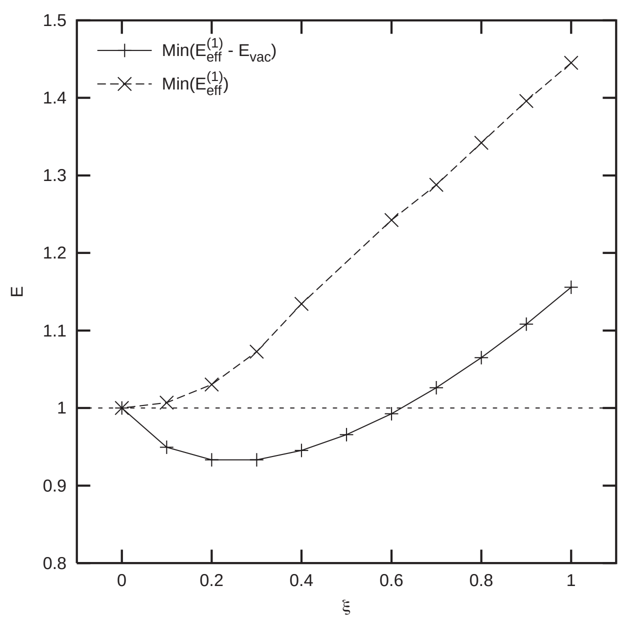
$$a_1 = p_2 \frac{r/w}{1+(r/w)^2} e^{-r/w}$$

$$\rho = 1 + p_3 \frac{(r/w)^2}{1+(r/w)^3} e^{-r/w}$$

★ Sample interpolation from trivial to twisted configuration:

$$\Sigma e^{i\eta} = 1 - \xi + \xi \exp\left(-i\pi e^{-r/w}\right)$$

with $f = 10$(!)



Another example: A Path over the Sphaleron

- ⋆ Note there is a fermion zero mode in the background of a sphaleron. Suggestive.

- ⋆ Sphaleron interpolation:

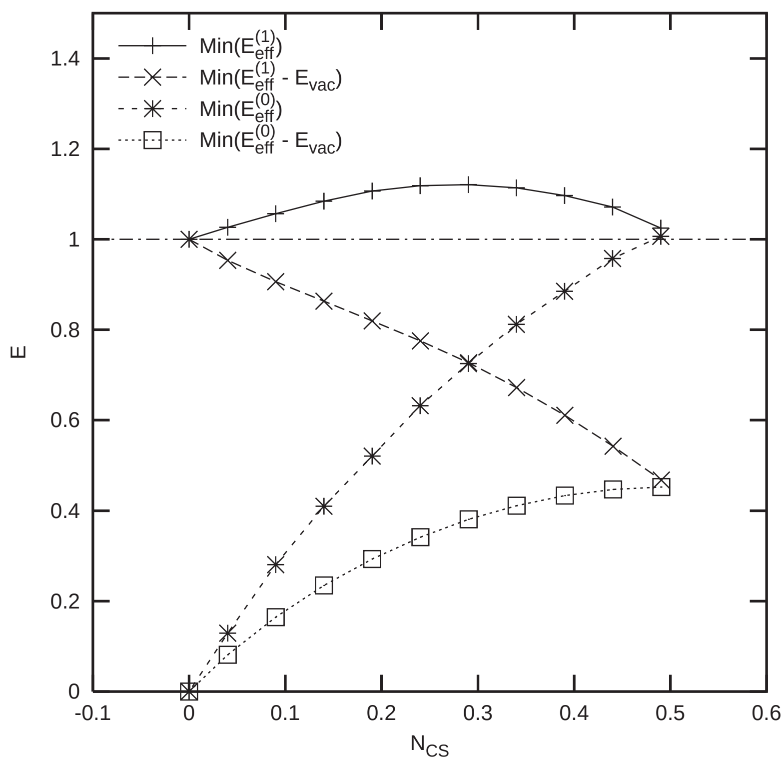
$$\Phi = v(1-\xi)\mathbb{1} + \xi v U^{(1)}$$
$$A_j = \xi \frac{i}{g} U^{(1)} \partial_j U^{(1)\dagger}$$

where

$$U^{(1)}(\vec{x}) = e^{i f(r) \tau_j \hat{x}_j / 2}$$

- ⋆ As ξ goes from 0 to 1 configuration goes from trivial vacuum to winding number 1 vacuum with sphaleron at $\xi = 1/2$.

