Parton distributions at low x

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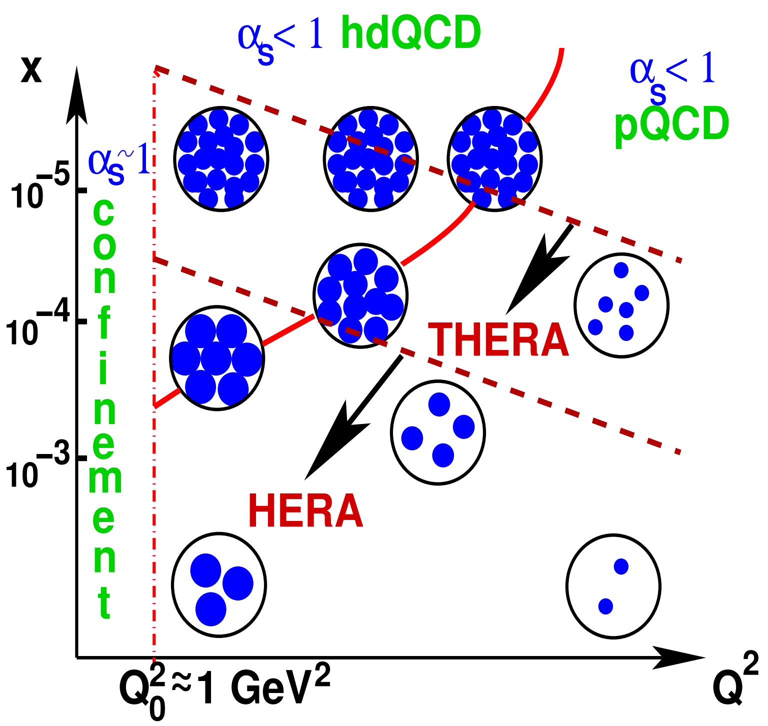
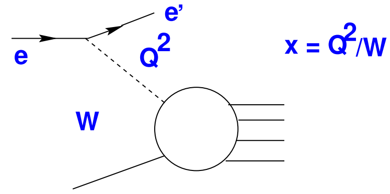
Low x WS, Cracow, June 27 -30, 2001

Fighting prejudices

Our team: **E.Gotsman, E.L. M. Lublinsky and U. Maor**

Our papers: **hep-ph : 0102321; 0104108; 0106112**



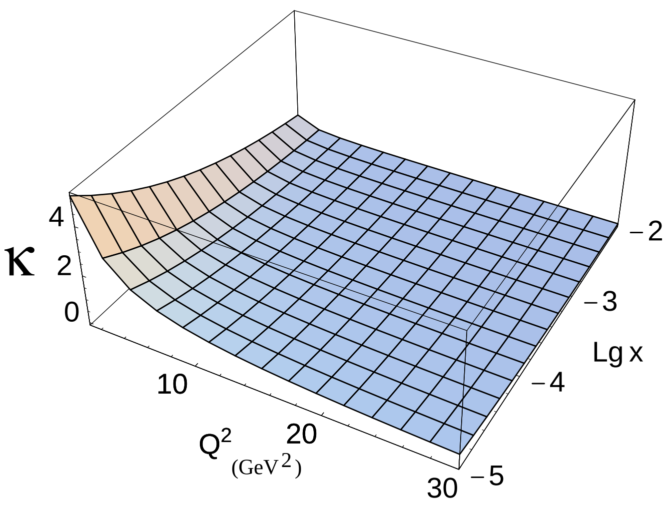


Packing factor:

$$\kappa(Q^2, x) \;=\; \frac{3\,A\,\pi^2\alpha_S}{Q^2}\,\frac{1}{\pi R_A^2}\,xG^{DGLAP}(Q^2, x)$$

Lessons from HERA:

- **The parton density is high at HERA energies**



- **All predictions of asymptotic hdQCD have already been seen in HERA data;**

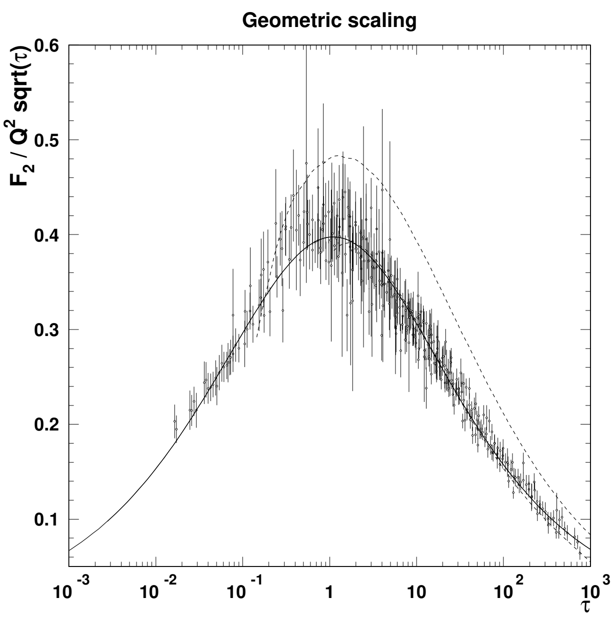
- **All HERA data can be described by models including parton saturation;**

