Fits to Structure Functions at Small-x

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I will present an outline of the most up-to date fits within the conventional framework of NLO-in-α_s perturbation theory, highlighting some features which cause concern at small x. I will also discuss a resummation relevant for small x which combines the leading $\ln(1/x)$ resummation with a resummation of running coupling constant contributions, i.e. a resummation in β_0.

We (A.D. Martin, R.G. Roberts, W.J. Stirling and R.S.T.) perform a global fit to all available structure function and related data in order to obtain best possible determination of NLO parton distributions, and **also** to determine the success of NLO QCD in describing the data which are fit. Hence, include all data within the feasible range of applicability.

There is a wide range of data used, but that most relevant for small x is

H1 $F_2^{e^+p}(x,Q^2)$ 1996-97 moderate Q^2 and 1996-97 high Q^2, and $F_2^{e^-p}(x,Q^2)$ 1998-99 high Q^2 New.

ZEUS $F_2^{e^+p}(x,Q^2)$ 1996-97 New.

NMC $F_2^{\mu p}(x,Q^2), F_2^{\mu d}(x,Q^2), (F_2^{\mu n}(x,Q^2)/F_2^{\mu p}(x,Q^2))$ - goes down to $x = 0.0045$.

CDF Inclusive jet data New

D0 Inclusive jet data New

Fit currently performed by minimizing χ^2 to all data adding statistical and systematic errors in quadrature (other than jet data) - treatment of errors to improve. Start evolution at $Q_0^2 = 1\mathrm{GeV}^2$.

Cut data at $Q^2 = 2\mathrm{GeV}^2$ and at $W^2 = 12.5\mathrm{GeV}^2$ to avoid higher twist. Use Thorne-Roberts VFNS. Improves global fit compared to ZM-VFNS and FFNS.

High x gluon is important to the fit for small x structure functions because the momentum sum-rule constraint means the high x gluon places constraints on the possible low-x gluon, and also contributes to small x evolution via convolution.

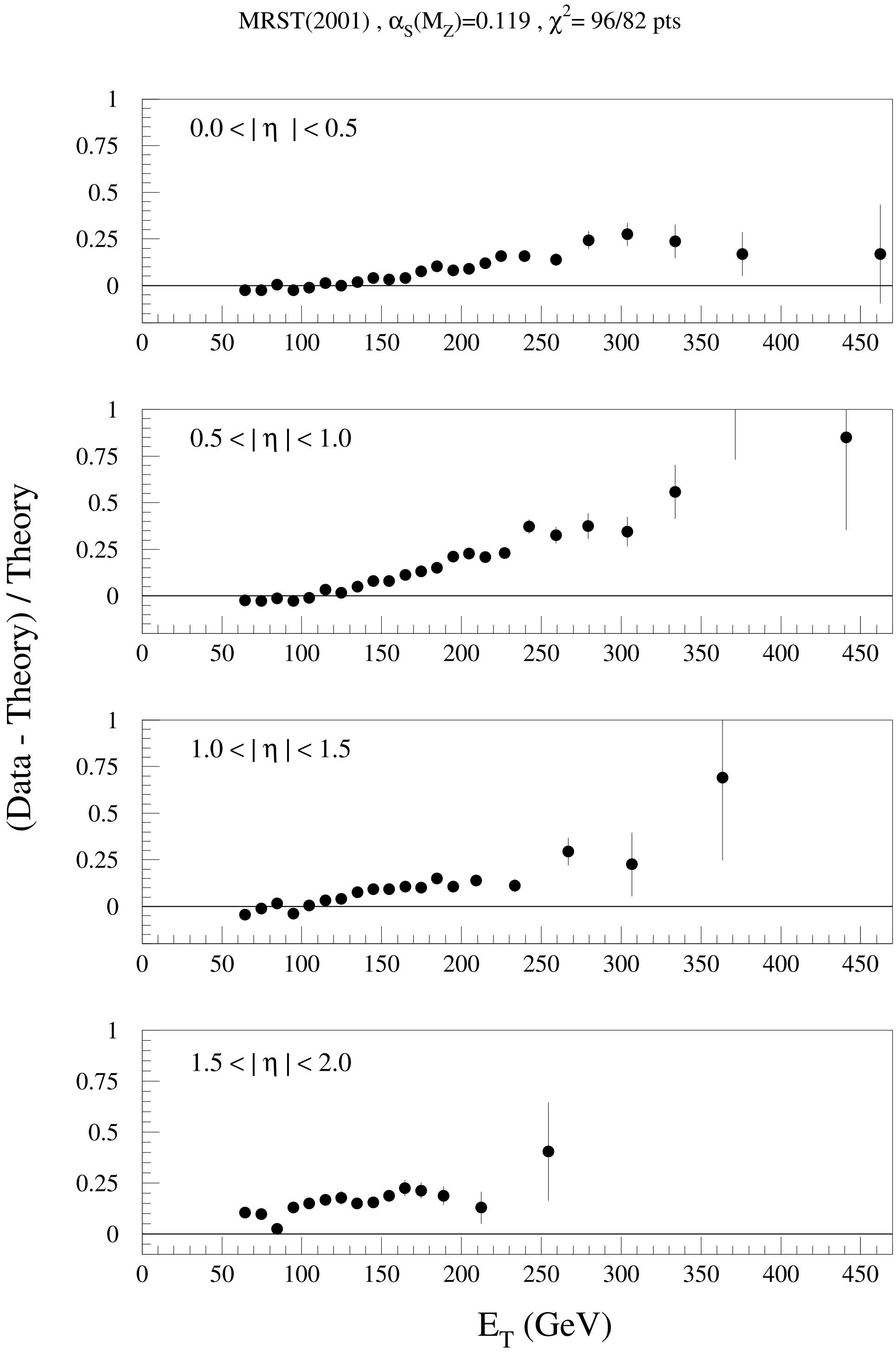
Previously used prompt photon data with $g\uparrow$, g and $g\downarrow$ corresponding to no intrinsic k_T, moderate k_T and max k_T. Both uncertainty in stability of theory and in consistency of experiments. Now use constraint of reasonable fit to jet data.

$\rightarrow g(x,Q^2)$ somewhere between previous $g\uparrow$ and g (slightly nearer $g\uparrow$). $xg(x,Q^2) \sim 0.1$ at $x = 0.4, Q^2 = 20\mathrm{GeV}^2$. Uncertainty of $\sim 15\%$ in $g(x,Q^2)$ for $x \sim 0.4$ $Q^2 \sim 20\mathrm{GeV}^2$.

Complicated interplay between jets and DIS data. Once we enforce high x constraint on gluon DIS data prefer $\alpha_S = 0.121$, and are happy with form of gluon. However $\rightarrow$ poor fit to jets - jet data alone prefer $\alpha_S(M_Z^2) = 0.115$. Forcing lower $\alpha_S \rightarrow$ DIS fit deteriorating and much preferring smaller high x gluon. Fit achieved by experimental groups. Compromise between all data $\rightarrow \alpha_S = 0.119$.

Good fit to jet data. Normalization about correct.

Fit to new D0 jet data:



New HERA data with larger coverage and much smaller errors have far more significant weight in fit.

Data require gluon to be negative!

$$xg(x,Q_0^2) = A(1-x)^{\eta}(1+\epsilon x^{0.5}+\gamma x)x^{\delta} - A_-(1-x)^{\eta_-}x^{-\delta_-}$$

$\eta_- \sim 10$, $\delta_- \sim 0.3$.

Not using this extended parameterization $\rightarrow \Delta\chi^2 \approx 100$.