- ⋆ For example, $f^{(1)}(r) = -2\pi e^{-r/w}$.



Conclusion on Standard Model

- ⋆ Quantum corrected sphaleron is heavier than classical sphaleron by an amount of order the perturbative fermion mass.

- ⋆ This generates a barrier that stabilizes heavy fermions even when perturbative fermion mass is greater than spaleron energy.

- ⋆ Heavy enough fermions are still unstable.

- ⋆ No sign of residual light fermion to resolve Witten anomaly.

  - Anomaly saturated by states without a particle interpretation?

  - Beyond the spherical ansatz?

Application 2: Interfaces

Interface $\equiv$ a field configuration

Nontrivial in m dimensions $\otimes$ Trivial in n dimensions

Examples:

- ⋆ Domain walls in lattice simulations.
- ⋆ Fluctuations of bulk fields in braneworld models.
- ⋆ Casimir induced cosmological constant?

Restrictions:

- ⋆ One-loop – $\mathcal{O}(\hbar)$
- ⋆ Renormalizable theory
- ⋆ Symmetric in m space.

Notation:

- ⋆ μ – mass

  p – momentum in trivial directions

  k – momentum in non-trivial directions

- ⋆ $\omega(p,k) = \sqrt{\mu^2 + k^2 + p^2} \quad \mu(p) = \sqrt{\mu^2 + p^2}$

- ⋆ $E_m[\phi] \to \mathcal{E}_{n,m}[\phi] \equiv E_{n,m}[\phi]/L^n$

Illustrate with $g\bar{\psi}\phi\psi$ where $n + m + 1 = d$ heads toward value where only first Born (i.e., tadpole graph) diverges.

$$\mathcal{E}_{n,m}[\phi] = \pm \int \frac{d^n p}{(2\pi)^n} \sum_\ell D_m^\ell \Bigg[\int_0^\infty \frac{dk}{2\pi} (\omega(k,p) - \mu(p)) \otimes \frac{d}{dk} \left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k) \right) + \frac{1}{2} \sum_j (|\omega_{j,m}^\ell(p)| - \mu(p)) \Bigg] + \overline{\mathcal{F}}_{n,m}^{(1)}[\phi] \,.$$

Note: because phase shift does not depend on p, the p integration looks trivial, but one cannot interchange it with k integration in physical dimension:

Perform p integration using dimen. regularization,

Result:

$$\mathcal{E}_{n,m}[\phi] = \mp \frac{\Gamma(-\frac{1+n}{2})}{2(4\pi)^{\frac{n+1}{2}}} \sum_\ell D_m^\ell \Bigg[\int_0^\infty \frac{dk}{\pi} (\omega^{n+1}(k) - \mu^{n+1}) \otimes \frac{d}{dk} \left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k) \right) + \sum_j (|\omega_{j,m}^\ell|^{n+1} - \mu^{n+1}) \Bigg] + \overline{\mathcal{F}}_{n,m}^{(1)}[\phi]$$

$\Gamma(-\frac{n+1}{2})$ diverges at $n = 1, 2, \ldots$!

But theory is renormalizable at those dimensions by construction!

Resolution?

Coefficient of Γ must vanish as $n \to 1, 3, \ldots$. Implies finite energy sum rules, which generalize Levinson's theorem:

$$\int_0^\infty \frac{dk}{\pi}\frac{d}{dk}\delta_m^\ell(k) + \sum_j 1 = 0$$

$$\int_0^\infty \frac{dk}{\pi}k^2\frac{d}{dk}\left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k)\right) - \sum_j (\kappa_{j,m}^\ell)^2 = 0$$

$$\int_0^\infty \frac{dk}{\pi}k^4\frac{d}{dk}\left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k) - \delta_m^{(2)\ell}(k)\right) + \sum_j (\kappa_{j,m}^\ell)^4 = 0$$

where first is Levinson's theorem and the last is the identity required to go to a dimension where Γ^2 diverges.

Graham, RLJ, Quandt, & Weigel,
Phys. Rev. Lett. **87**, 131601 (2001) [hep-th/0103010].
Ann. Phys. **293** 240 (2001) [quant-ph/0104136].
R. D. Puff, Phys. Rev. **A11** (1975) 154.

Then with these identities in hand, continuation to integer n is smooth and straightforward.