- **All HERA data can also be described without assuming saturation, but assuming that matching between "soft" and "hard" interactions occurs at rather large $Q^2 \approx 2 \div 4\ GeV^2$;**

Outline:

- Extrapolation from HERA to THERA, LHC and eRHIC;

- New scaling $\sigma = R^2 F(Q^2/Q_s^2(x))$;



  - K. Golec-Biernat, J. Kwiecinski and A. Stasto

- Saturation scale for ep and eA DIS;

Our approach:

Two steps:

- Solving the non-linear evolution equation
  - which takes into account all twists contributions;
  - but only in leading log approximation of QCD;

- Solving the DGLAP-type linear equation for the correcting function ΔN. It turns out that $\Delta N \to 0$;

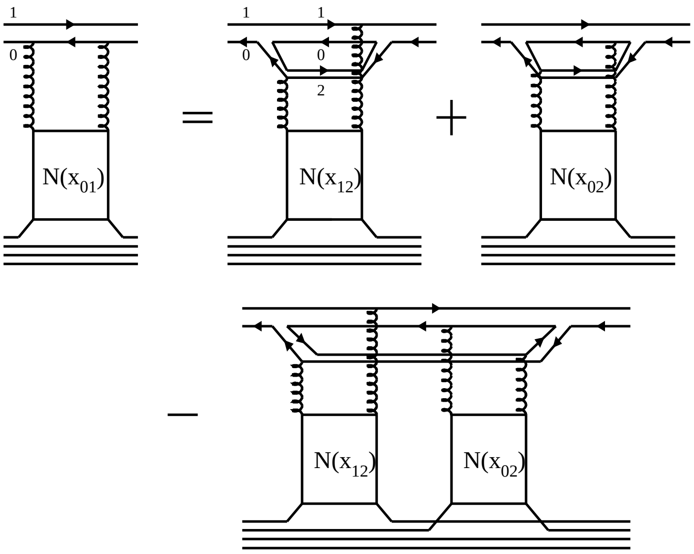
The full solution is

$$N(r_\perp^2, x; b_t) = \tilde{N}(r_\perp^2, x; b_t) + \Delta N(r_\perp^2, x; b_t)$$

$\tilde{N}(r_\perp^2, x; b_t) \to$ non-linear equation;

$\Delta N(r_\perp^2, x; b_t) \to$ DGLAP linear equation;

Non-linear evolution:



GLR (81)
Mueller & Qiu (86)
....
....
....
Balitsky (95)
Kovchegov (99)
Braun (00)
Iancu, Leonidov & McLerran (00)

$$\frac{d\tilde{N}(x_{01},y;b_t)}{dy} = -\frac{2\,C_F\,\alpha_S}{\pi^2}\,\ln(\frac{x^2_{01}}{\rho})\,\tilde{N}(x_{01},y;b_t)\,+$$

$$\frac{C_F\,\alpha_S}{\pi^2}\times\int_\rho d^2x_2\frac{x^2_{01}}{x^2_{02}\,x^2_{12}}$$

$$\Big(\,2\,\tilde{N}(x_{02},y;b_t) - \tilde{N}(x_{02},y;b_t)\tilde{N}(x_{12},y;b_t)\Big)$$

Comments:

- The equation sums:

$$\left(\alpha_S^2\left(\frac{1}{x}\right)^{\Delta}\right)^n \text{ with } \Delta \;\propto\; \alpha_S$$

These corrections are important starting from

$$\alpha_S \ln(1/x) \;\approx\; \ln(1/\alpha_S)$$

Higher order corrections give $\Delta \;=\; C_1\alpha_S + C_2\,\alpha_S^2 + \ldots$ and they are essential only for

$$\alpha_S \ln(1/x) \;\geq\; 1/\alpha_S \;\gg\; \ln(1/\alpha_S)$$

The calculation of higher order corrections in the DGLAP equations can be considered as a model without solving the master equation.

