A Different Approach to Δg

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Steven raised two controversional questions in his talk.

I. What is the correct definition of the operator describing the quark spin?

Everyone agrees that the correct operator to leading order is

$$J_{\mu 5} = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5 q$$

with

$$\Delta q S_\mu = \langle PS|J_{\mu 5}|PS\rangle$$

The question is whether it is useful to make a modification at NLO.

$$\Delta q \to \Delta q_0 - \frac{\alpha}{2\pi}\Delta G$$

This is a standard question of choice of scheme. It is familiar in many other contexts where the definition of an operator becomes ambiguous at higher order in α (note that the modification is order α).

Standard renomalization theory tells us that if we use the same scheme to compute both the perturbative coefficients (eg. $\alpha/2\pi$) and the matrix elements, then the answer is scheme independent.

The assertion of Altarelli and Ross (and Carlitz, Collins and Mueller) is that this Δq_0 is closer in spirit to the quantity computed in quark models, however . . .

The quantity guestimated in quark modes is not computed with any QCD radiative corrections (to say nothing of NLO), so we don't know which scheme to assign to a quark computation, and therefore cannot verify the asssertion.

II. What is the operator to identify with the total gluon contribution to the spin of the proton, Δg?

This is the question I would like to focus on.

First, why does one expect to find such an operator?

Since the gluon spin is a piece of one of the Lorentz generators in QCD, one would expect, via Noether's theorem, to find a local operator corresponding to it. It need not be G.I., and in fact it is generally agreed that the Noether process does not yield a local gauge invariant operator interpolating the gluon spin (see RLJ and A. Manohar, and X. Ji).

A second reason for looking for a local operator is that this is what typically occurs in the OPE treatment of DIS. Consider a typical quark distribution, like $\delta q^a(x)$

$$\Delta q^a(x) S^+ \propto \int d\xi e^{ix\xi} \langle PS|\bar{q}(0)\gamma^+\gamma_5\lambda^a I(0,\xi\eta) q(\xi\eta)|PS\rangle$$

This formula requires some explanation. First, it is manifestly gauge invariant. It is not in $A^+ = 0$ gauge. The ξ integral is taken along a direction on the null plane specified by the unit vector $\eta^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$. $I(0,\xi\eta)$ is a Wilson link between 0 and ξn, necessary to preserve gauge invariance. Finally, I have suppressed the Q^2 dependence which smears this integral out around the light-cone by an amount $|\Delta\vec{\xi}_\perp| \sim 1/Q$

Then when we take a moment of $\Delta q(x)$ we use $\int_{-\infty}^{\infty} dx e^{i\xi x} = 2\pi\delta(\xi)$ to relate the moments of δq to local operators, eg.

$$\int dx \Delta q(x) S^+ = \langle PS|\bar{q}(0)\gamma^+\gamma_5\lambda^a q(0)|PS\rangle$$

Notice that the Wilson Link goes away and the operator is local.

The important properties of this operator are

- Local
- Gauge invariant
- Spin n (for $\int dx x^{n-1} \ldots$)
- Twist 2, ie. dimension $n+2$.

What about gluons?

When interest turned to the gluon spin, theorists sought an operator to describe it, in exact analogy to the operator $\bar{q}\gamma_\mu\gamma_5 q$ which descibes quark spin.

We are not talking about scheme dependence here. That issue arises only when we try to extend the definition of an operator beyond leading, trivial, order in QCD perturbation theory.

Because the gluon field strength has engineering dimension 2, it is not possible to find an operator satisfying all those constraints. Anything local, with two factors of $F^{\mu\nu}$, has dimension 4. If it had spin 1, then it would have twist 3 and would not contribute in the scaling limit.

In response to this dilemma Altarelli and Ross, and Efremov and Teryaev *guessed* that the operator K_μ, the Kogut-Susskind current, should be the correct interpolating field. This was a guess. It raises fundamental questions about gauge invariance and it brings in the physics of the anomaly, instantons, etc.

However it is not the only proposal, and in my opinion, it is obviously the wrong proposal.

Manohar's proposal – 1990

[Following up our paper to which Steven referred.]

Consider the lightcone description of the x dependent gluon polarization structure function. This can easily be worked out by "inverse Mellin transform" of the standard operators that are connected to the higher moments $n > 0$ of Δg. The result is

$$\Delta g(x) S^+ \propto \frac{1}{x}\int d\xi e^{ix\xi} \langle PS|\tilde{F}^+_\alpha(0) I(0,\xi\eta) F^+_\alpha(\xi\eta)|PS\rangle$$

Note that this is gauge invariant, etc.

One can check this (and see how it was derived) by taking the moments with $n > 1$, for example, take $n = 2$:

$$\int dx\ x\ \Delta g(x) S^+ \propto \langle PS|\tilde{F}^+_\alpha(0) F^+_\alpha(0)|PS\rangle$$

which is the expected twist-two, spin-two axial operator which measures the spin weighted momentum fraction of the gluons in the proton.

So what happens when one takes the zeroth moment, which measures the total spin?

Note that

$$\int \frac{dx}{x} e^{i\xi x} \propto \text{sign}(\xi)$$

So one obtains

$$S^+ \Delta G \propto \int d\xi\ \text{sign}(\xi)\ \langle PS|\tilde{F}^+_\alpha(0) I(0,\xi\eta) F^+_\alpha(\xi\eta)|PS\rangle$$

Which is Gauge invariant, twist-two, axial, and spin-one,

however it is not local.

Comments:

- It is hard to see how you could get a non-gauge invariant operator out of manipulations of a gauge invariant gluon distribution function.

- It is not surprising that it is not local, because the Noether analysis did not yield a local gauge invariant operator corresponding to gluon spin.

What then is the relation to K_μ, and how did A&R, E&T make their mistake?

Easy:

The correct, gauge invariant operator coincides with K^+ in $A^+ = 0$ gauge

To see this:

$$F^{+\alpha} = \partial^+ A^\alpha - \partial^\alpha A^+ - g[A^+, A^\alpha] \stackrel{A^+=0}{\rightarrow} \partial^+ A^\alpha \propto \frac{\partial}{\partial \xi} A^\alpha$$

Then integrate by parts,

- $\frac{d}{d\xi}\text{sign}(\xi) = 2\delta(\xi)$

- and ignore surface terms at $\xi = \pm\infty$.

and obtain

$$\Delta g(x) S^+ \propto \langle PS|\tilde{F}^+_\alpha(0) A^\alpha(0)|PS\rangle \propto \langle PS|\, K^+(0)\big|_{A^+=0}\, |PS\rangle$$

So A&R, E&T made the wrong generalization beyond $A^+ = 0$ gauge.

They guessed that gluon spin was described by a local, gauge non-invariant operator, but in fact it's a non-local, gauge invariant operator.

Comments:

- Connection between correct operator and K^+ is still clouded – even in $A^+ = 0$ gauge by possible surface terms in integration by parts, related to Steven's $\delta(x)$ terms, so I am not even confident that $K^+(0)|_{A^+=0}$ is an acceptible operator.

- There is no obvious connection to the anomaly because the operator of interest does not obey $\partial_\mu G^\mu \propto F\tilde{F}$.

- Furthermore there is no connection to K-S ghost poles etc.