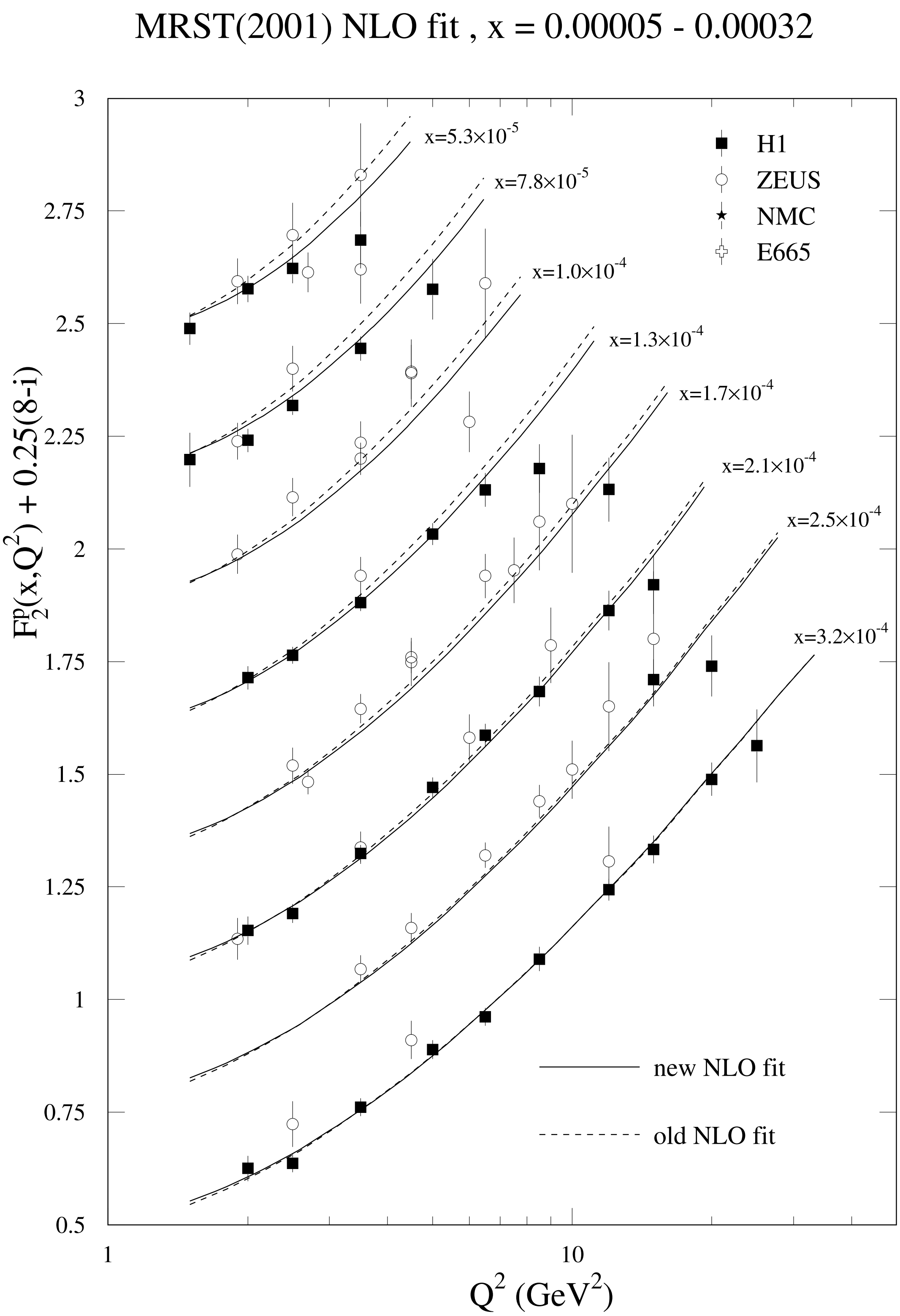
Good fit to HERA data, but some problem at highest Q^2 at moderate x, i.e. in $dF_2/d\ln Q^2$.

Not just low x low Q^2 data that require negative gluon. Moderate x data need lots of gluon to get reasonable $dF_2/d\ln Q^2$. Must be compensated for elsewhere.

$F_L(x,Q^2)$ dangerously small at smallest x, Q^2.

All data sets acceptable fits. (Miss large $\ln(1-x)$ terms by using slightly higher W^2 cut).

Comparison of MRST(2001) and MRST(1999)



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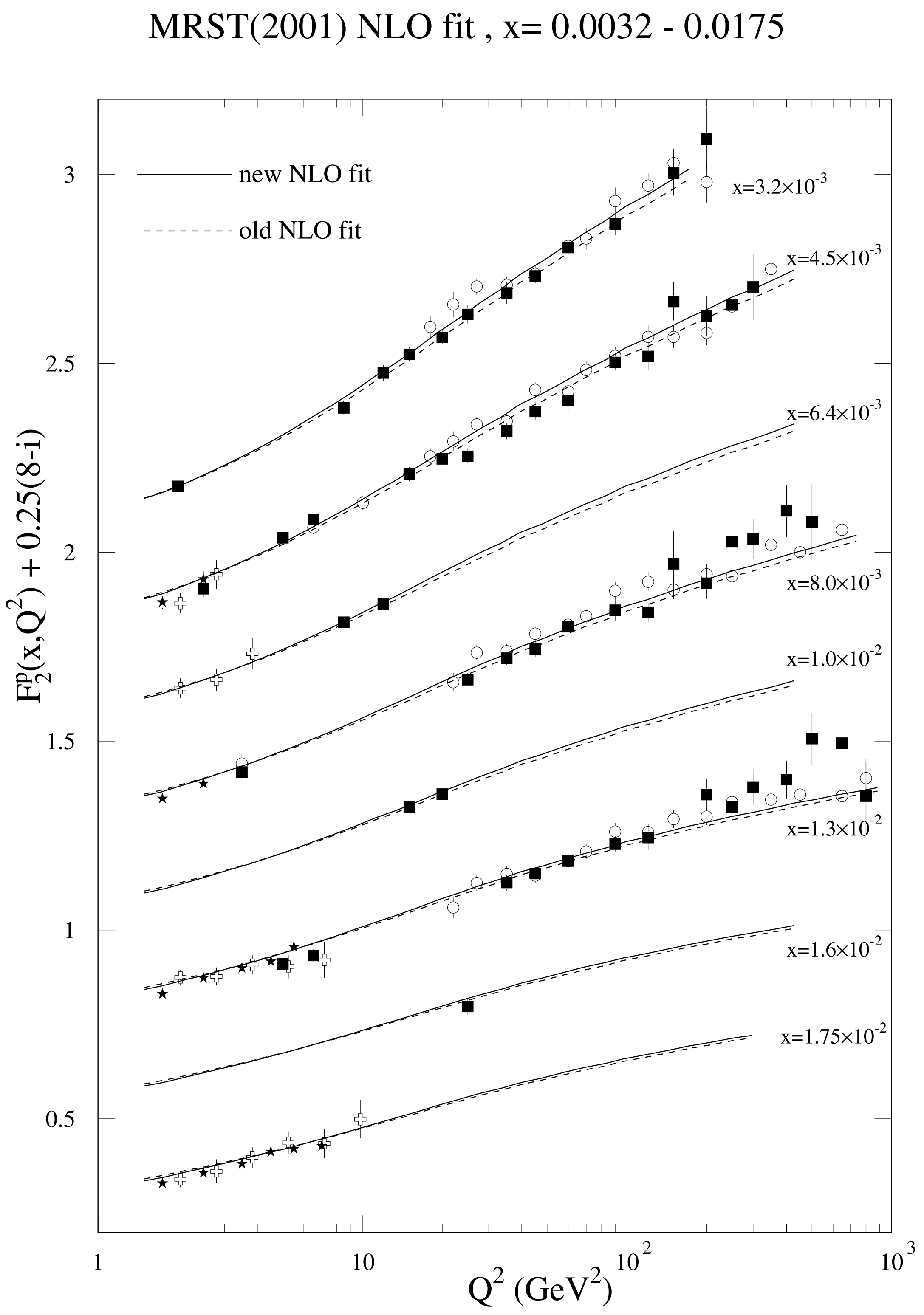