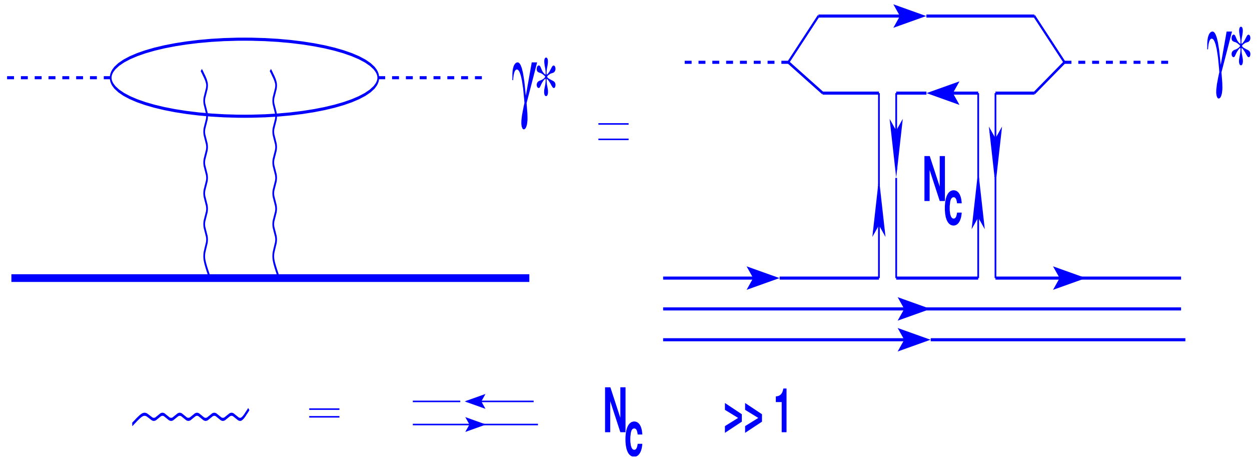
- $N_c \;\gg\; 1$ (Bartels (92)) but the regular procedure of taking into account $1/N_c^2$ corrections has been developed (Laenen & E.L (95), Balitsky (95), Iancu & McLerran (00)),

- **Two large N_c limits:**

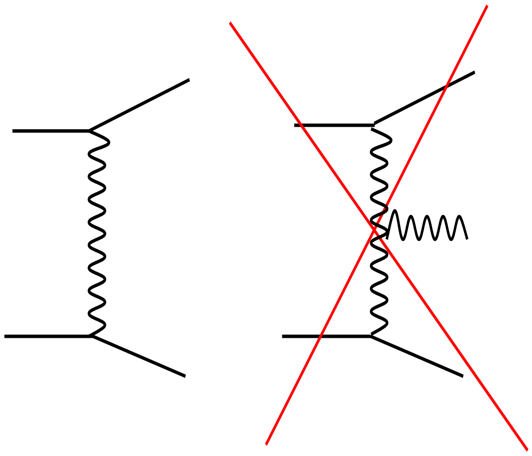
$$\alpha_S N_C \approx 1; \text{but } \alpha_S \ll 1$$

- $N_c \Longrightarrow \infty$ **but** $N_f/N_c \approx 1$ **(G. Veneziano (1975)) ;**
- $N_c \Longrightarrow \infty$ **but** $N_f/N_c \longrightarrow 0$ **(t'Hooft (1974)) ;**

The master equation leads to correct Veneziano large N_c limit.



Initial conditions:



