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Introduction to Low x Physics and Saturation

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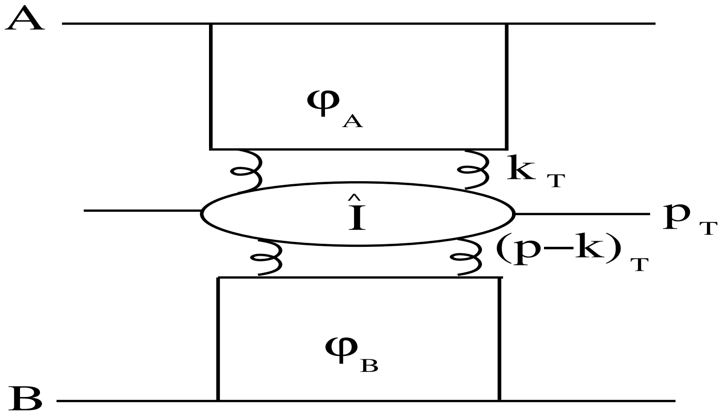
1. Introduction: small x and saturation



• To compute inclusive particle production: Different evolution equations have been designed for different situations (factorization schemes) (Andersson '02):

A) $\Lambda^2_{\rm QCD} \ll p_T^2 \sim s \Longrightarrow$ collinear factorization, DGLAP evolution equations in $\ln Q^2$.

B) $\Lambda^2_{\rm QCD} \ll p_T^2 \ll s \Longrightarrow k_T$-factorization, BFKL evolution equation in $\ln{(1/x)}$.

• Relation between gluon distributions in both schemes (Collins '03, Hoyer '03):

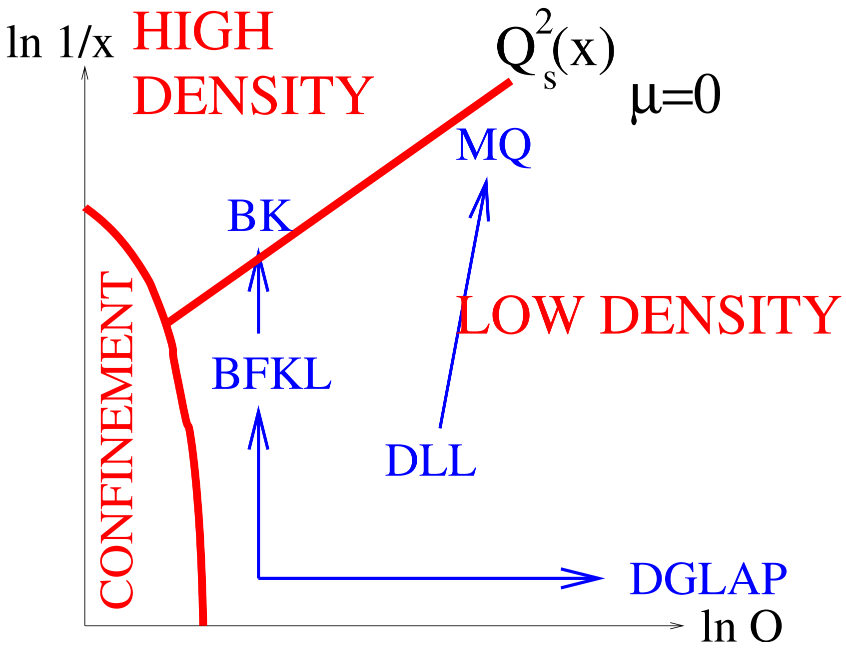
$$xG(x,Q^2) = \int_{Q_0^2}^{Q^2} dk_T^2 \, \varphi(x,k_T^2), \quad \Lambda_{\rm QCD}^2, Q_0^2, Q_s^2, \ldots \ll Q^2. \qquad (1)$$

• But $xG(x,Q^2) \propto x^{-\lambda}, \quad \lambda \sim 0.3$, so at small x high partonic densities are expected: parton interaction or, alternatively, multiple scattering (unitarity) must play a role.

• Saturation: the number of partons (the field associated to them) gets bounded; the S-matrix becomes 0 (the cross section, geometrical): important at high energies or small x.

• Appeal: old ideas (pomeron interaction, multiple scattering in Reggeon Field Theory) (e.g. Boreskov '01, Armesto '03) in a new (QCD) language; link of small x and heavy ion physics (initial stage of a HIC).

• Basic aim: to study the formulation and consequences of unitarity in terms of the QCD fields.



