Nonlinear evolution and saturation

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- MODELS FOR HIGH-ENERGY INCLUSIVE PROCESSES AND SCREENING IN DEUTERIUM. J.Kwieciński, L.Leśniak, K.Zalewski, 1974.
- IMPLICATIONS OF THE GLAUBER MODEL FOR INELASTIC HADRON - NUCLEUS SCATTERING. T.Jaroszewicz, J.Kwieciński, L.Leśniak, K.Zalewski, 1979.
- MINI - JETS, QCD AND UNITARITY, A.Capella, J. Tran Thanh Van, J.Kwieciński, 1987.
- ESTIMATE OF THE SCREENING CORRECTIONS TO GLUON DISTRIBUTIONS IN THE SMALL X REGION. J.Kwieciński, 1985.
- UNIFIED DESCRIPTION OF NUCLEAR SHADOWING OF VIRTUAL PHOTONS. J.Kwieciński, B.Badełek, 1988.
- SHADOWING EFFECTS IN NUCLEAR PARTON DISTRIBUTIONS FOR SMALL VALUES OF X. J.Kwieciński, 1990.
- SHADOWING IN GLUON DISTRIBUTIONS IN THE SMALL X REGION. J.C.Collins, J.Kwieciński, 1990.

- PARTON DISTRIBUTIONS AT SMALL X. J.Kwieciński, A.D.Martin, W.J.Stirling, R.G.Roberts, 1990.
- SEMIHARD QCD EXPECTATIONS FOR PROTON - (ANTI)-PROTON SCATTERING AT CERN, TEVATRON AND SSC COLLIDERS. J.Kwieciński, A.D.Martin, 1991.
- THE SMALL X BEHAVIOR OF THE GLUON. J.Kwieciński, A.D.Martin, P.J.Sutton, 1991.
- SHADOWING EFFECTS IN PARTON DISTRIBUTIONS FOR SMALL VALUES OF X. J.Kwieciński, 1991.
- QCD PREDICTIONS FOR THE BEHAVIOR OF THE GLUON DISTRIBUTION AT SMALL X. J.Kwieciński, A.D.Martin, P.J.Sutton, 1991.
- SHADOWING IN INELASTIC LEPTON - DEUTERON SCATTERING. B.Badełek, J.Kwieciński, 1992.
- QCD PREDICTIONS FOR DEEP INELASTIC STRUCTURE FUNCTIONS AT HERA. A.J.Askew, J.Kwieciński, A.D.Martin, P.J.Sutton, 1993.

- SHADOWING IN THE DEUTERON AND THE NEW $F2(N)/F2(P)$ MEASUREMENTS. B.Badełek, J.Kwieciński, 1994.
- GEOMETRIC SCALING FOR THE TOTAL $\gamma^* p$ CROSS-SECTION IN THE LOW X REGION. A.M. Staśto, K. Golec-Biernat, J. Kwieciński, 2001.
- GLUON SHADOWING IN THE LOW X REGION PROBED BY THE LHC. M.A.Kimber, J.Kwieciński, A.D.Martin, 2001.
- SATURATION MODEL FOR TWO PHOTON INTERACTIONS AT HIGH-ENERGIES. N.Timneanu, J.Kwieciński, L.Motyka, 2002.
- GEOMETRIC SCALING AND QCD EVOLUTION. J.Kwieciński, A.M.Staśto, 2002.
- SCREENING EFFECTS IN THE ULTRAHIGH-ENERGY NEUTRINO INTERACTIONS. K.Kutak, J.Kwieciński, 2003.

Introduction

We are interested in high-energy behaviour of the hadronic cross sections. Froissart - Martin bound gives:

$$\sigma(s) \leq \frac{\pi}{\mu^2} \ln^2 \frac{s}{s_0}$$

where $\mu^2 \sim m_\pi^2$ is a hadronic scale.
Unitarity is a property of short+long distance QCD.
From pQCD in the limit $s \gg t > \Lambda^2$ we know

$$\sigma(s) \sim s^\lambda$$

where $\lambda = \bar{\alpha}_s \chi$ being the LL(NLL) BFKL eigenvalue.