$$\tilde{N}(x_{01}, y_0 = \ln(1/x_0); b_t) \;=$$

$$???$$

$$e^{-\frac{1}{\alpha_S}} \;\leq\; x_0 \;\leq\; \frac{1}{2mR}$$

- **For nucleon**
  - **Models;**
  - **But**

$$\frac{dF_2(Q^2,x)}{dlnQ^2} \propto \int d^2 b_t\, \tilde{N}$$

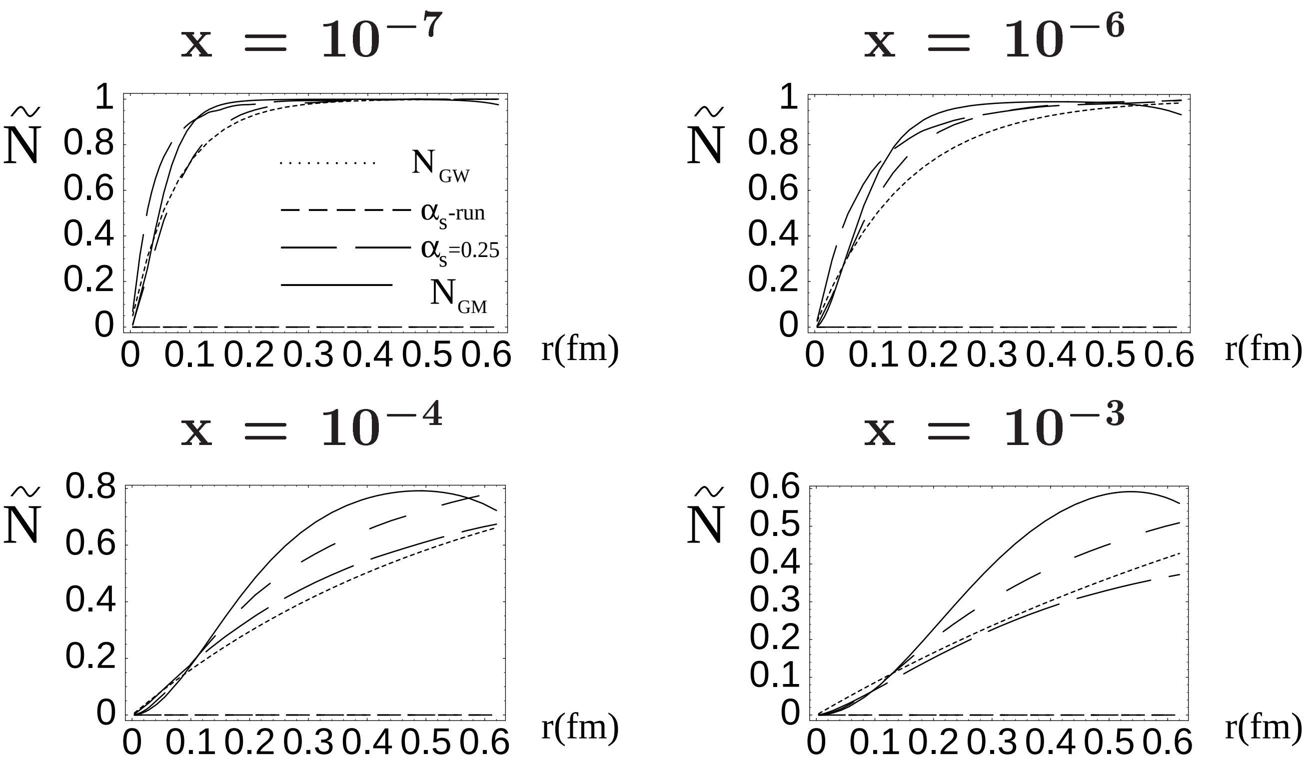
- **For nuclei**

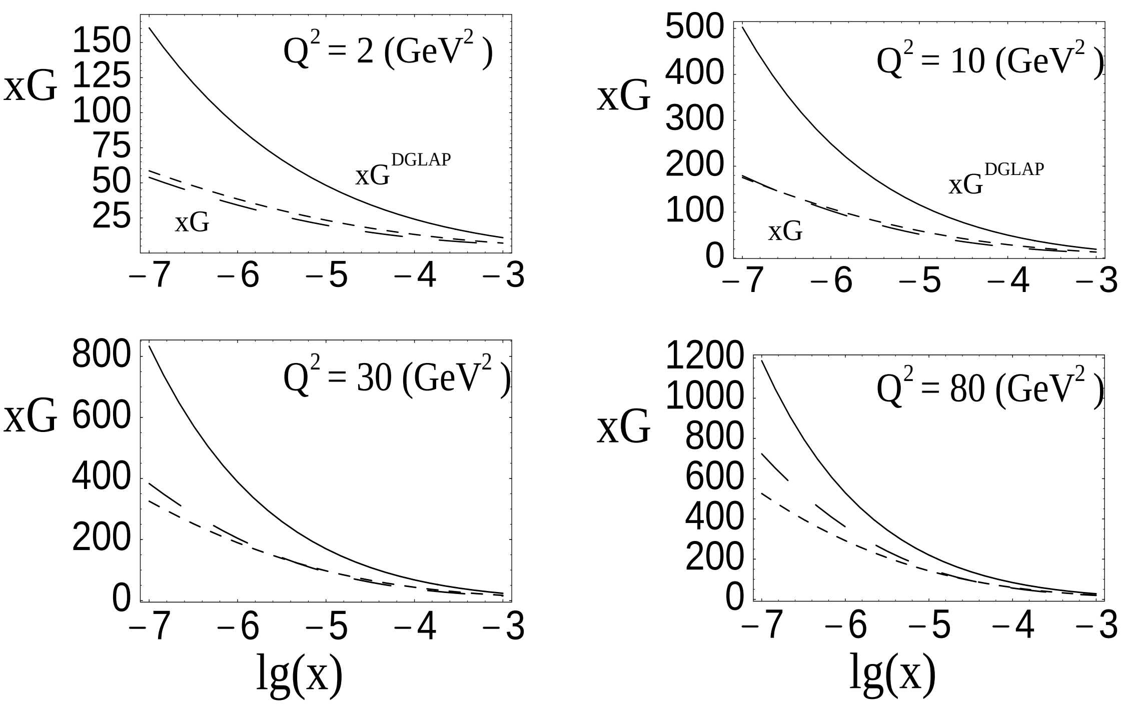
Glauber - Mueller formula:

$$N_{GM}(x_{01}, x; b) \;=$$
$$1 \;-\; \exp\left[-\frac{\alpha_S \pi A x_{01}^2}{2\, N_c\, R_A^2}\, xG^{DGLAP}(x, 4/x_{01}^2)\, S(b_t)\right]$$