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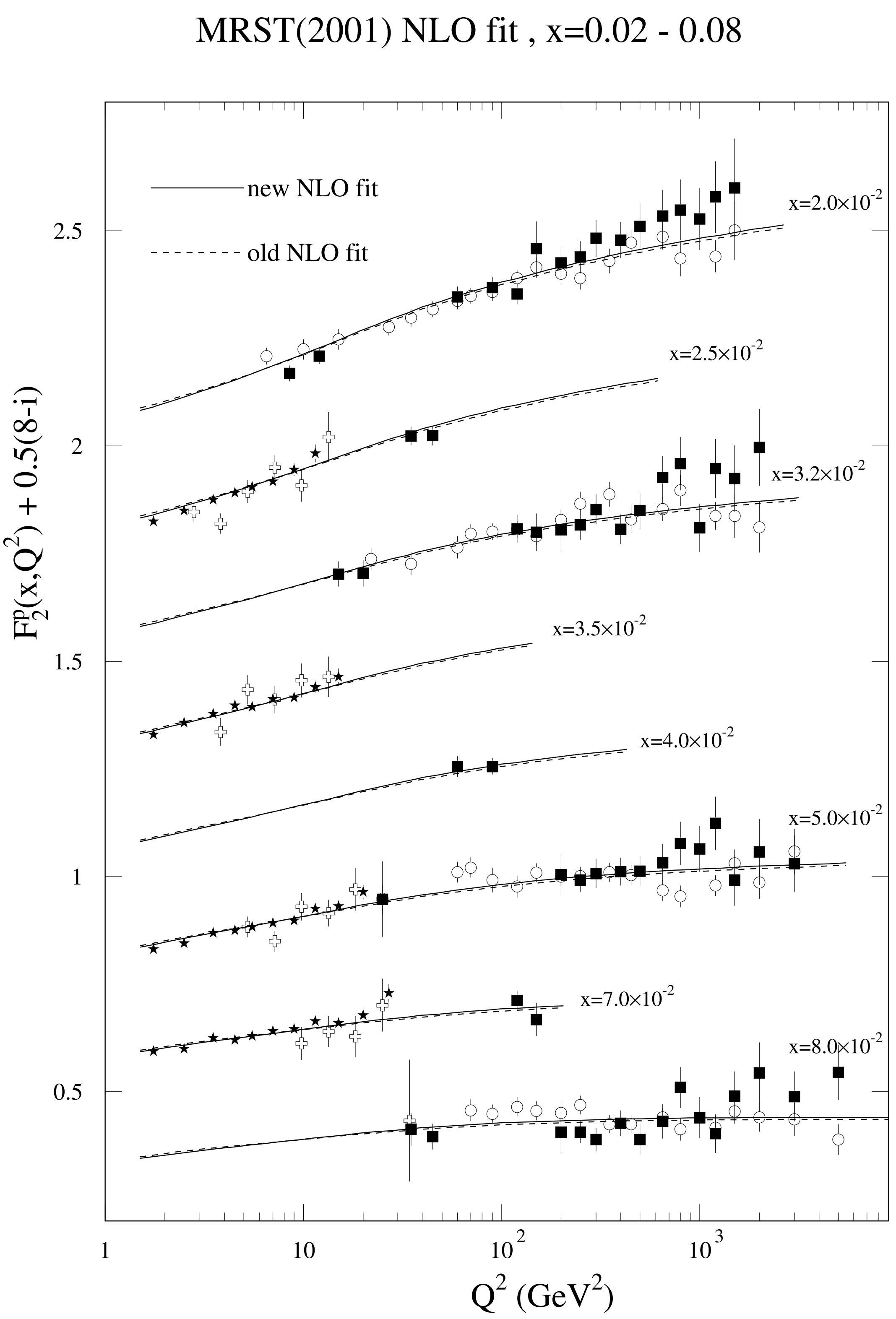
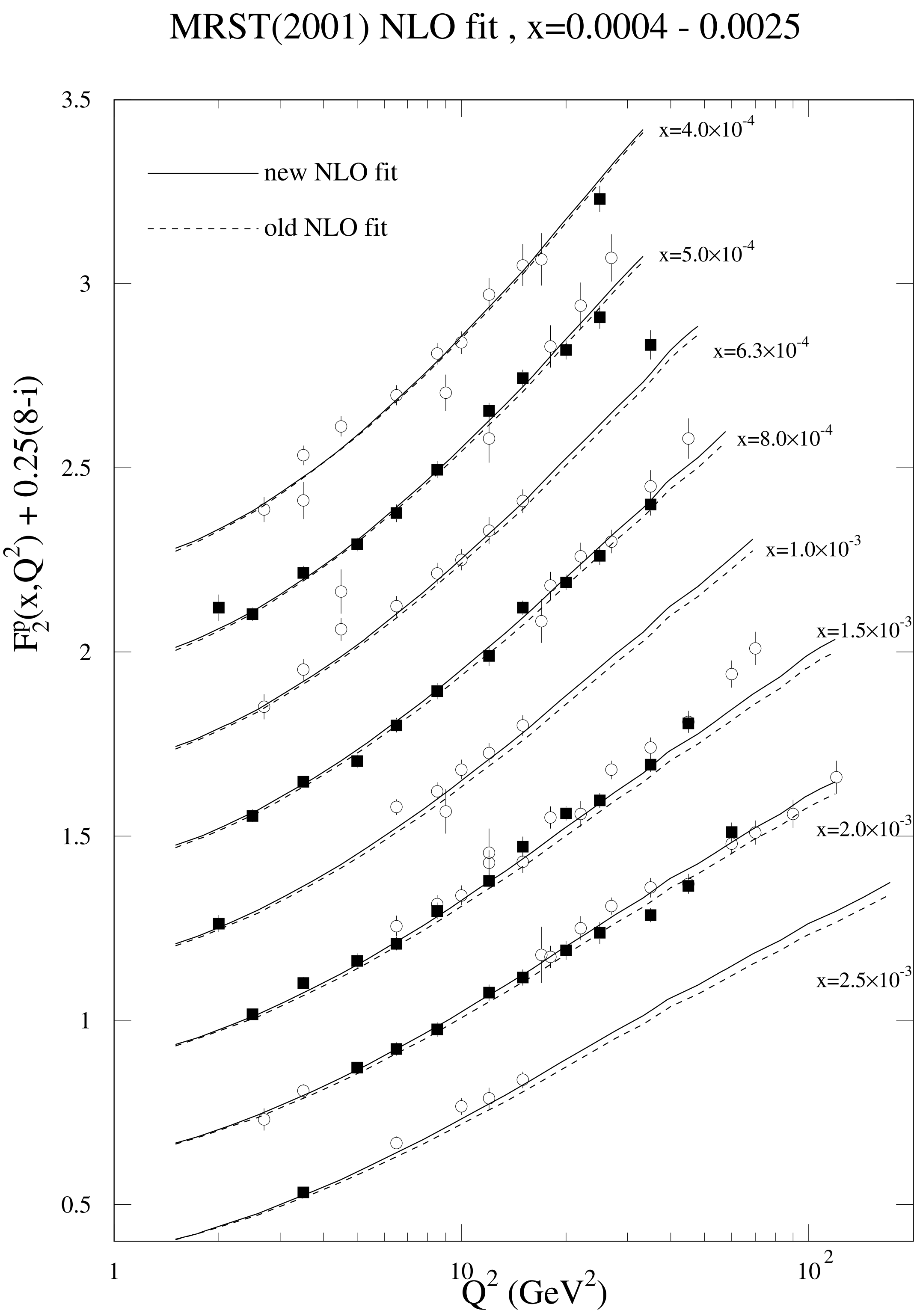
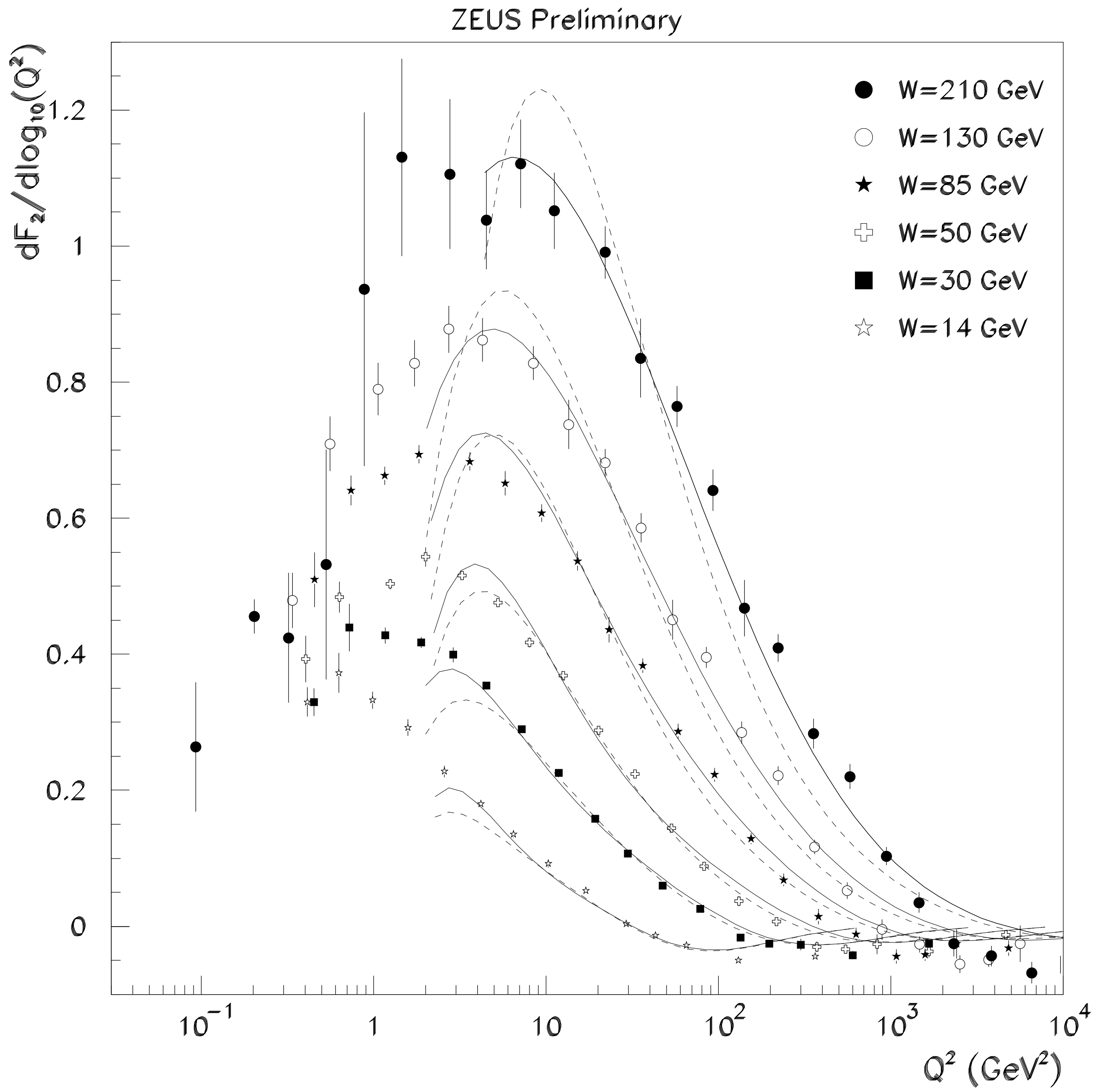