Perturbative saturation

Perturbative saturation takes into account gluon recombination and reduces the growth of the cross section. For example GLR equation in DLLA approximation: $\ln Q^2/\Lambda^2 \ln 1/x \gg 1$

$$\frac{\partial^2 xG(x,Q^2)}{\partial \ln \frac{1}{x} \partial \ln \frac{Q^2}{\Lambda^2}} = \bar{\alpha}_s xG - \frac{4\alpha_s^2 N_c}{3C_f R^2 Q^2}(xG)^2$$

Gluon production and recombination. Fixed order expansion

+ density expansion.

Approaches

Theory

- McLerran, Venugopalan, Jalilian-Marian, Kovner, Leonidov, Weigert, Iancu: Color Glass Condensate
- Mueller
- Balitsky, Kovchegov: nonlinear evolution equation $\rightarrow$
- Bartels, Wüsthoff: $2 \rightarrow 4$ gluon transition vertex

Models

- Golec-Biernat, Wüsthoff
- Levin *et al*
- Kowalski, Teaney
- . . .

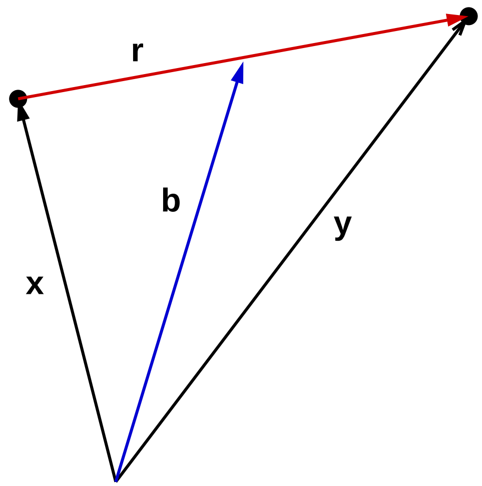
Balitsky-Kovchegov equation

$$\frac{\partial N_Y(\mathbf{x},\mathbf{y})}{\partial Y} = \bar{\alpha}_s \int \frac{d^2\mathbf{z}}{2\pi} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2} [N_Y(\mathbf{x},\mathbf{z}) + N_Y(\mathbf{y},\mathbf{z}) - N_Y(\mathbf{x},\mathbf{y}) - N_Y(\mathbf{x},\mathbf{z})N_Y(\mathbf{y},\mathbf{z})]$$

$N_Y(\mathbf{x},\mathbf{y})$ forward amplitude for scattering of the $q\bar{q}$ dipole on a nucleus target. Linear + rescattering term.

$$Y = \ln(\tfrac{1}{x}) \qquad \mathbf{r} = \mathbf{x} - \mathbf{y}$$

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi} \qquad \mathbf{b} = \frac{\mathbf{x}+\mathbf{y}}{2}$$



Symmetries

The kernel:

$$d^2\mathbf{z}\frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{y}-\mathbf{z})^2}$$

has Möbius symmetry

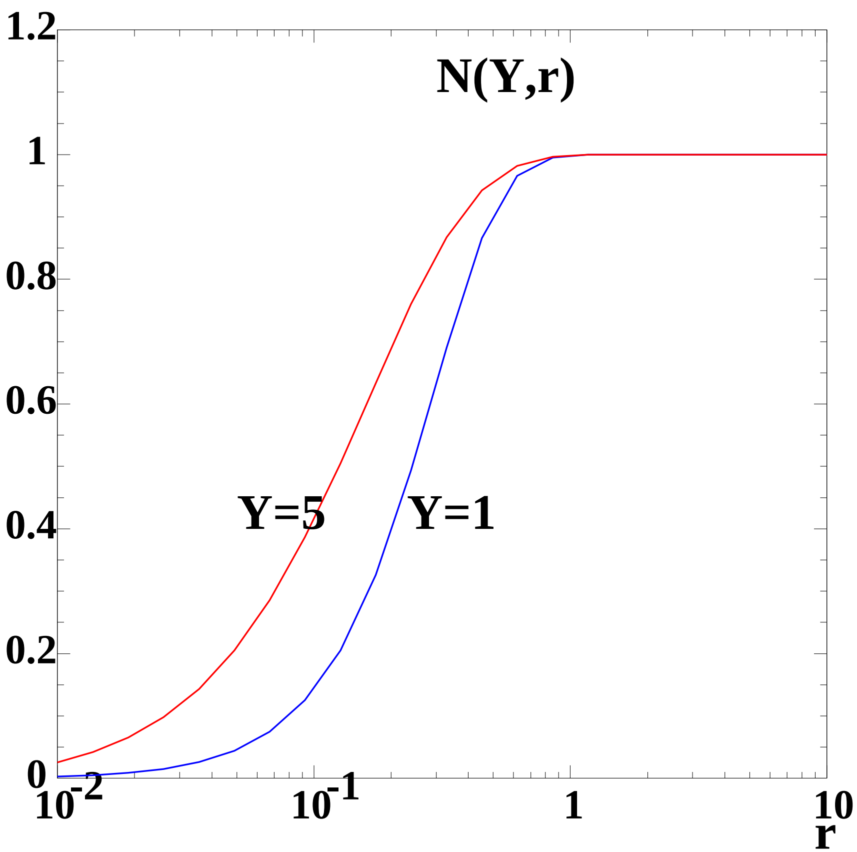
$$\mathbf{x}=(x_1,x_2) \qquad x=x_1+ix_2 \qquad x\rightarrow\frac{ax+b}{cx+d}$$