• Not discussed (an incomplete list):

A) NLL developments (Altarelli '03, Andersen '03, ...) (parametrically less important than unitarity corrections: $y_{ms} \sim \ln(1/\alpha_s)/\alpha_s$, $y_{rc} \sim 1/\alpha_s^{5/3}$; also the sensitivity to the IR region is expected to be diminished by the non-linear evolution); nor running coupling in BK (Braun '03).

B) Non-linear equations in the collinear approach (Eskola '03) or the problems around the determination of the collinear gluon.

C) The relation of saturation with other realizations of collectivity such as percolation (Pajares here, Eskola '99) or Regge-Gribov ideas (Capella '99).

D) Drell-Yan and heavy flavor production or applications in $\gamma - \gamma$ or ultraperipheral nuclear collisions, in saturation models (Armesto '01, Betemps '03, Gonçalves '03); also non-forward extensions (Bartels '03) or diffraction (Golec-Biernat here).

E) Proposals to study some observables in pA, as γ production (Gelis '03).

2. Present realizations of saturation

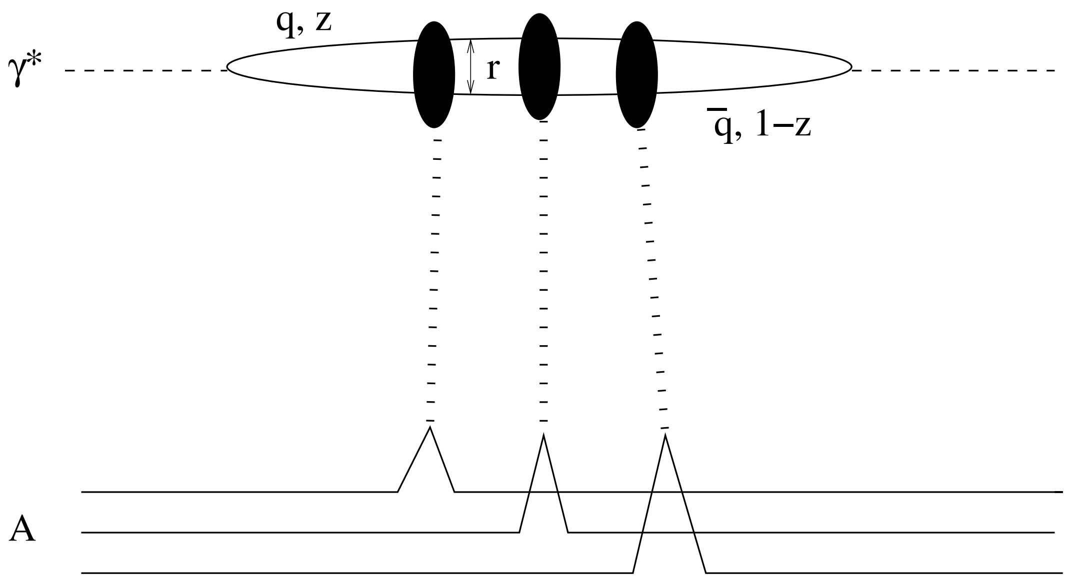
Phenomenological models

• The most simple and used model is GBW (Golec-Biernat '99):

$$N(r) = 1 - e^{-Q_s^2 r^2/4}, 2\int d^2b \equiv \sigma_0 \sim 20 \ \ \mathrm{mb}, Q_s^2 = \left(\frac{x_0 \sim 3 \cdot 10^{-4}}{x}\right)^{\lambda \sim 0.3}, \tag{2}$$

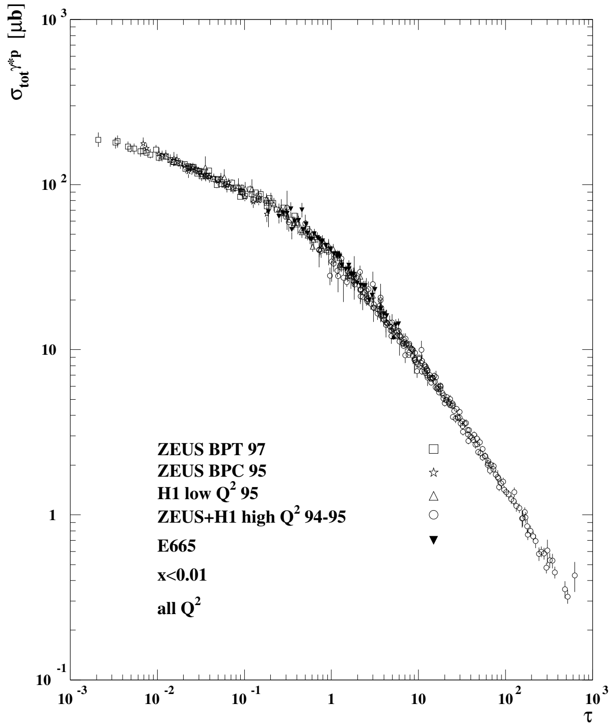
in the dipole model ($2\int d^2b\ N \equiv \sigma(r)$)