- $x_0 = 10^{-2}$

$\tilde{N}$ for nucleon:





ΔN for nucleon:

- $\Delta N(x_{01}, x; b) \;=\; x_{01}^2 \cdot B(Q = 2/x_{01}, x; b_t)$
-

$$\frac{dB(Q,x;b_t)}{d(\ln Q^2)} = \frac{C_F\alpha_S}{\pi}\int_{x/x_0}^{1} P_{g\to g}(z)\; B(Q,\frac{x}{z};b_t)\,dz - \frac{2\,C_F\alpha_S}{\pi}\int_{x/x_0}^{1}\frac{dz}{z}\,\tilde{N}(2/Q,\frac{x}{z};b_t)B(Q,\frac{x}{z};b_t) + \frac{Q^2 C_F\alpha_S}{4\pi}\int_{x/x_0}^{1}\left(P_{g\to g}(z) - \frac{2}{z}\right)\tilde{N}(2/Q,\frac{x}{z};b_t)\,dz$$

- where

$$P_{g\to g}(z) = 2\left[\frac{1-z}{z} + \frac{z}{(1-z)_+} + z(1-z) + \left(\frac{11}{12} - \frac{n_f}{18}\right)\delta(1-z)\right]$$

- At any value of Q^2 but at $x \to 0$ $\tilde{N} \to 1$

- Therefore at low x we have

$$\frac{dB(Q,x)}{d(\ln Q^2)} = \frac{C_F \alpha_S}{\pi}$$

$$\int_{x/x_0}^{1} \left(P_{g \to g}(z) - \frac{2}{z} \right) \left(\frac{Q^2}{4} \tilde{N}(2/Q, \frac{x}{z}) + B(Q, \frac{x}{z}) \right) dz$$

- Since $\left(P_{g \to g}(z) - \frac{2}{z} \right) < 0$, ΔN is decreasing function of Q^2

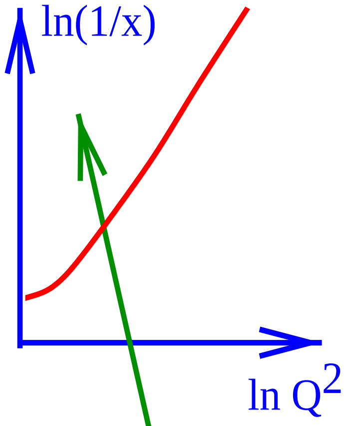
- Since $\left(P_{g \to g}(z) - \frac{2}{z} \right) \to 0$ $\Delta N \to 0$ at $x \to 0$

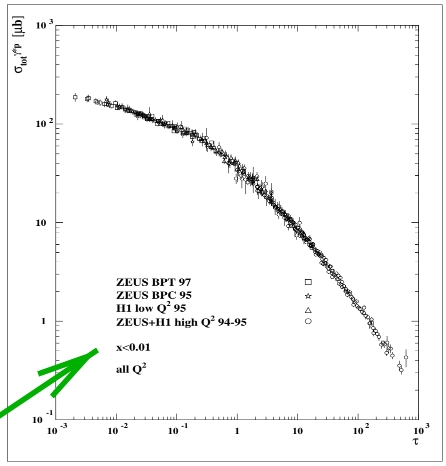
- Using $\gamma(\omega) = \frac{\alpha_S N_c}{\pi} \left(\frac{1}{\omega} - 1 \right)$, we can easily find that

-

$$\Delta N(r_\perp, x) \simeq -\frac{\bar{\alpha}_S}{1 + \bar{\alpha}_S} \tilde{N}(r_\perp, x) + C(x) \, (r_\perp^2)^{1 + \bar{\alpha}_S}$$

A new scaling





- Stasto, Golec-Biernat & Kwiecinski

$\sigma(\gamma^* p) = F(\tau = Q^2 / Q_s^2(x))$

- Bartles & E.L. (1992) ; McLerran & Venugopalan (94)
  E.L. & Tuchin (1999) ;McLerran & Kovchegov (98);

Simple arguments for a new scaling

-

$$\frac{dN(Q,y,b_t)}{dy} = \bar{\alpha}_S \left(\int K(Q,Q')N(Q',y;b_t) - N^2(Q,y;b_t) \right)$$

- **For moments** $N(\omega,Q) = N(\omega,Q_0)\,e^{\gamma(\omega)\,\ln Q^2}$ **we have**

$$\omega e^{\gamma(\omega) lnQ^2} = \chi(\gamma) e^{\gamma(\omega) lnQ^2} - N(\omega,Q_0) \int d\omega' e^{(\gamma(\omega') + \gamma(\omega-\omega') lnQ^2}$$