Born + dimensional regularization is

- ⋆ Simple
- ⋆ Unambiguous
- ⋆ Correct

Examples

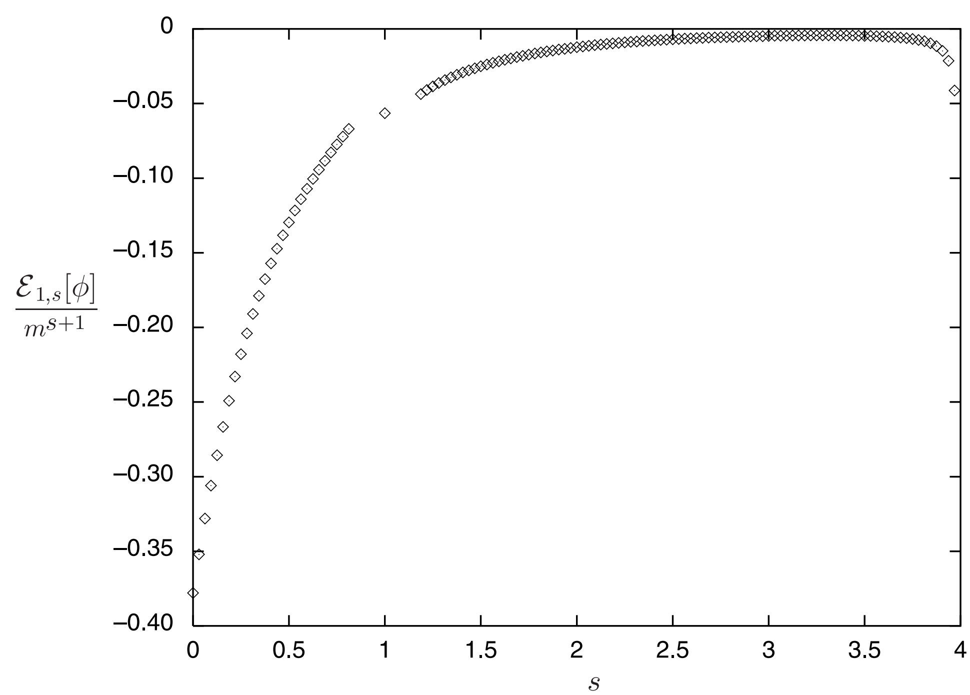
$n \to 1$:

$$\mathcal{E}_{1,m} = \pm\frac{1}{4\pi}\sum_\ell D_m^\ell\Bigg[\int_0^\infty \frac{dk}{\pi} k \log\frac{\omega(k)^2}{\mu^2}\left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k)\right) - \frac{1}{2}\sum_j (\omega_{j,m}^\ell)^2 \log\frac{(\omega_{j,m}^\ell)^2}{\mu^2} + (\kappa_{j,m}^\ell)^2\Bigg].$$

$n \to 3$:

$$\mathcal{E}_{3,m} = \pm\frac{1}{32\pi^2}\sum_\ell D_m^\ell\Bigg[-\int_0^\infty \frac{dk}{2\pi} 4k\omega(k)^2 \log\frac{\omega(k)^2}{\mu^2} \left(\delta_m^\ell(k) - \delta_m^{(1)\ell}(k) - \delta_m^{(2)\ell}(k)\right) + \frac{1}{2}\sum_j\left((\omega_{j,m}^\ell)^4 \log\frac{(\omega_{j,m}^\ell)^2}{\mu^2} + \mu^2(\kappa_{j,m}^\ell)^2 - \frac{1}{2}(\kappa_{j,m}^\ell)^4\right)\Bigg].$$

Applications to branes, vortices, etc. have yet to be explored.

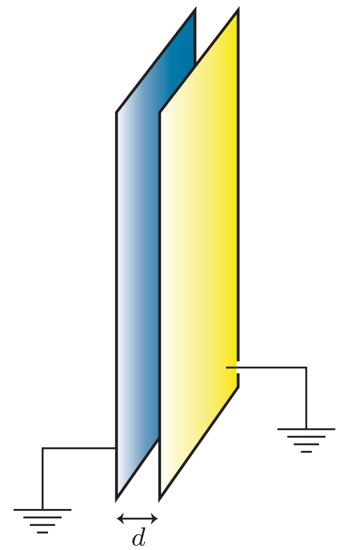


Application 3: Boundary Conditions & the "Casimir Problem"