$$\sigma_{T,L}^{\gamma^* A}(x,b) = \int_0^1 dz \int d^2r |\Psi_{T,L}(z,r,Q^2)|^2 \sigma_{dA}(x,r,b). \tag{3}$$



• This model leads to Bjorken scaling (DGLAP evolution included (Bartels '02)) and has no impact parameter dependence (included (Kowalski '03)).

• As $N(r) = Re(1 - S)$, the saturation scale Q_s^2 indicates the approach to the unitarity limit.



• It gives a good description of inclusive and diffractive DIS nucleon data for $x < 10^{-2}$, $Q^2 < 450$ GeV^2. It implies a $[\tau = Q^2/Q_s^2(x)]$-scaling (Staśto '01), which has been searched in nuclear data (Freund '01) but the naive expectation $Q_{sA}^2 \propto AT_A(b) \propto A^{1/3}$ (2/3 in DLL (Levin '01)) is not fulfilled at available x, Q^2 (Armesto '02, Kowalski '03).

Theoretical developments

• The MV model (McLerran '94) treats classical radiation from color sources moving ultrarelativistically through a large nucleus. With a form for the color correlators in the target and taking into account the non-abelian gluon interaction, $dxG/d^2bd^2l \propto \int d^2r\ e^{-il\cdot r}N(r)/r^2$.

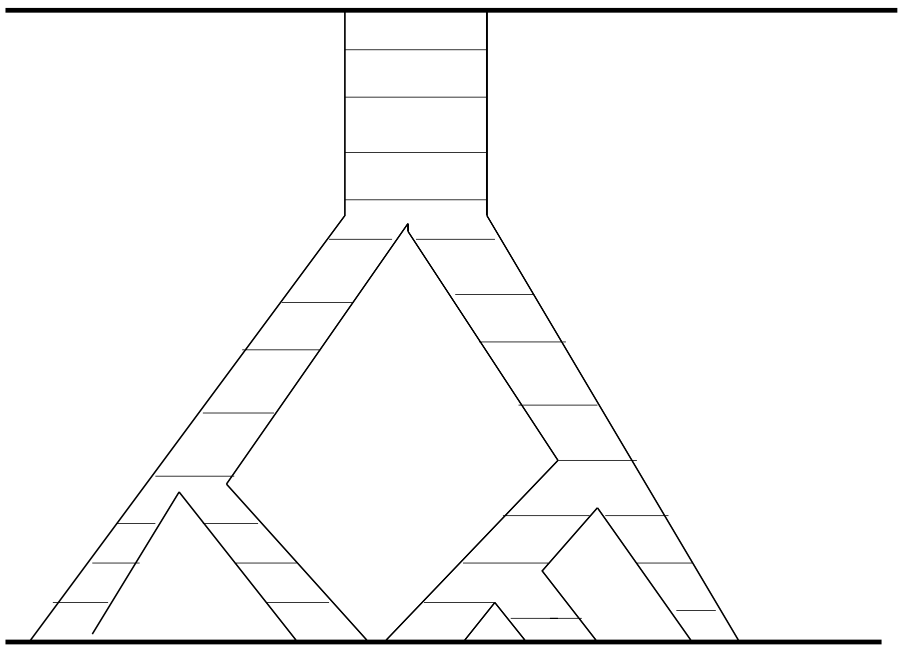
• Gluon radiation (quantum evolution) of the color sources leads to an evolution equation: CGC (Jalilian-Marian '97, Iancu '01). The rescattering of the projectile in the nucleus is described through Wilson lines whose average on target configurations gives the S-matrix (Balitski '96, Kovner '01).

• Point: fields (occupation numbers) become large ($F_{\mu\nu}, f_g \propto 1/g^2$) $\Rightarrow$ classical arguments, but coupling small $\Rightarrow$ perturbative methods.