- **Taking integral over ω' by Saddle Point Method we obtain**

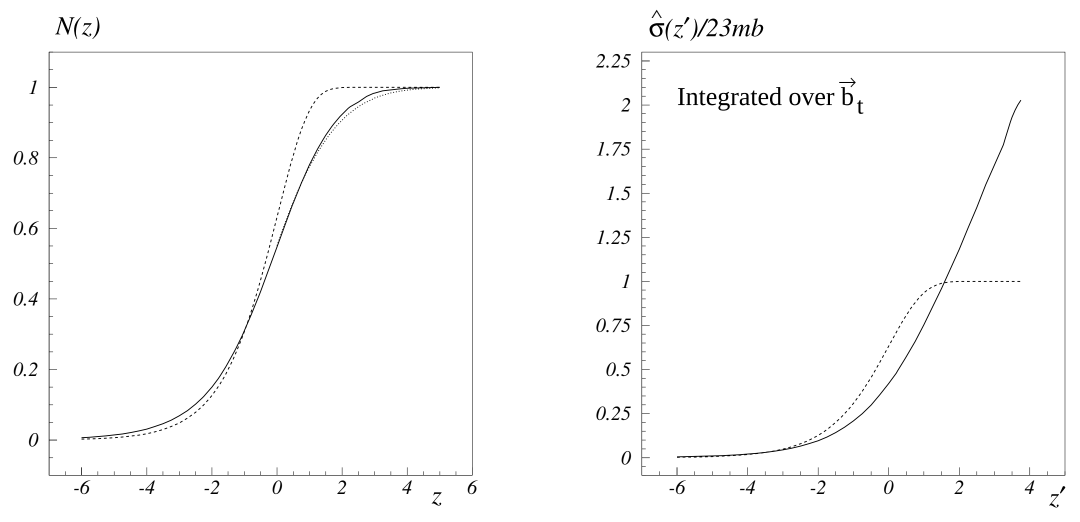
$$\int d\omega' e^{(\gamma(\omega')+\gamma(\omega-\omega')ln Q^2} \Longrightarrow e^{2\gamma(\omega/2) ln Q^2}$$

- **Therefore $\gamma(\omega) = 2\gamma(\omega/2) \rightarrow \gamma(\omega) = C\omega$**
- **Calculating $N(y, Q^2)$ we obtain**

$$N(y, Q^2) = \int d\omega\, e^{-\omega y + \gamma(\omega)\ln Q^2} = \int d\omega e^{\omega(-y + C\ln Q^2)}$$

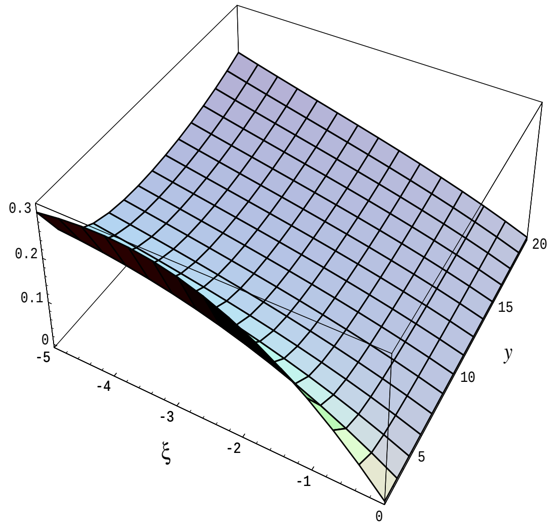
- $-y + C\ln Q^2 = \ln(Q_s^2(x)/Q^2) = \xi$