where $ad-bc\neq 0$.
translations, rotations, rescaling, inversions
Well studied are solutions with translational invariance:

$$\mathbf{x},\mathbf{y},\mathbf{z}\longrightarrow\mathbf{x}+\mathbf{c},\mathbf{y}+\mathbf{c},\mathbf{z}+\mathbf{c} \qquad N_Y(\mathbf{x},\mathbf{y})=N_Y(|\mathbf{x}-\mathbf{y}|)=N_Y(r)$$

Approximation of infinitely large nucleus.

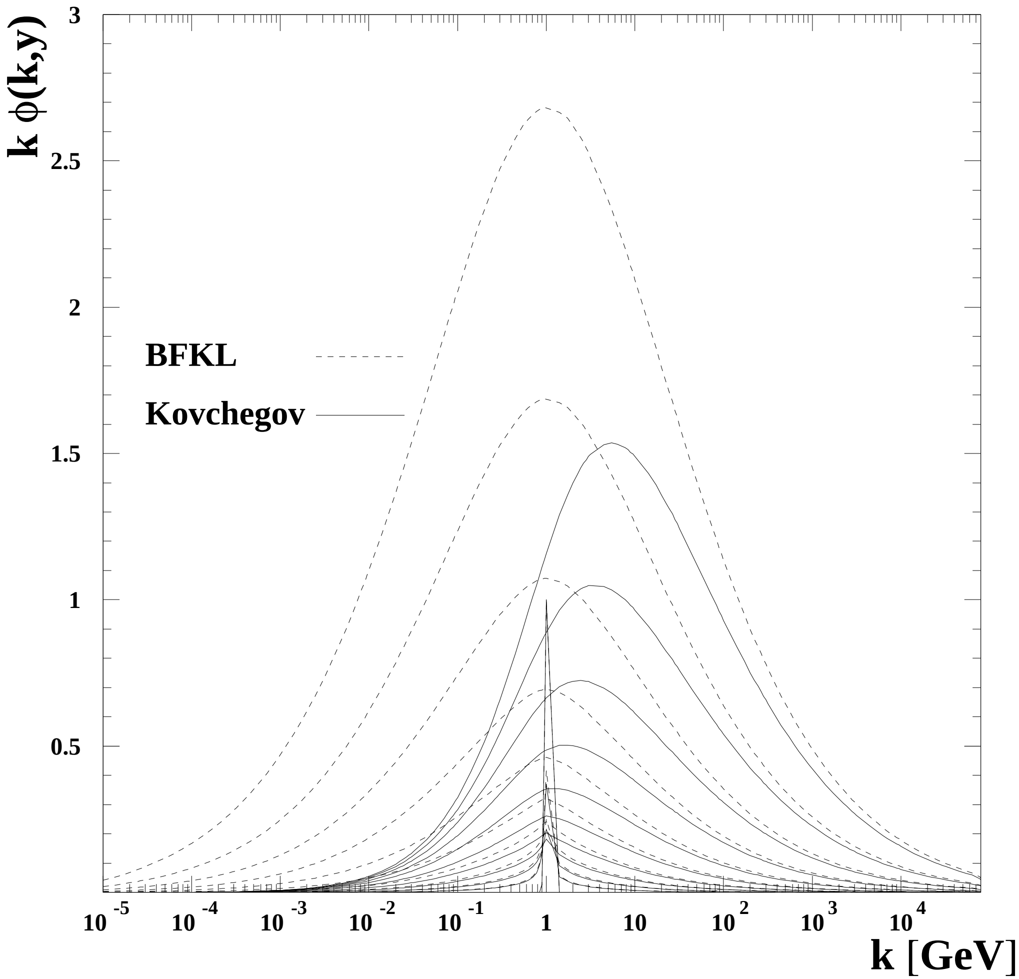
Solution without b dependence



- Nonlinearity tames the growth and the amplitude is always $N < 1$.