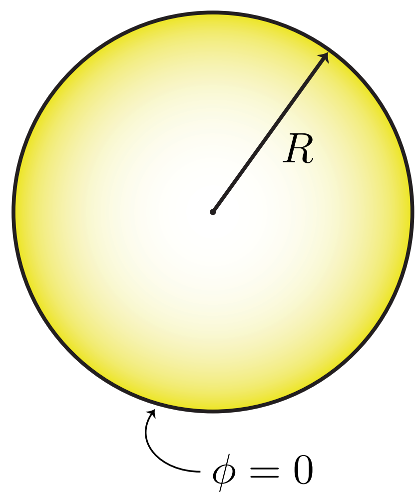
⋆ (a) Classic example: Electromagnetic field between parallel plates. Measure force/area:

$$\frac{F}{A} = -\frac{\pi^2}{240}\frac{\hbar c}{d^4}$$

Confirmed by experiment.



⋆ (b) Mathematical physics problem: The Dirichlet Sphere: What is the energy (relative to the true vacuum) of a fluctuating field constrained to vanish on the surface of a sphere? In particular, how does this energy change with R, the radius of the sphere?



⋆ Important distinction: (a) is a force between rigid bodies; (b) is a surface stress, measurable only by deforming the surface, $R \to R + \delta R$.

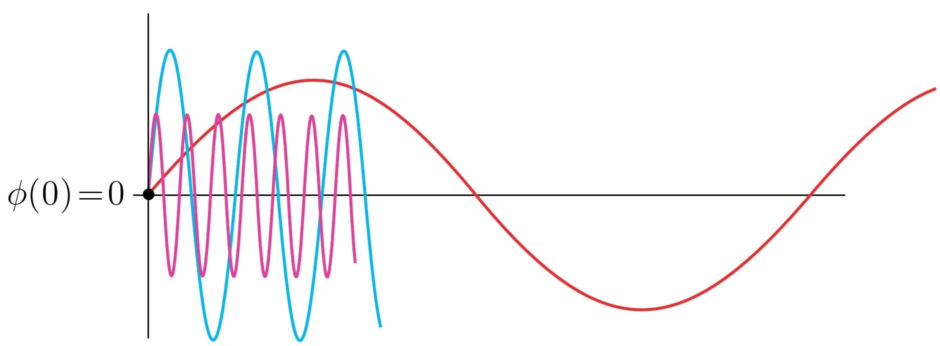
What is the problem?

- ⋆ "Let $\phi(\vec{x}, t)$ be a scalar field that vanishes on a surface $\mathcal{S}$. . . "

- ⋆ Casimir Effect
  Bag models
  Brane worlds
  Lattice quantum field theories

- ⋆ **hep-th/0207205** and forthcoming.

A Boundary Condition on All Modes is a serious affair. . .



$$-\phi'' + \lambda\sigma(x)\phi + m^2\phi = \omega^2\phi$$

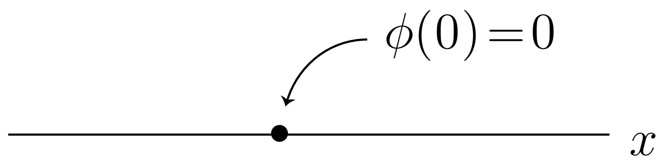
1.	$\sigma \to \delta(x)$	Sharp	Dirichlet
2.	$\lambda \to \infty$	Strong	

. . . that one would expect to have a significant effect on the theory

Elementary Considerations

Dirichlet point in 1-dimension

Standard Result:



$$\widetilde{E}_1 = \frac{1}{2}\sum \hbar\omega - \hbar\omega_0 = 0$$

- Free: $\phi \sim \sin kx \quad k > 0$
  $\phi \sim \cos kx \quad k > 0$

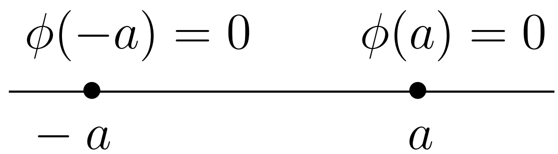
- Constrained: $\phi \sim \sin kx \quad k > 0$
  $\phi \sim \sin k\,|x| \quad k > 0$

Identical spectra $\longrightarrow \widetilde{E}_1 = 0$

Surprised? It costs no energy to constrain the field on all scales.