Scaling solution



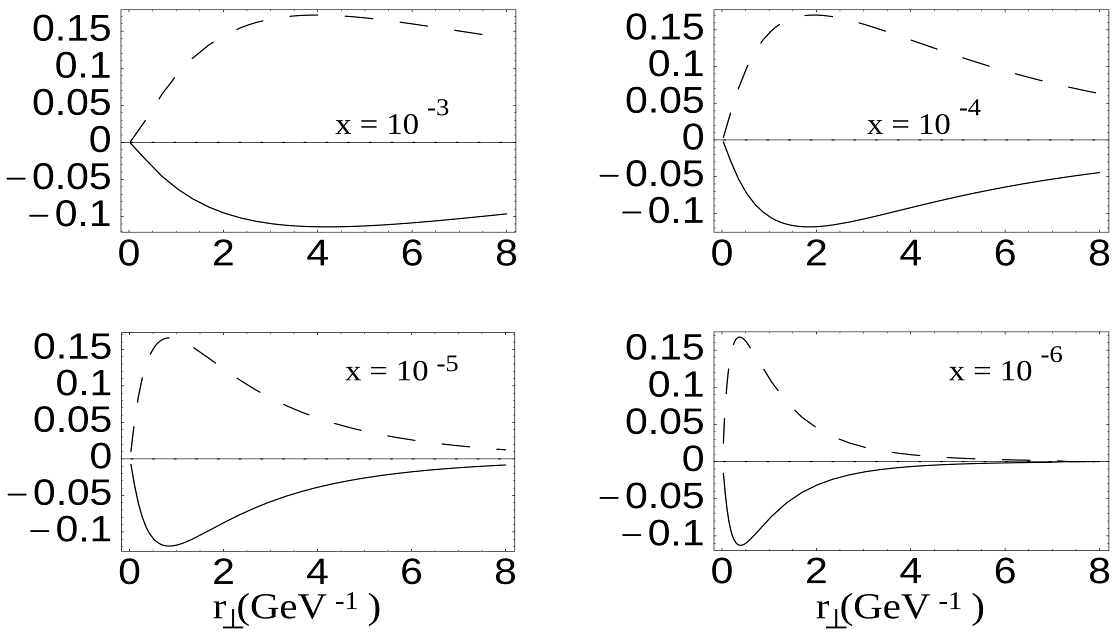
$$z = \ln(x_\perp^2\, Q_s^2(x))$$

Corrections to scaling solution



$$\delta \tilde{N}(y,\xi)/\tilde{N}(\xi)$$

Where is scaling violation ?



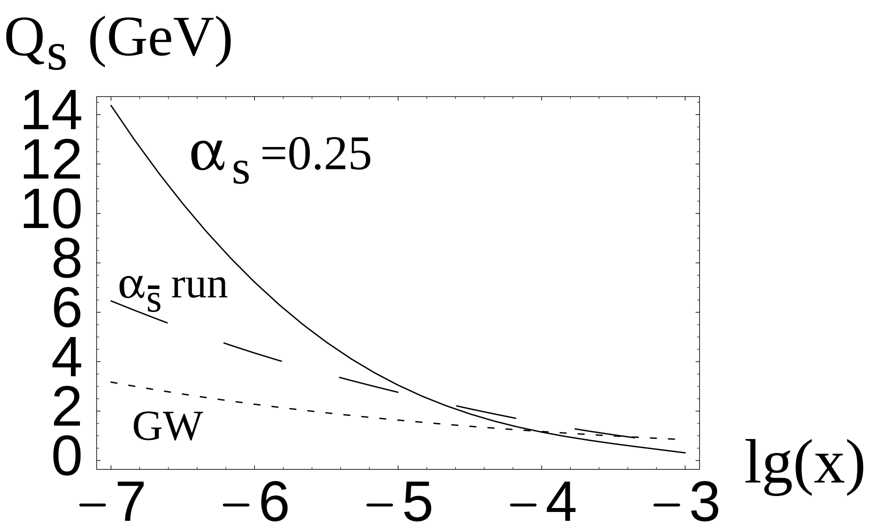
$$\frac{\partial N}{\partial \ln r_{\perp}^{2}} \Big/ \frac{\partial N}{\partial \ln(1/x)} = \frac{d \ln Q_{s}^{2}(x)}{d\ \ln(1/x)}$$

Saturation scale $Q_s^2(y)$ (realistic estimates)

- **Definition of $Q_s^2(y)$:**

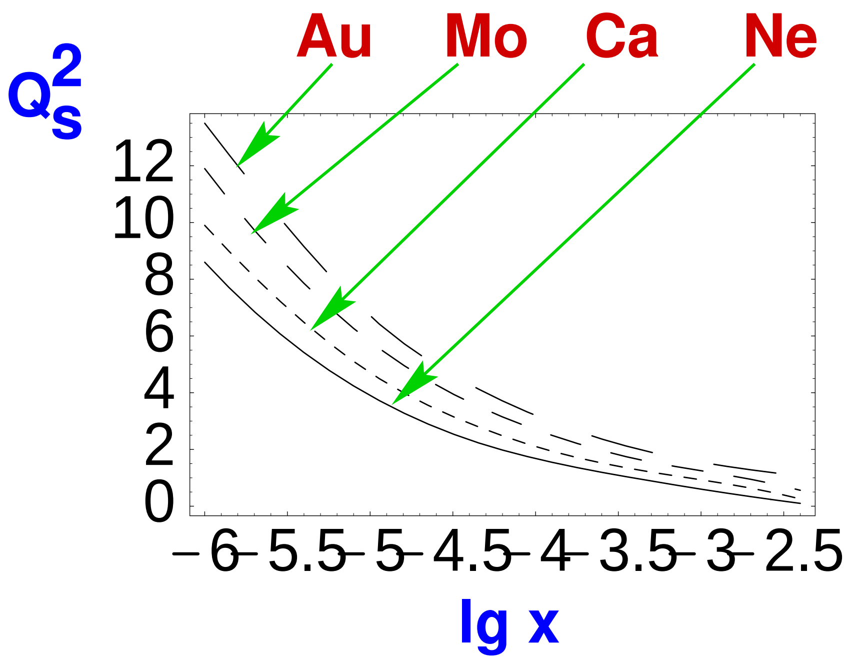
$$\boxed{N\left(2/Q_s(x), x; b=0\right) \;=\; 1/2}$$