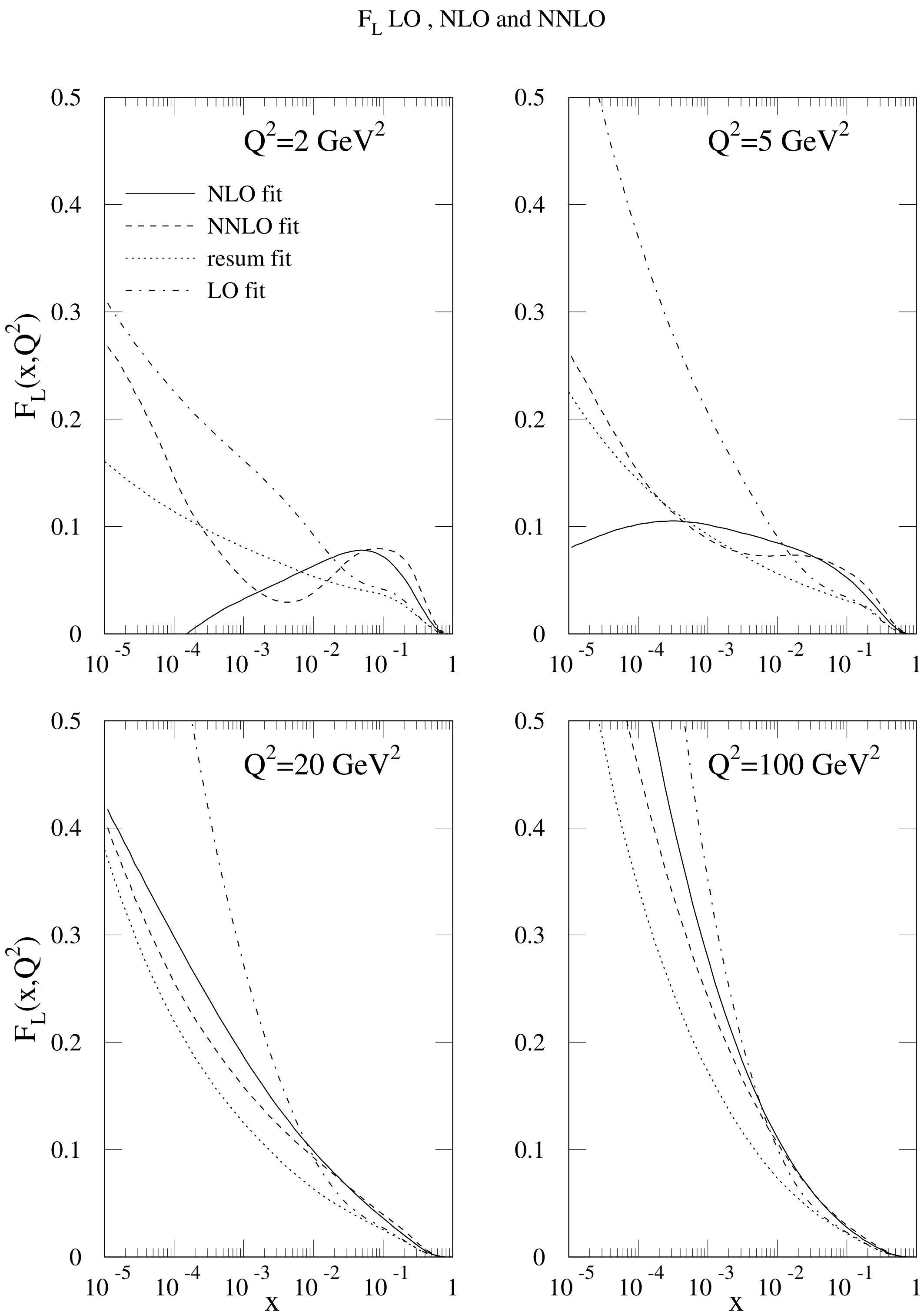
Comparison of MRST(2001) and MRST(1999)



Comparison of MRST(2001) (dashed) and small-x resummed (solid) values of $dF_2(x,Q^2)/d\ln Q^2$ with ZEUS data.



Comparison of $F_L(x, Q^2)$ using various approaches.



Quality of NLO-in-α_S fit.

Data set	No. of data pts	MRST
H1 ep	400	377
ZEUS ep	272	253
BCDMS μp	167	205
NMC μp	126	136
NMC μd	126	100
SLAC ep	53	71
E665 μp	53	52
E665 μd	53	60
CCFR $F_2^{\nu N}$	63	57
CCFR $F_3^{\nu N}$	63	57
NMC n/p	156	144
Jets	113	152

And about 1 per point for smaller sets. $\alpha_S(M_Z^2) = 0.119$. Compromise between different data sets. Total $\chi^2 = 1754/1723$ - but errors overestimated. Some areas where theory perhaps needs to be improved.

In order to investigate real quality of fit and regions with problems we try varying kinematic cuts on data. Procedure - change W^2_{cut}, Q^2_{cut} and x_{cut}, re-fit and see if quality of fit to remaining data improves and/or input parameters change dramatically.

For W^2_{cut} raising from $12.5\mathrm{GeV}^2$ has no effect.