Two Dirichlet Points

⋆ Casimir energy for two Dirichlets:



$$-\phi''(x) = k^2\phi(x), \quad \text{with} \quad \phi(-a) = \phi(a) = 0.$$

⋆ Traditional answer:

$$E_{\text{Casimir}} = -\frac{\pi}{48a}$$

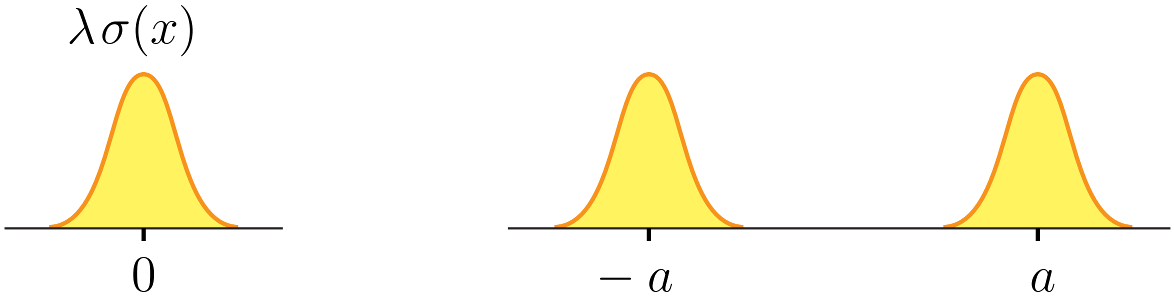
⋆ Critique:

- Limit $a \to \infty$ gives $E_{\text{Casimir}} = 0$?
  Expect $2 \times E_1$ So it costs no energy to force a field to vanish at a point!

- Limit $a \to 0$ gives $E_{\text{Casimir}} \to \infty$?
  Expect E_1 So it costs infinite energy to force a field to vanish at a point?!

- E_{Casimir} should diverge like $\ln m$ as scalar mass $\to$ zero in 1-dimension.

Embed in renormalized, continuum QFT



★ Field theoretic approach:

$$\mathcal{L}_{\text{int}} = \lambda\sigma(x)|\phi(x)|^2 + C_1\sigma(x)$$

with

$$\sigma(x) = \delta(x+a) + \delta(x-a)$$

$C_1\sigma(x)$ is counterterm in expectation of logarithmic divergence of effective action in $1+1$ dimensions.

★ Renormalization condition:

$$\langle\sigma(x)\rangle_{\text{vac}} = 0$$

★ Boundary condition limit is $\lambda \to \infty$.

★ Note: One could introduce arbitrary action for σ – with some justification. But we ignore this in the spirit of the mathematical physics problem.

Approach to Dirichlet Point

Sharp $\quad \sigma(x) \to \delta(x)$

$$E_1(\lambda) = \frac{1}{2\pi}\int_m^\infty dt\, \frac{t\log\left(1+\frac{\lambda}{2t}\right) - \frac{\lambda}{2}}{\sqrt{t^2-m^2}}$$

$$E_2(\lambda,a) = \frac{1}{2\pi}\int_m^\infty dt\, \frac{t\log\left(1+\frac{\lambda}{t}+\frac{\lambda^2}{4t^2}(1-e^{-4at})\right) - \lambda}{\sqrt{t^2-m^2}}$$

- $a \to \infty \quad E_2(\lambda,a) \to 2E_1(\lambda)$
- $a \to 0 \quad E_2(\lambda,0) = E_1(2\lambda)$

$\surd\surd$

But

$$\text{Dirichlet} \longrightarrow \lambda \to \infty$$

$$E_2(\lambda,a) \longrightarrow -\lambda\log\lambda$$

So Dirichlet limit doesn't exist for the energy

Force?

- Boundary condition: $\widetilde{F}_2(a) = -\frac{d\widetilde{E}_2}{da} = -\frac{\pi}{48a^2}$

- QFT: $F_2(\lambda, a) = -\frac{dE_2(\lambda, a)}{da}$

$$\lambda \to \infty, \; F_2(\lambda, a) \longrightarrow \widetilde{F}_2(a)$$

Conclude

From this simple example

⋆ Energy – relative to the vacuum cannot be reliably calculated with boundary condition method

⋆ Renormalized energy in continuum QFT diverges in boundary condition limit
i.e., cutoff dependent
i.e., material dependent

⋆ Force – change in energy with rigid displacement is finite & cutoff independent & agrees with boundary condition method

⋆ Also, energy density agrees with boundary condition method at points away from "surface".