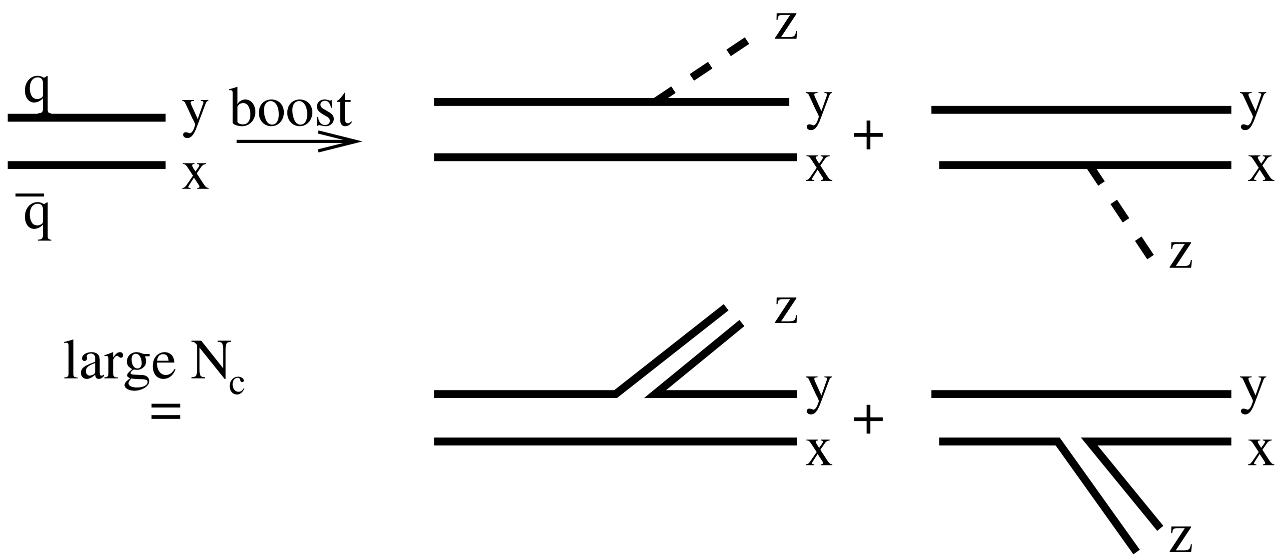
• The coupled equations for n-gluon correlators decouple in the large N_c limit: the Balitski-Kovchegov equation appears for $n = 2$ (Kovchegov '99); also deduced in BFKL (Braun '00) as a sum of fan diagrams in LL1/x.

• BK: one of the very few quantitative tool we have to study the non-linear regime at small x: all-twist equation.

• As in BFKL, the coupling constant is fixed.



$$\frac{\partial N(x,y)}{\partial \ln y} = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 z}{2\pi} \frac{(x-y)^2}{(x-z)^2 (y-z)^2} \tag{4}$$
$$\times [N(x,z) + N(y,z) - N(x,y) - N(x,z)N(z,y)].$$



• Analytical solution unknown: numerical studies. Estimations on the behavior of the solution using BFKL, DLL or modifications (Iancu '02, Mueller '02, '03). E.g. $Q^2_{sA} \propto A^{1/3}$ in fixed coupling but $Q^2_{sA} \propto \ln A$ in running.

3. Balitski-Kovchegov equation

Numerical solutions

• With $b = (x+y)/2 \gg r = x - y$ (so b is a parameter in the initial condition),

$$\left(\frac{\partial}{\partial y} + H_{\mathrm{BFKL}}\right)\phi(y,k,b) = -\phi^2(y,k,b), \tag{5}$$

$$y = \frac{\alpha_s N_c}{\pi}\ \ln\left(x_0/x\right), \quad \varphi = \frac{N_c}{2\pi^2\alpha_s}\ k^2\nabla_k^2\phi = \frac{N_c}{2\pi^2\alpha_s}\ h, \tag{6}$$

$$\phi_A(x,k,b) = \int \frac{d^2r}{2\pi r^2}\ \exp\left(ik\cdot r\right)N_{dA}(x,r,b), \tag{7}$$