- Generation of rapidity dependent saturation scale
  $Q_s(Y) = Q_0 e^{\lambda \bar{\alpha}_s Y}$
  $\lambda \simeq 2$
- In the saturated regime the amplitude depends only on one variable - scaling:
  $N_Y(r) = N(rQ_s(Y))$

Solution without b dependence

K.Golec-Biernat, L.Motyka,A.S.



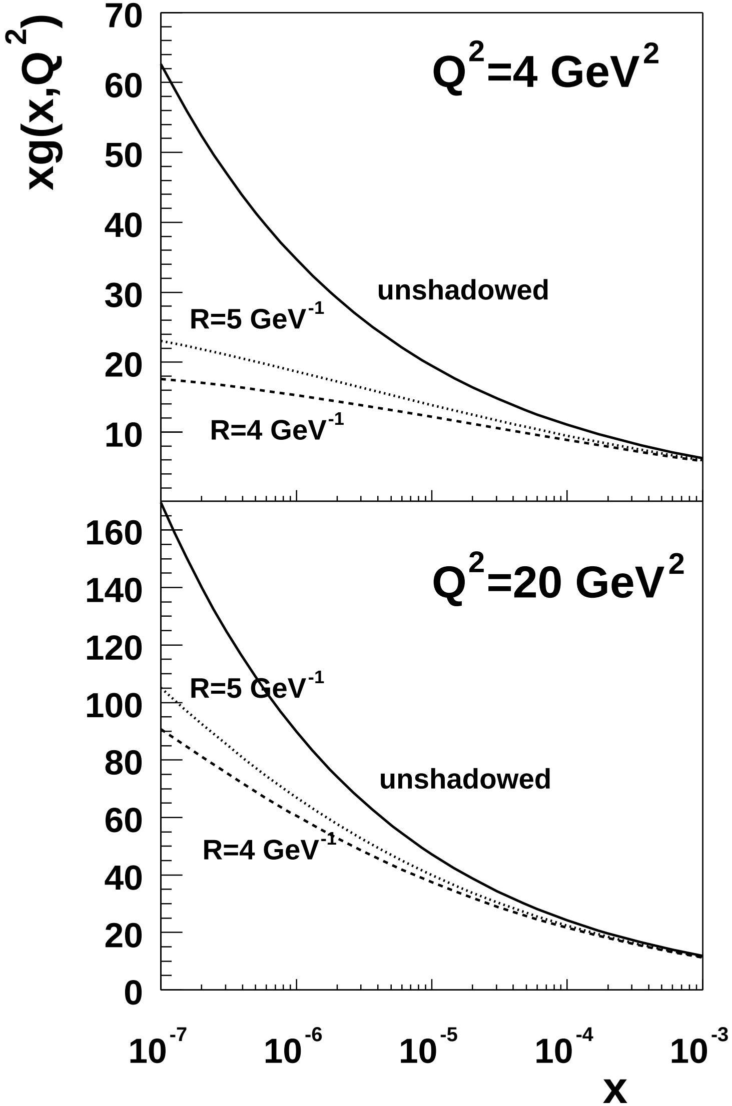
Momentum space.

$\phi(k,Y) =$
$\int_0^\infty \frac{dr}{r} J_0(kr) N_Y(r)$

Rapidities from $Y = 1$ to $Y = 10$.

The maximum of the $k\phi$ distribution moves with the saturation scale $Q_s(Y)$.