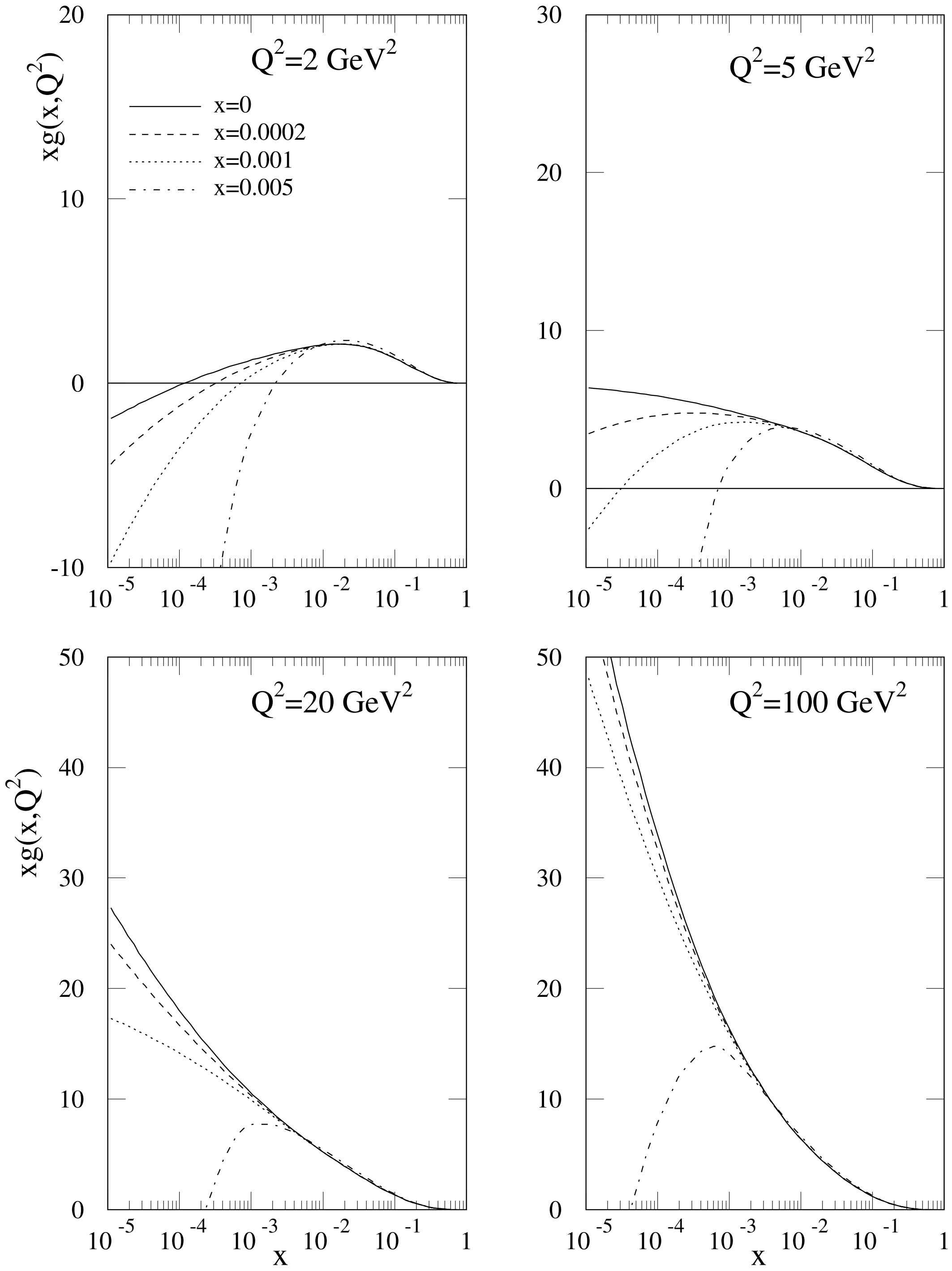
Raising Q^2_{cut} from $2\mathrm{GeV}^2$ to $3\mathrm{GeV}^2$ very small effect - $\Delta\chi^2 = 12$ input gluon slightly more negative. No further improvement above this.

Raising x_{cut} from 0 to 0.0002 to 0.001 to 0.005 continuous improvement - $\Delta\chi^2 = 48$, $\delta_- \to 0.6$ and $\alpha_S(m^2_Z) \to 0.1175$. Stability reached at $x = 0.005$. At each step moderate x gluon becomes more positive and $\alpha_S(M^2_Z)$ more in line with jets and high x data sets.

Data set	No. of data pts	MRST	$x_{cut} = 0.005$
H1 ep	287	292	271
ZEUS ep	141	130	148
NMC μp	125	135	123
NMC μd	125	100	86
Jets	113	152	141

When $W^2_{cut} = 12.5$, $Q^2_{cut} = 3\mathrm{GeV}^2$ and $x_{cut} = 0.005$, no real tension between data sets.

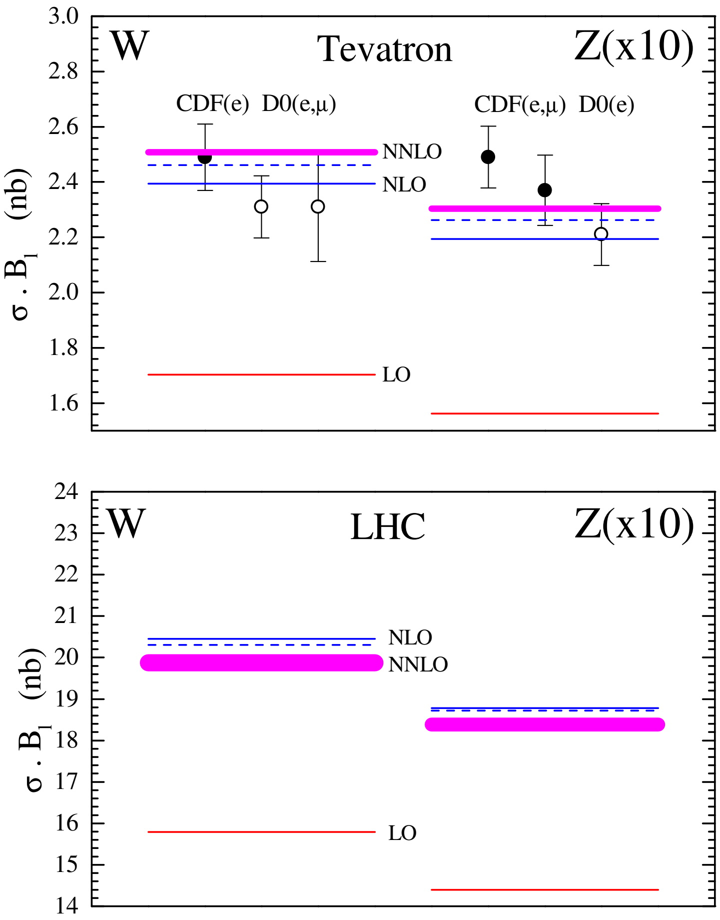
Comparison of gluons obtained when x_{cut} is varied.



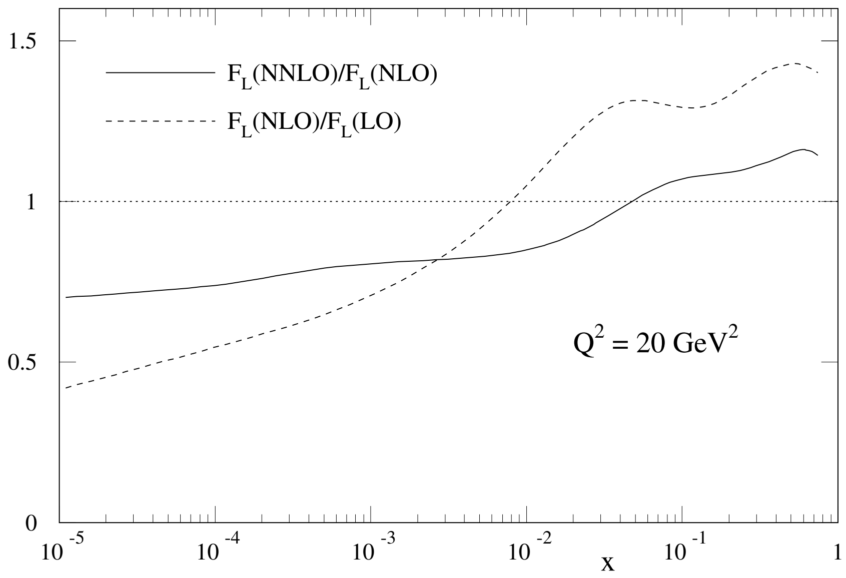
In order to investigate stability of perturbative expansion can also use what we already know about NNLO.