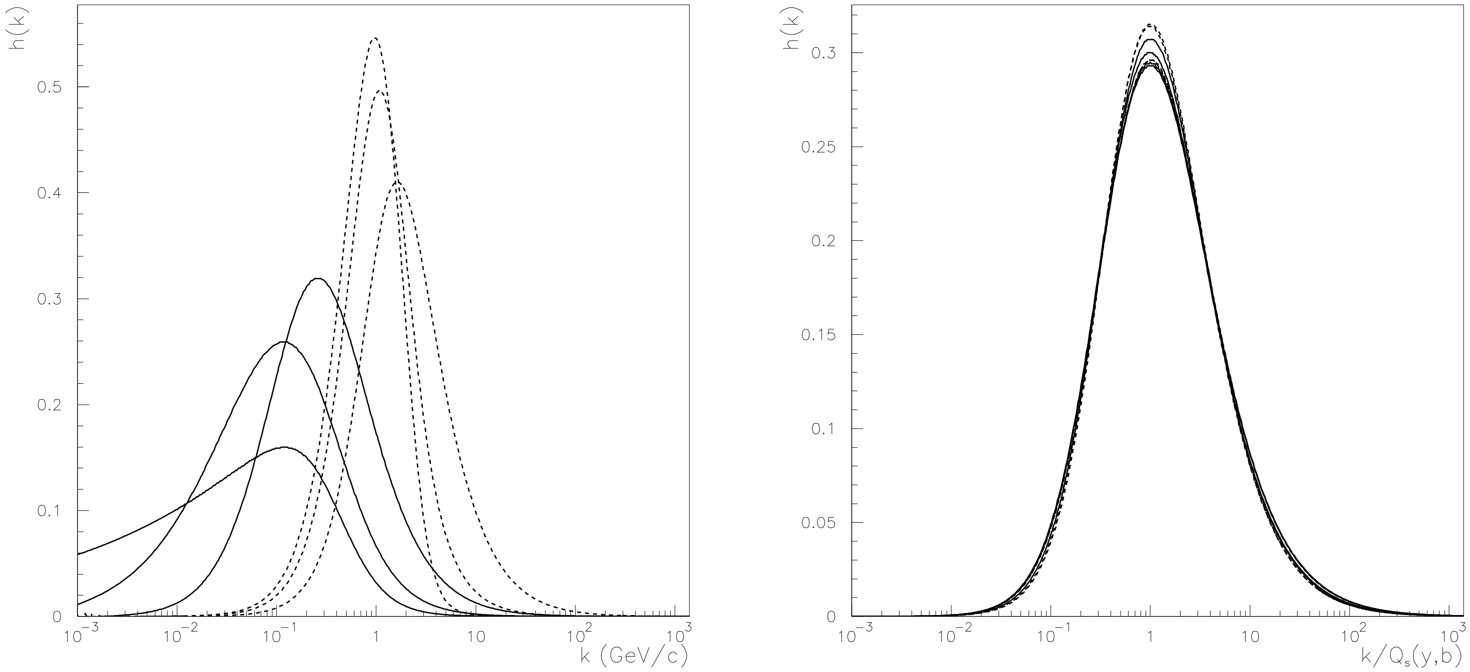
$$\varphi_A(x,k,b) = -\frac{N_c}{4\pi^2\alpha_s}k^2\int \frac{d^2r}{2\pi}\ \exp\left(ik\cdot r\right)\sigma_{dA}(x,r,b), \tag{8}$$

• Several numerical works available: Levin-Tuchin-Lublinsky, Golec-Biernat-Motyka-Staśto, Armesto-Braun, Kimber-Kwieciński-Martin.