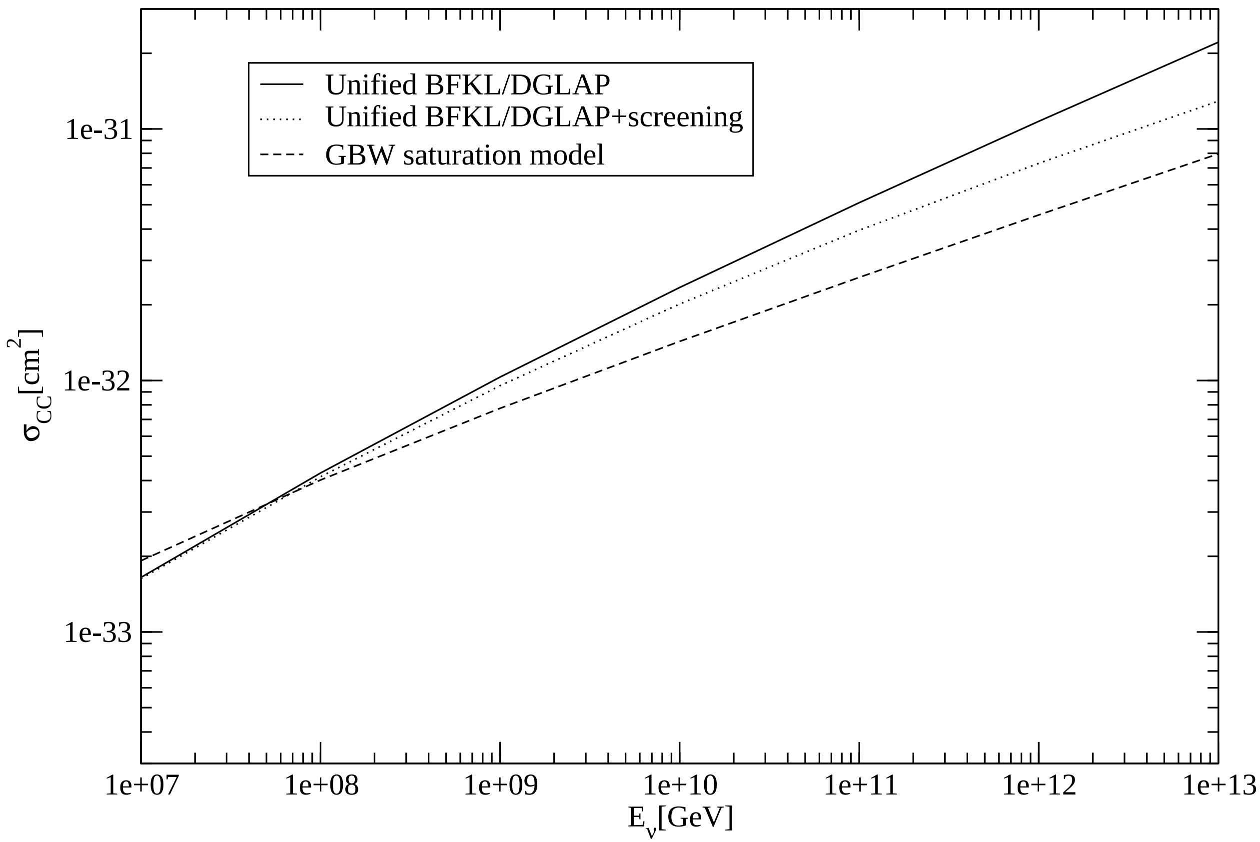
Predictions for LHC



J. Kwieciński, A.D. Martin, M. Kimber

Predictions for LHC energies. Gluon density obtained from (modified) BK equation.

Predictions for UHE ν's



K.Kutak, J.Kwieciński

$\sigma^{CC}(E_\nu)$ for the DIS νN interaction as a function of E_ν at high energies. Average $Q^2 \sim M_W^2 \rightarrow$ rather small saturation.

Impact parameter dependence from BK

One would like to know the b dependence of $N_Y(r,b)$.
Amount of saturation depends on b:

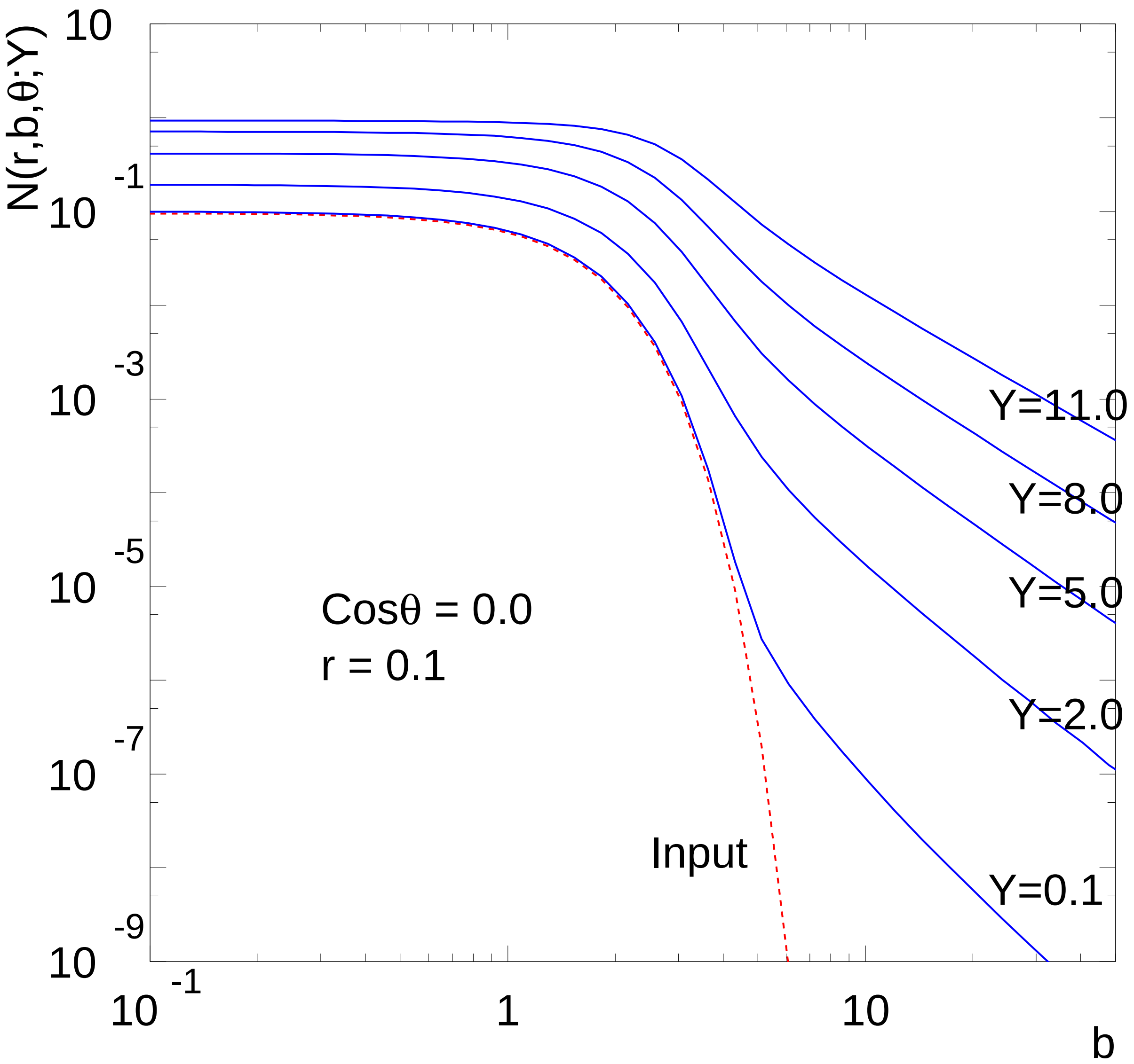
$$\sigma(s) = 2Re \int d^2\mathbf{b}(1 - S(s,\mathbf{b}))$$

where S is scattering matrix.
Nontrivial problem in BK equation even numerically $\longrightarrow$ 5 variables.
Cyllindrical symmetry of the target $\longrightarrow$ 4 variables.
Take the initial condition of the Glauber-Mueller form:

$$N_0 = 1 - e^{-c_r r^2 F(b)}$$

where $F(b) = \exp(-c_b b^2)$

$N_Y(r,b)$ vs impact parameter b



K. Golec-Biernat, A.S.

Input Glauber-Mueller:

$N_0 = 1 - e^{-c_r r^2 e^{-c_b b^2}}$

Evolution in Y: saturation at small b, power tails at large b.