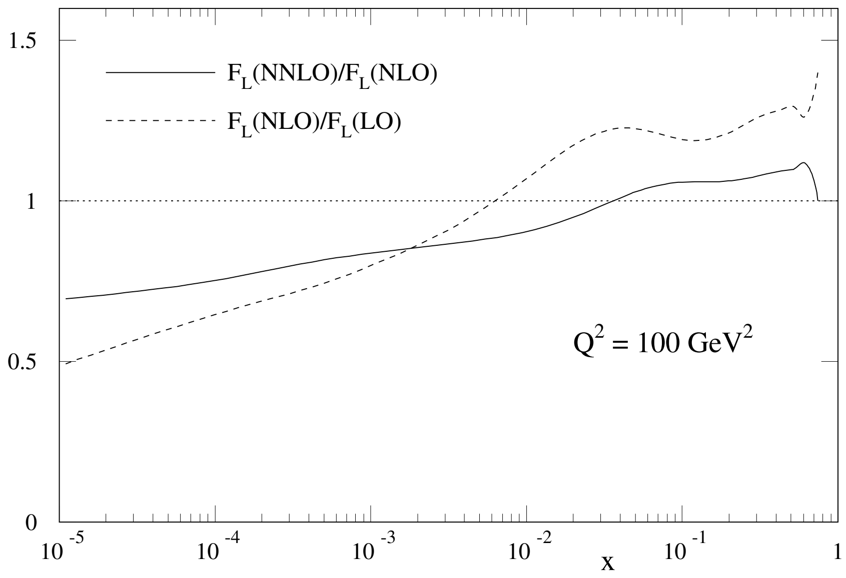
Coefficient functions known at NNLO. Singular limits $x \to 1$, $x \to 0$ known for NNLO splitting functions as well as limited moments (Larin, Nogueira, van Ritenberg Vermaseren, Retey). Approximate NNLO splitting functions devised by van Neerven and Vogt.

Used in global fit by MRST. Improve quality of fit. Reasonable stability for (quark-dominated) W and Z cross-sections. ($dF_2(x,Q^2)/d\ln Q^2$) problem?)



More danger in gluon dominated quantities - gluon not directly constrained by data. Possible/probable poor convergence order by order for $F_L(x, Q^2)$ at small x.





Note K-factor for Higgs production at LHC ~ 1.6 Spira, Djouadi, Graudenz and Zerwas.

Therefore, although NLO fit seems acceptable there are very clear hints that there are problems at small x, and that this is not very correlated to small Q^2.

Try alternative perturbative organization - resummation of leading $\ln(1/x)$ terms and running coupling effects (resummation of β_0 terms). Obtained by solving running-coupling BFKL equation. Impossible to obtain normalizations and inputs without regularization, but possible to obtain splitting functions unambiguously.

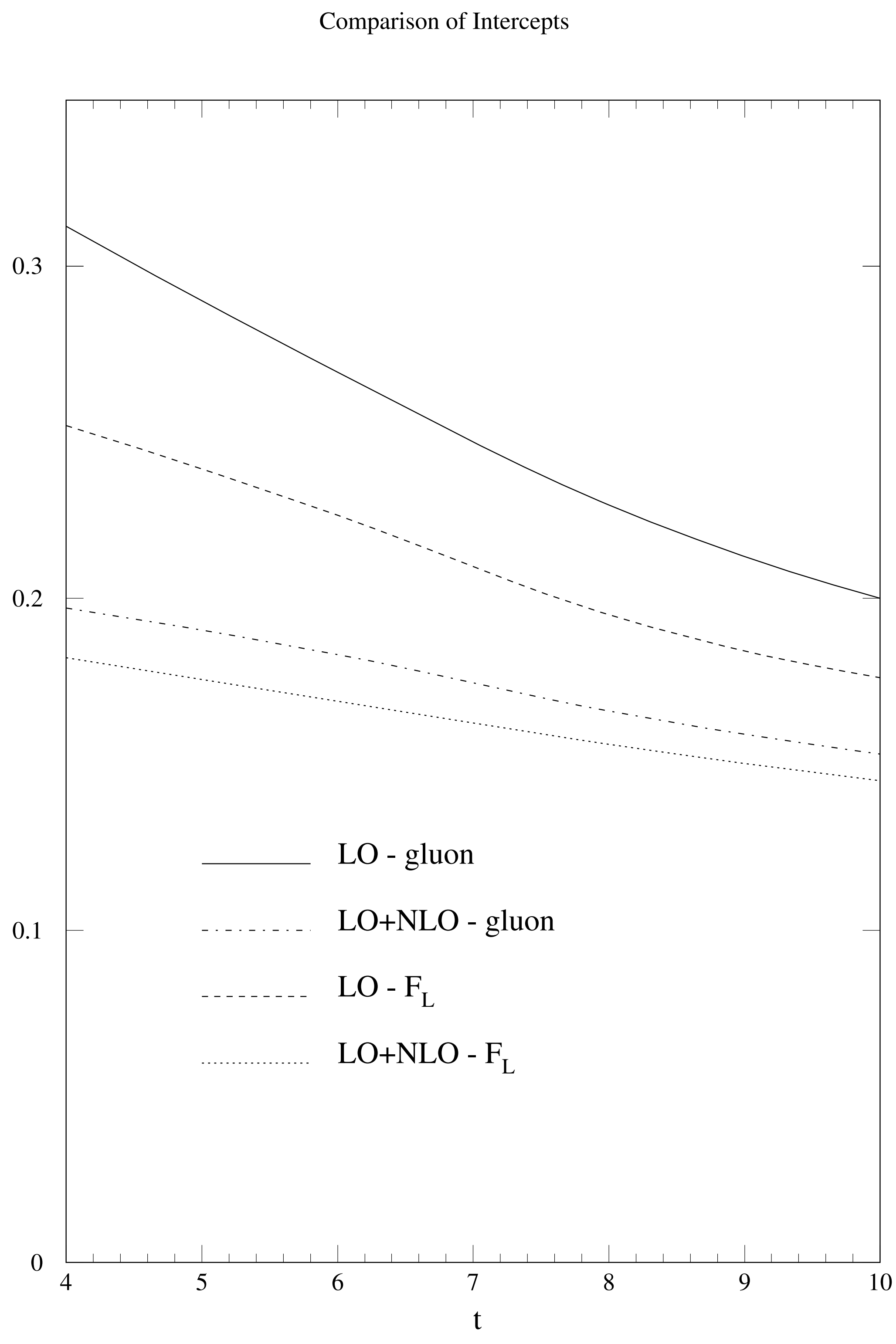
$$xP_{gg}(x,Q^2) = \sum_{n=1}^{\infty}\sum_{m=0}^{n-1} a_{nm}\alpha_S^n(Q^2)\ln^{n-1-m}(1/x)\beta_0^m$$

Solving running coupling equation reduces "LO" effects significantly, but stabilizes perturbative series.