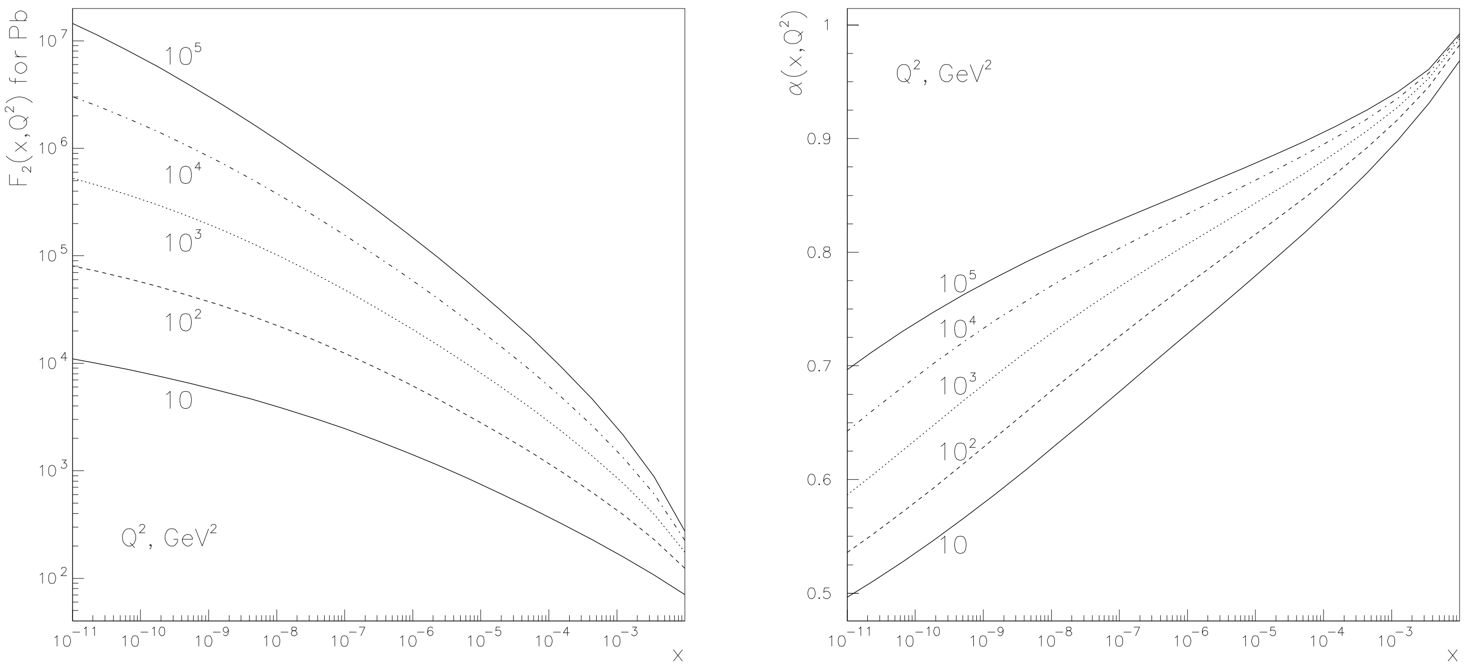
- Features of the solution: soliton-like behavior of h and scaling.

- Solution to the IR diffusion problem.

- Saturation scale $\equiv$ maximum of h.



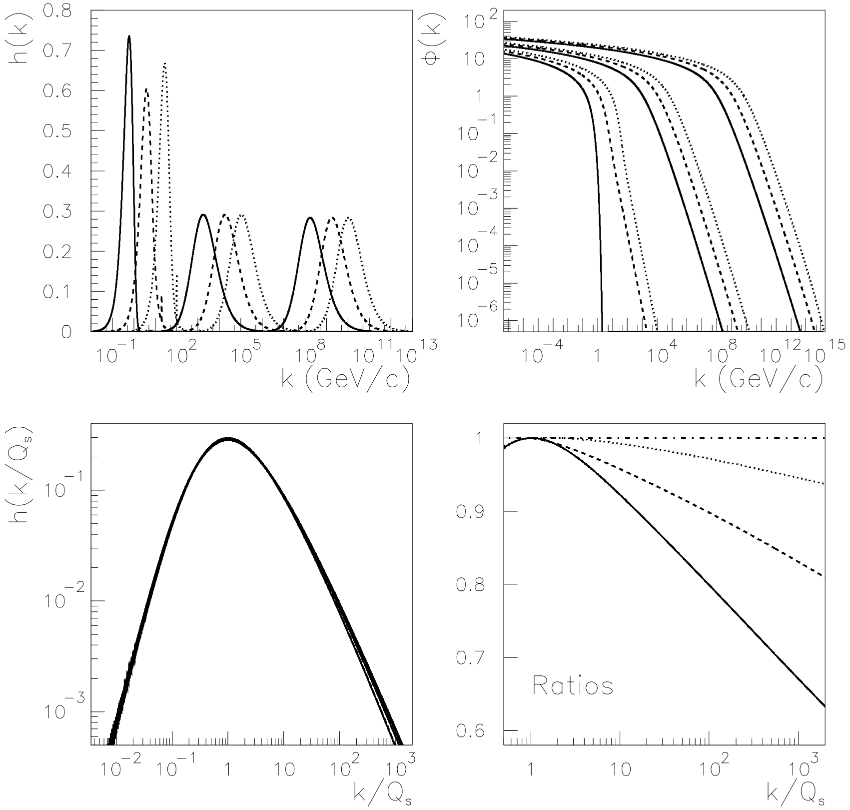
- Predictions starting from some 'realistic' initial function (Armesto '01).



$$F_{2A}(x,Q^2) = A^{\alpha(x,Q^2)} F_2(x,Q^2) \propto Q^2 \ln^2(1/x), \quad x \to 0.$$

Scaling; total cross sections

- Scaling with $k^2/Q_s^2(x)$ appears $(h = k^2\nabla_k^2\phi)$ (Armesto '01, Lublinsky '01, Golec-Biernat '02, Albacete '03).