Origin of power tails in b

Input Glauber-Mueller: $N_0 = 1 - e^{-c_r r^2 e^{-c_b b^2}}$

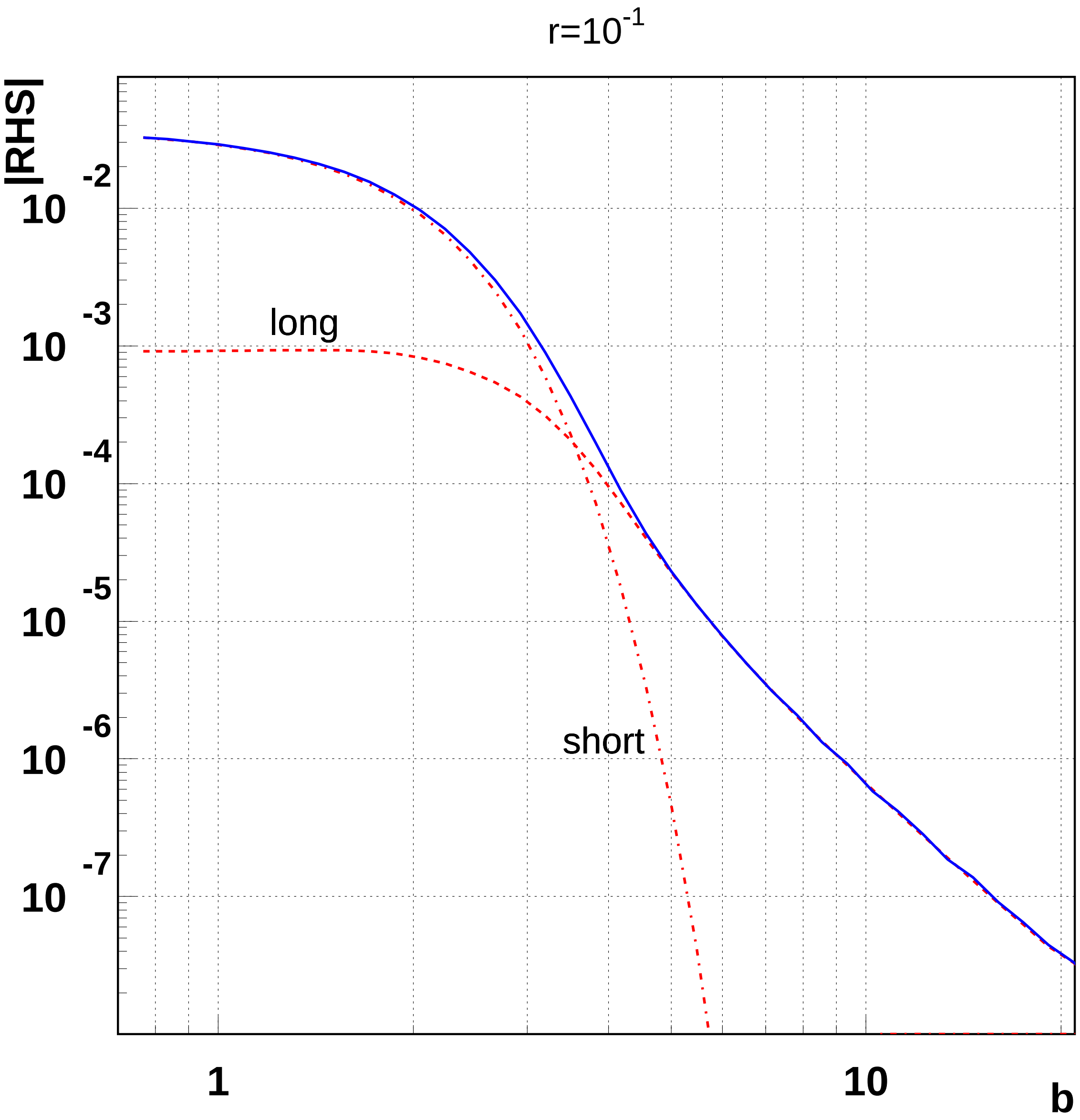
$$\left[\overbrace{\int \Theta(r_0 - |\mathbf{z} - \mathbf{b}|)}^{short} + \overbrace{\int \Theta(|\mathbf{z} - \mathbf{b}| - r_0)}^{long} \right] \frac{d^2\mathbf{z}(\mathbf{x} - \mathbf{y})^2}{(\mathbf{x} - \mathbf{z})^2(\mathbf{y} - \mathbf{z})^2}$$

$$\cdot \left(N^0_{xz} + N^0_{yz} - N^0_{xy} - N^0_{xz} N^0_{yz}\right)$$

- short $\rightarrow$ exponential behaviour, factorisation of initial profile at small b
- long $\rightarrow$ power behaviour, $\sim 1/b^4$ at large b

Emergence of power tails in b leads to the exponential increase of the cross section and violation of unitarity despite the saturation.