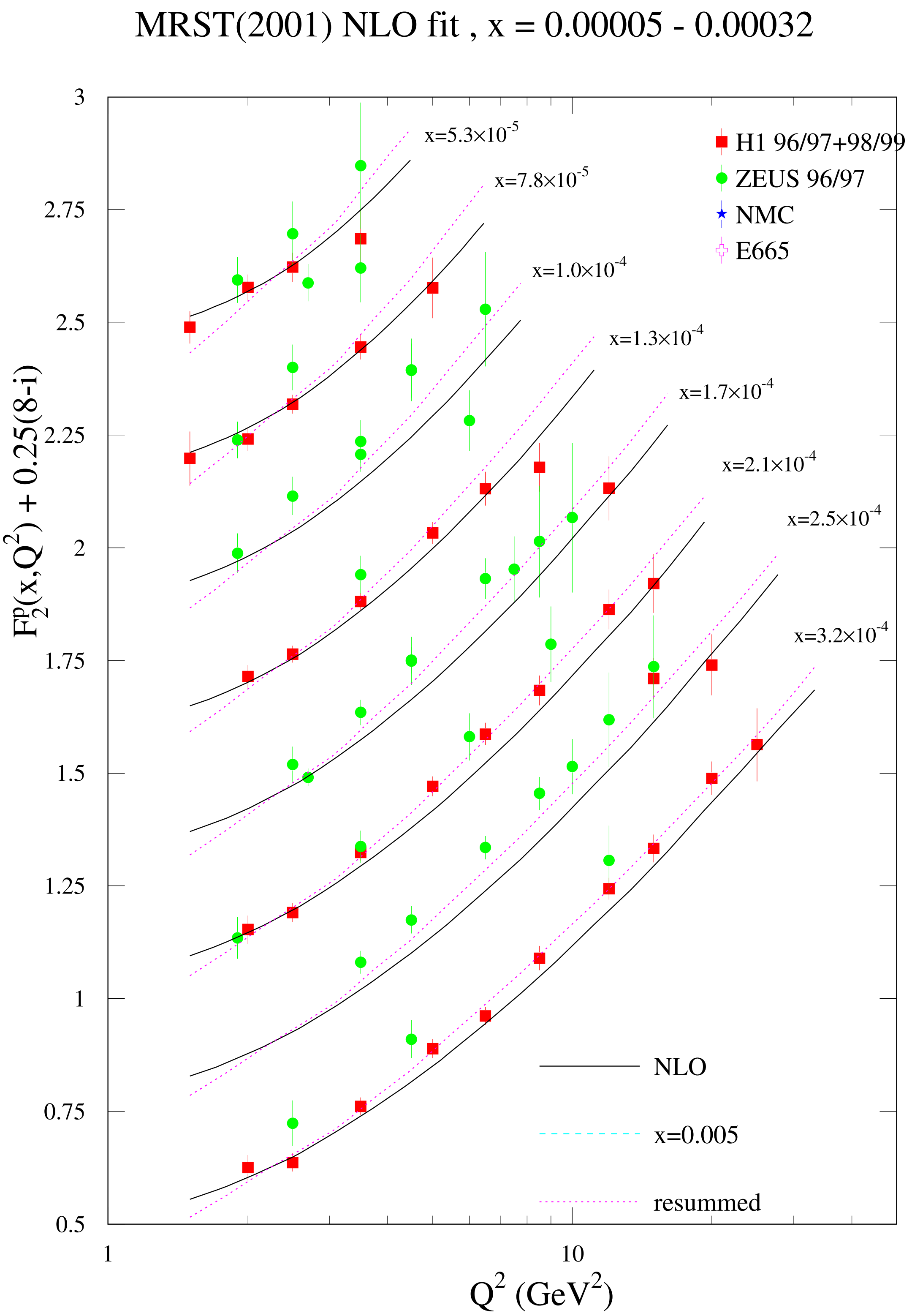
LO resummed calculation leads to better fit than NLO-in-α_S fit (jets not completely incorporated yet), particularly in terms of $dF_2(x,Q^2)/d\ln Q^2$. Stability under changes in x_{cut}, $\alpha_S(M_Z^2) = 0.116$, and much more stable result for $F_L(x,Q^2)$ at low-moderate Q^2.

Data set	No. of data pts	MRST	resummed
H1 ep	400	377	336
ZEUS ep	272	253	272
NMC μp	126	136	126
NMC μd	126	100	94

Stability of asymptotic power-like behaviour of splitting function $\sim x^{-\lambda}$ in resummed calculation.



Comparison of MRST(2001), $x_{cut} = 0.005$ and resummed fit.



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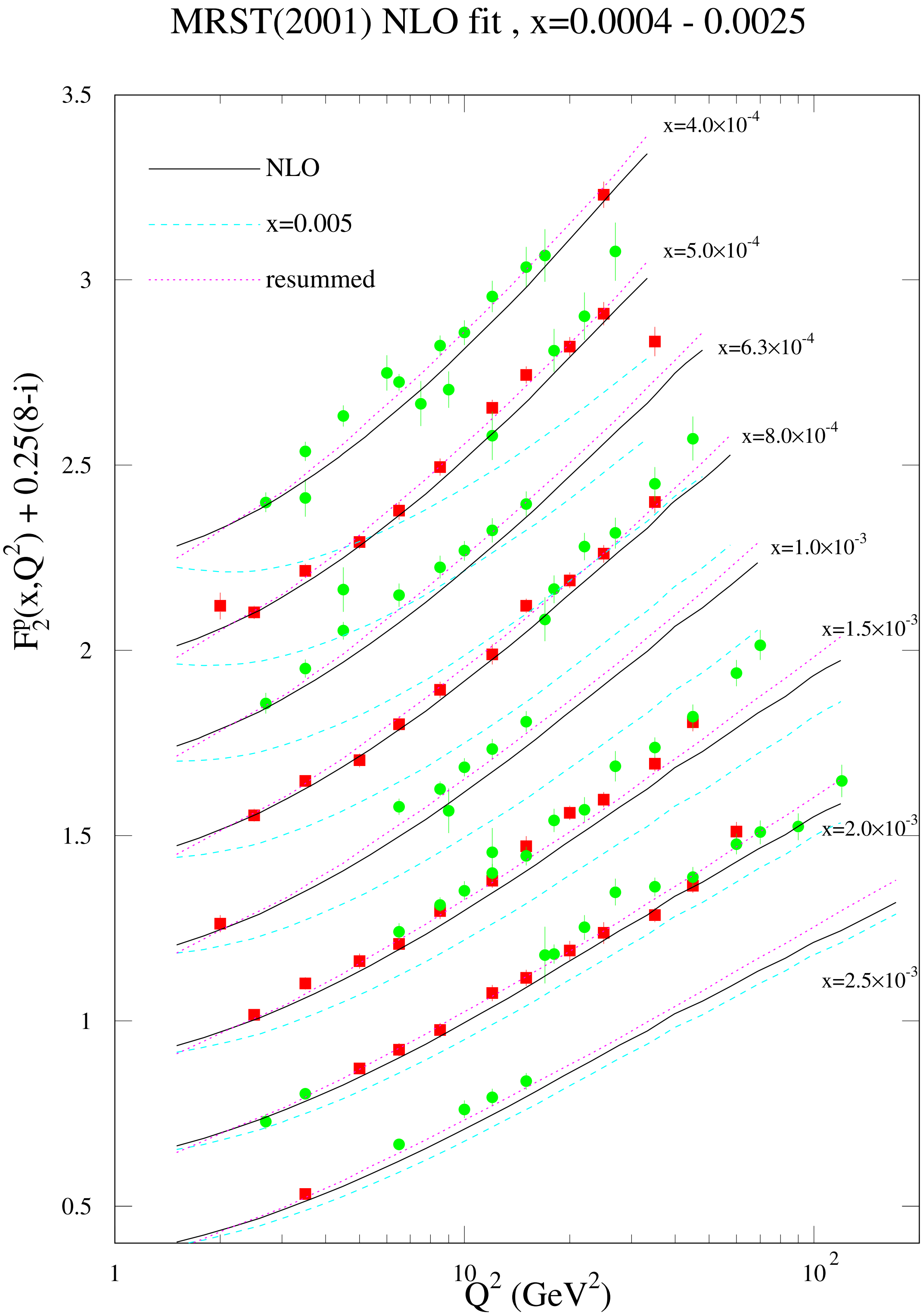