• A universal function of k/Q_s appears, independent of the initial condition (even if it has a completely 'wrong' large k-behavior).

• Fits to $s_1(\rho) = a\rho^{2(1-\bar{\lambda})}$ and $s_2(\rho) = a\ln(b\rho)\rho^{2(1-\bar{\lambda})}$ for $\rho = k/Q_s > 5$ have been performed.

• For BFKL, s_1 with $\bar{\lambda} = 0.37$ as expected (Iancu '02) gives a good fit.

• For BK, s_1 never gives a good fit, while s_2 with $\bar{\lambda} = 0.32$ does (Mueller '02).

• If $Q_s^2(y) \propto \exp(\Delta N_c \alpha_s/\pi)$, $\Delta = 4.82$ for BFKL (expected $4.85 \div 4.88$) and $\Delta = 4.6$ for BK (expected $\Delta > 4$ (DLL) and < 4.88 (BFKL)); with $\alpha_s = 0.2$, this means $\lambda \sim 0.85 \gg 0.3$ in GBW.

• These fits to the region $k > Q_s$ have phenomenological implications (Kharzeev '02, Baier '03).

• Froissart bound, $\sigma \propto \ln^2 s$, recovered? (Ferreiro '02, Kovner '03): relation with the b-dependence, (Stásto here). Relevant for the implications on soft physics.

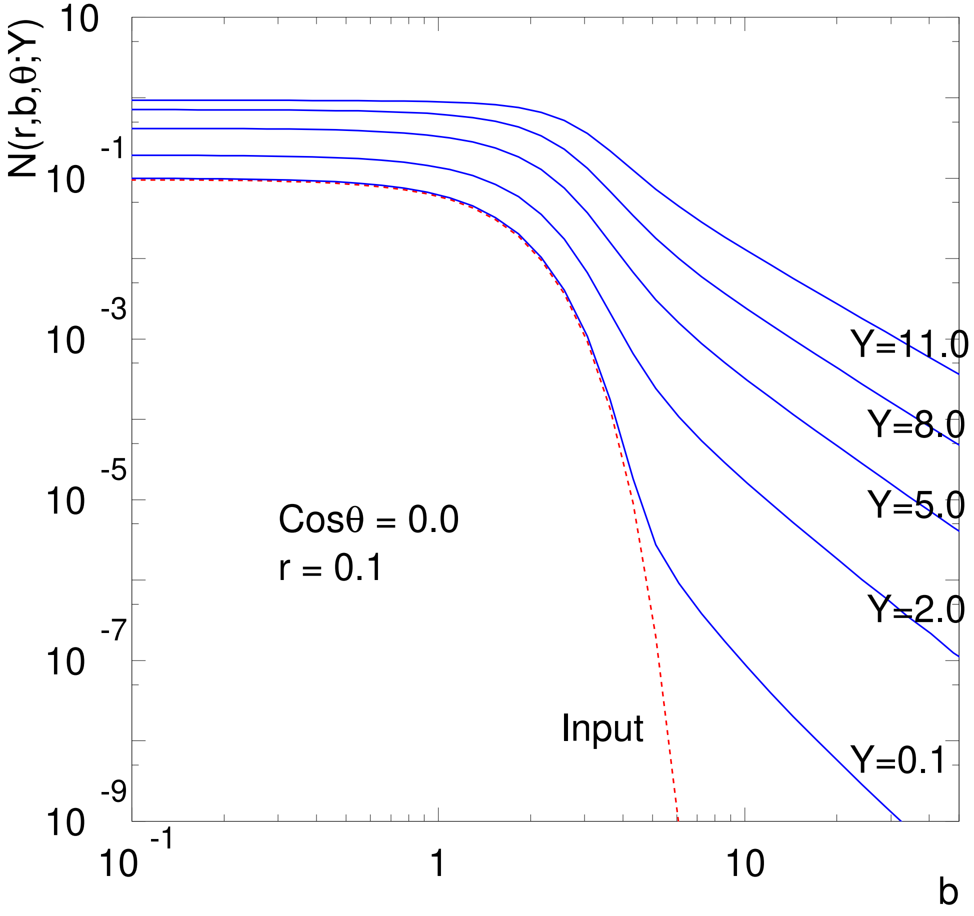
• First, no Froissart bound should be expected: unitarity, analyticity but no mass gap.

• But one could hope that the initial condition in b, $\propto \exp{(-Ab)}$ for large b, would cure the problem.