Short vs long distance contributions



Saturation without unitarisation

Kovner, Wiedemann

Perturbative saturation does not lead to unitarisation.

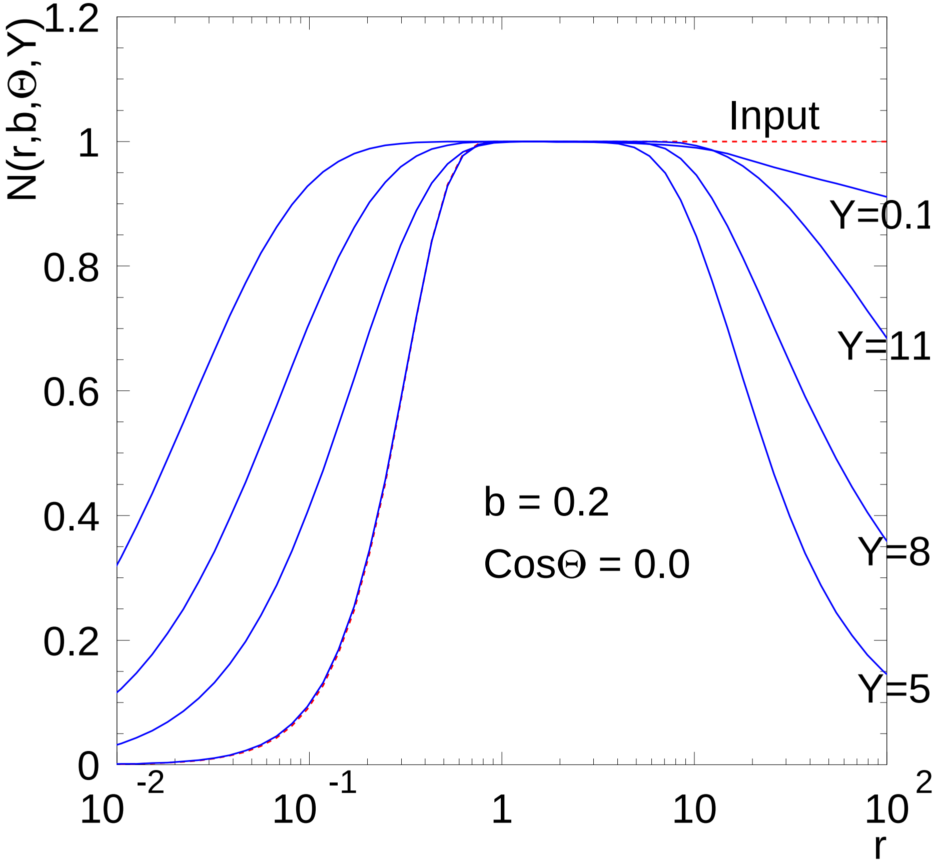
Argument by Heisenberg

$$\rho(b) \sim e^{-mb} \longrightarrow \sigma(s) \sim \ln^2 s$$

$$\rho(b) \sim \frac{1}{b^\gamma} \longrightarrow \sigma(s) \sim s^\lambda$$

It is not enough to start with the exponential profile in the initial condition. Need to limit the range of the forces by introducing the nonperturbative scale directly in the evolution $\rightarrow$ break the conformal invariance.

Dependence on the dipole size r



Input Glauber-Mueller: $N_0 = 1 - e^{-c_r r^2 e^{-c_b b^2}}$

Fixed impact parameter $b = 0.2$.

Amplitude saturates in the limited range:

$1/Q_s(Y,b) < r < R_H(b,Y)$

At large r: the big dipole misses the target.

Conclusions and outlook

- Impact parameter profile from BK equation.
- Emergence of power-like tails $\sim 1/b^{\gamma}$.
- Violation of unitarity despite the saturation.
- Decreasing amplitude for large dipole sizes r.

- Modification of BK equation $\rightarrow$ scale in the evolution.
- Phenomenology: t dependence.
- Running of the coupling etc.
- $\dots$