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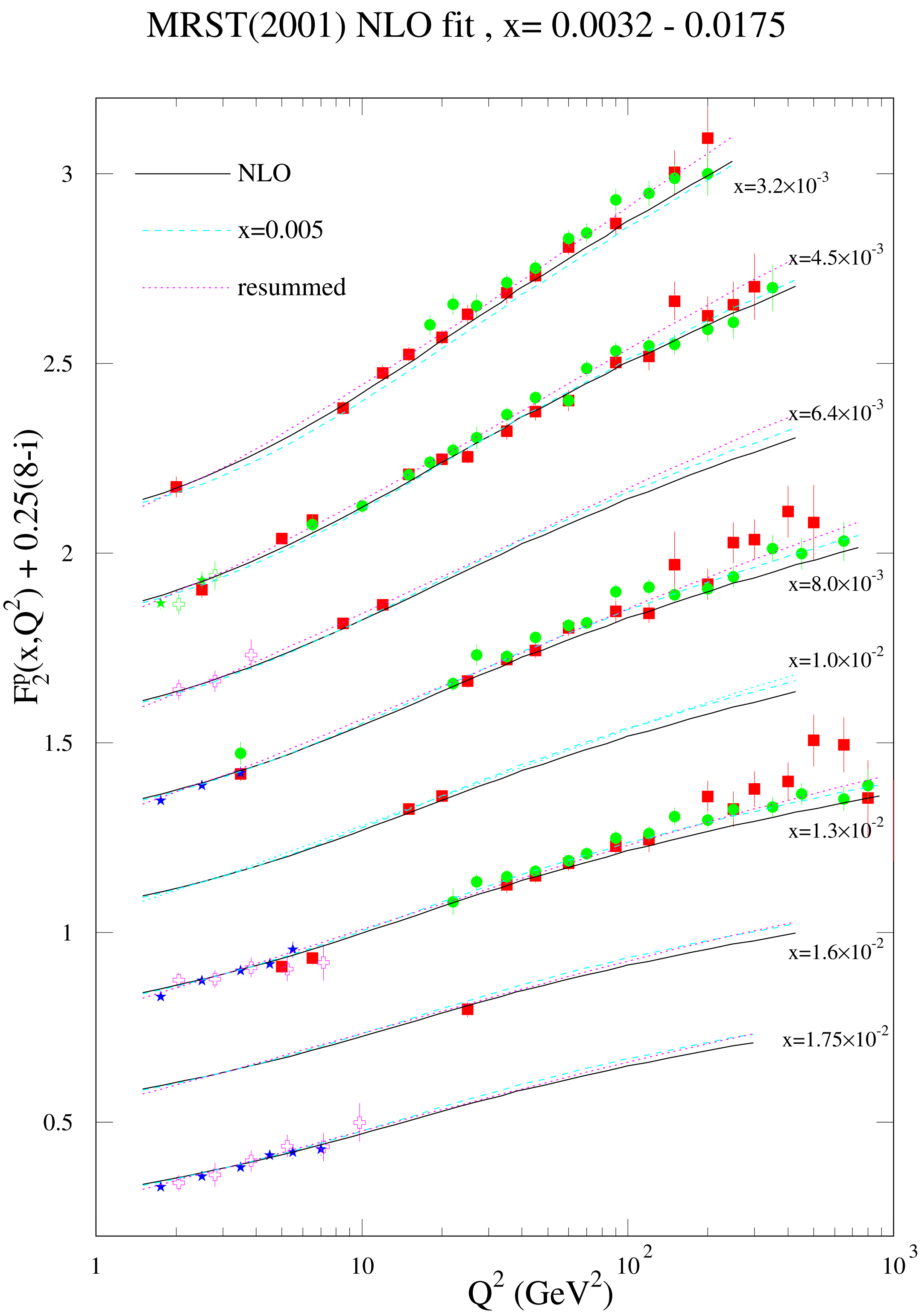

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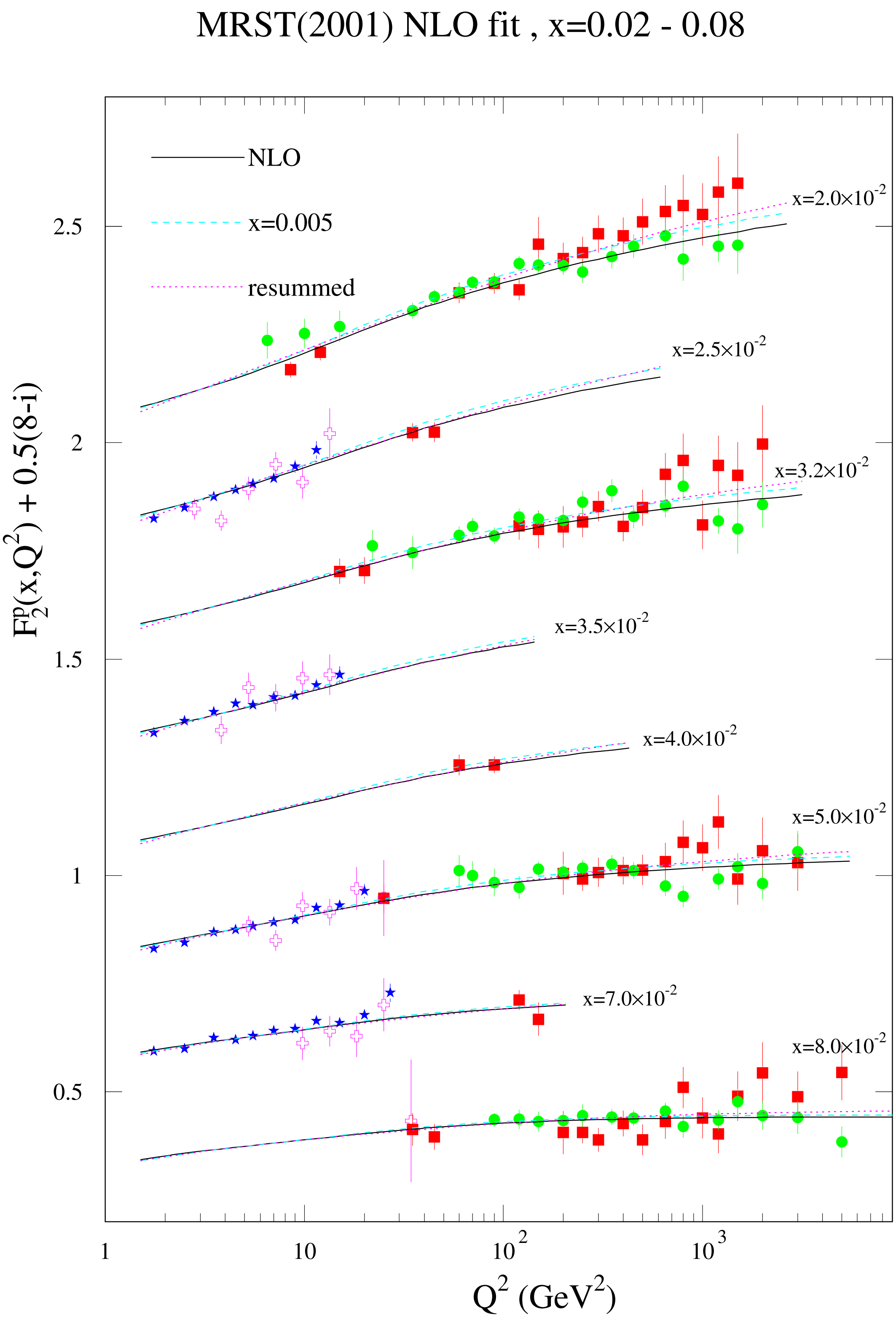

Conclusions

Perform global fits to all up-to-date data over wide range of parameter space. NLO-in-$\alpha_S(Q^2)$ fit fairly good.

High x gluon constrained (to 15%) by new jet data. Upper part of previous range. Affects (worsens) small-x fit. Pulls in opposite directions on $\alpha_S(Q^2)$.

Serious problem: $dF_2/d\ln Q^2$ at moderate x. Also needs negative small x gluon $\rightarrow$ strange form for $F_L(x,Q^2)$ at small x until fairly high Q^2.

Quality of $dF_2/d\ln Q^2$ can be significantly improved by reducing x_{cut} at the expense of even more negative gluon below x_{cut}. Little evidence of improvements/modifications with Q^2 cuts.

LO resummed fit using splitting functions obtained from running-coupling BFKL equation improves total fit, provides stability and gives "sensible" prediction for $F_L(x,Q^2)$.