Generic!

★ All fields — scalar, Dirac, vector

★ All dimensions — divergences get worse as D grows

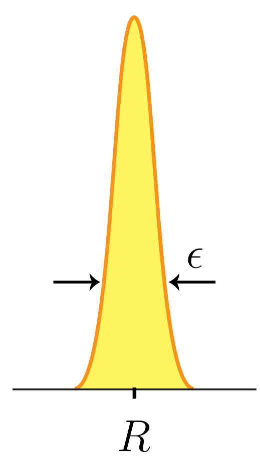
★ Origin — high momentum components of backgrounds; not loop divergences

So What?

★ Forces – between rigid bodies – are fine

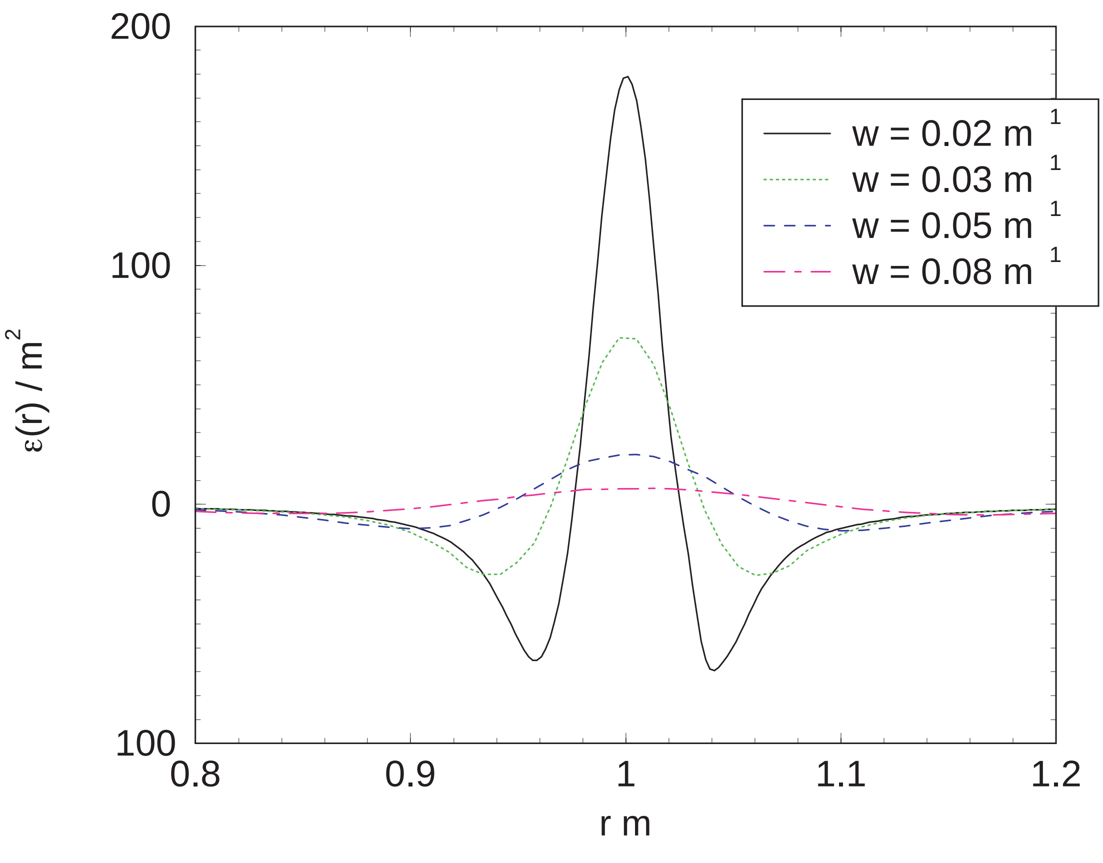
★ Stresses are not!

- "Dirichlet sphere" $\phi(\vec{x}) = 0 \quad |\vec{x}| = R$ in D-space dimensions
- $E(R)$ diverges like $\dfrac{\log \epsilon}{\epsilon}$ in sharp limit and so does $\dfrac{dE}{dR}$!
- Conducting sphere?