- **For proton:**

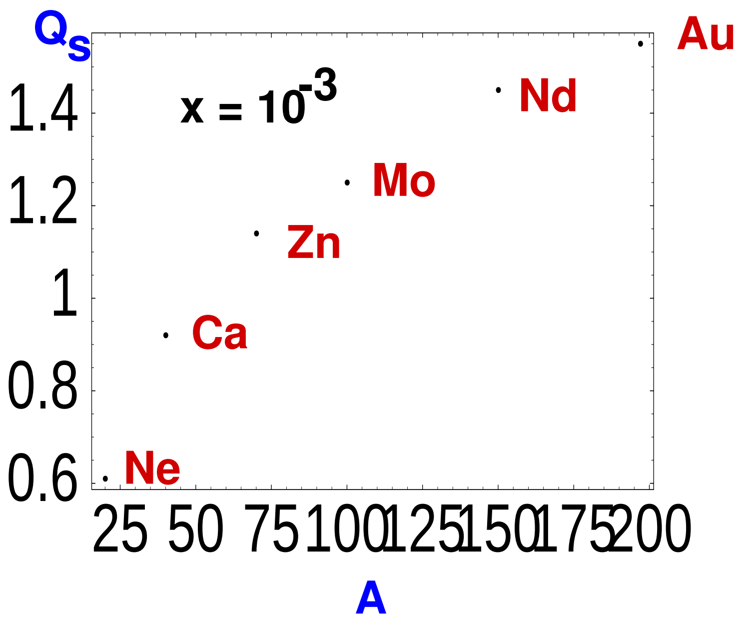


(a)

- **For nuclei:**



Au
Mo
Ca
Ne
Q_s^2
12
10
8
6
4
2
0
−6 −5.5 −5 −4.5 −4 −3.5 −3 −2.5
lg x

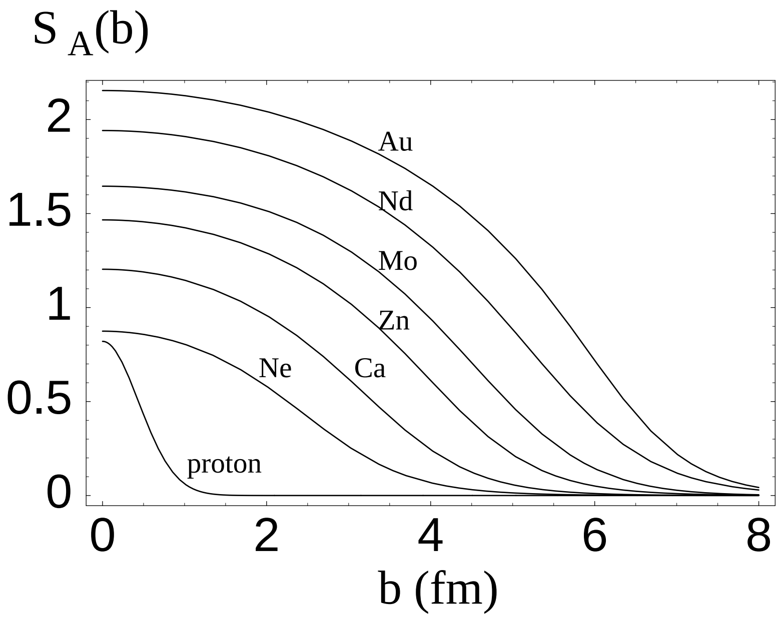


Q_s
$x = 10^{-3}$
Au
Nd
Mo
Zn
Ca
Ne
1.4
1.2
1
0.8
0.6
25 50 75 100 125 150 175 200
A

$$Q_s^2(A;x) \;\neq\; A^{\frac{1}{3}}\,Q_s^2(p;x)$$

but

$$Q_s^2(A;x) \;=\; (A/A_{Ne})^{\frac{1}{3}}\;Q_s^2(p;x)$$



Next steps:

- $\frac{1}{N_c^2}$ - corrections;
- ΔN in the LO DGLAP;
- Diffraction structure function as the solution to the non-linear equation (Kovchegov & Levin (1999));
- The relation between $k_\perp$ and $r_\perp$ in the saturation region;
- The matching between “soft” and “hard” processes in the saturation region;