Gaussian approximation to Dirichlet circle in 2-dimensions.

$$\sigma(r) = \lambda A e^{-\frac{(r-a)^2}{2w^2}}$$



Dirichlet Sphere and Planes in Light of QFT

⋆ $\sigma = 0$ on sphere or planes?
Couple $\phi(\vec{x},t)$ to a background field, $\sigma(\vec{x})$

$$E[\sigma] = \int d^n x \left\{ \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}|\vec{\nabla}\phi|^2 + \tfrac{1}{2}m^2\phi^2 + \lambda\sigma(\vec{x})\phi^2(\vec{x},t) \right\}$$

To obtain Dirichlet condition:

1. $\sigma \to \delta(r-R)$ (sphere) or
   $\sigma \to \delta(z+a/2) + \delta(z-a/2)$ (planes)
2. $\lambda \to \infty$

Both are highly singular and damage the high energy behavior of Green's functions.

Can the damage be absorbed by the counterterms available in a renormalizable QFT?

or

Does it signal a real divergence in the renormalized energy of this singular background field configuration?

⋆ Sphere and plane look similar, but are very different

Plane $\quad \dfrac{F}{A} = \dfrac{d\bar{E}(a)}{da} \qquad$ Sphere $\quad F/A = \dfrac{1}{4\pi R}\dfrac{d\bar{E}(R)}{dR}$

Results on the Dirichlet Casimir Problem

⋆ Consider for $n = 1, 2, 3$:

- 1-dimension Sphere/plane $\longrightarrow$ two points, $x = \pm a$
  - ⋆ Energy is finite as $\sigma \longrightarrow \delta(x \mp a)$, but diverges like $-\lambda \ln \lambda$ as $\lambda \longrightarrow \infty$
  - ⋆ Force is finite and agrees with naive calculation.
- 2-dimensions "Dirichlet circle" or lines

  Fourier components of σ fall too slowly with $\vec{p}$ as $\sigma \longrightarrow \delta$. So the Casimir energy is infinite in the sharp limit ($\epsilon \longrightarrow 0$), even before the Dirichlet limit is taken.
  - ⋆ Dirichlet circle: $E(R) \propto R\lambda^2 \ln \epsilon$ so dE/dR diverges in the sharp limit even at finite strength, λ.
  - ⋆ Force between lines is finite and agrees with naive calculation, even though the energy of the individual Dirichlet lines diverges in the sharp limit just as badly as the circle.
- 3-dimensions "Dirichlet sphere" or planes

  Worse than $n = 2$. Casimir energy again diverges in the sharp limit, $\epsilon \longrightarrow 0$, one power worse in ϵ.
  - ⋆ Dirichlet circle: $E(R) \propto (R^2\lambda^2 \ln \epsilon)/\epsilon$ so dE/dR diverges in the sharp limit even at finite strength, λ.
  - ⋆ Force between planes is finite and agrees with naive calculation, even though the energy of the individual Dirichlet planes diverges in the sharp limit just as badly as the sphere.

Insight from Continuum Renormalizable Quantum Field Theory

⋆ Conclusions:

- The "Dirichlet/Casimir" problem for the energy doesn't exist as a formal mathematical problem, even in simple cases in low dimensions.

- The total energy depends in detail on the interactions that constrain the fluctuating field on the boundary.

- Nevertheless, boundary condition methods give correct results for forces between rigid bodies and energy densities away from the boundaries.

- But boundary condition methods do not make sense for "Casimir stress" (energy per unit area) or for the total energy.

Summary and Outlook

★ Method is robust, unambiguous, computationally efficient, and makes contact with standard renormalization theory.

★ It is limited to one-loop, enough spatial symmetry to reduce to partial waves, and renormalizable theories.

★ Update on the Standard Model.

- Heavy quarks do not seem to form (spherically symmetric) non-topological solitons. Sphaleron and other classical objects receive large quantum corrections when quarks are heavy.
- Study of Z-strings – vortex-like solitions to standard model – are underway. Khemani & Schröder

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★ Traditional Casimir calculations: parallel plates, spheres, etc. Can be brought into the regime of traditional calculations in the context of renormalizable QFTs.

- Energy density calculations are robust and trustworthy.
- Forces between rigid bodies are well defined.
- Deformation stresses are not.

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- Gravity
- Energy densities
- Branes and vortices