• It is not so: a tail $\propto 1/b^4$ develops dynamically, so a regulation of the kernel is demanded (confinement).

• Beautifully shown in numerical solutions with b dependence (Golec-Biernat '03, Staśto here).

• They could determine the behavior of Q_s^2 with b.



Golec-Biernat '03

4. Phenomenology

Multiplicities and pseudorapidity distributions

• Multiplicities, their evolution with centrality and pseudorapidity distributions have been computed using semiclassical ideas (Kharzeev '00, Pirner '01, Kharzeev '01).

$$\frac{dN_{pA}}{dyd^2p\, d^2b} \propto \frac{1}{p^2}\int d^2k\, f^q(y,k)\, f^Q(y,\mathbf{p}-\mathbf{k})\,, \tag{9}$$

$$\frac{dN_{AA}}{dy\, d^2p\, d^2b} \propto \frac{A^{2/3}}{p^2}\int d^2k\, f^Q(y,k)\, f^Q(y,\mathbf{p}-\mathbf{k})\,, \tag{10}$$

(k_T-factorization, Szczurek here) with $f^{q,Q}=\phi$ (Gribov '83) or h (in the fundamental (Braun '00) or in the adjoint (Kovchegov '01)). Inclusive gluon distributions can be computed (LPHD). Justification pending (OK in pA?).

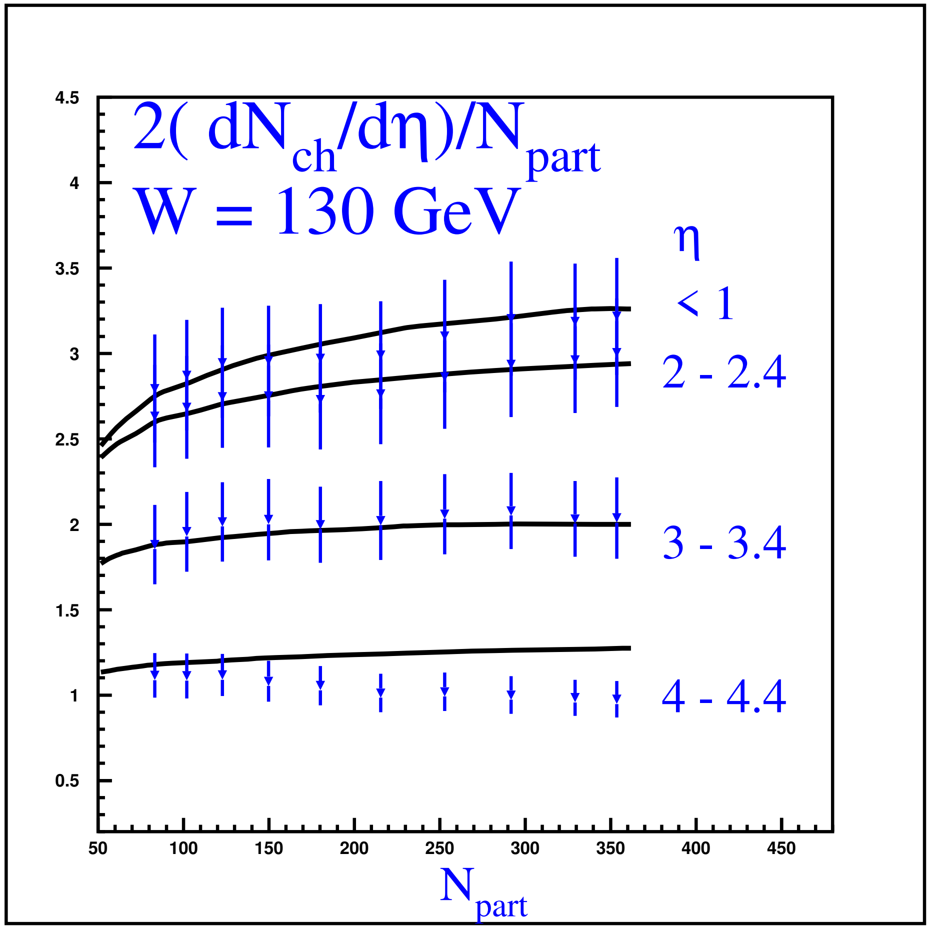
• A simple formula is deduced with some simplistic ansatz for f^Q:

$$\frac{dN}{dy} \propto s^{\lambda/2} N_{part} \exp\left(-\lambda|y|\right) \ln\left[(Q_s^2 \sim A^{1/3})/\Lambda^2\right]. \tag{11}$$

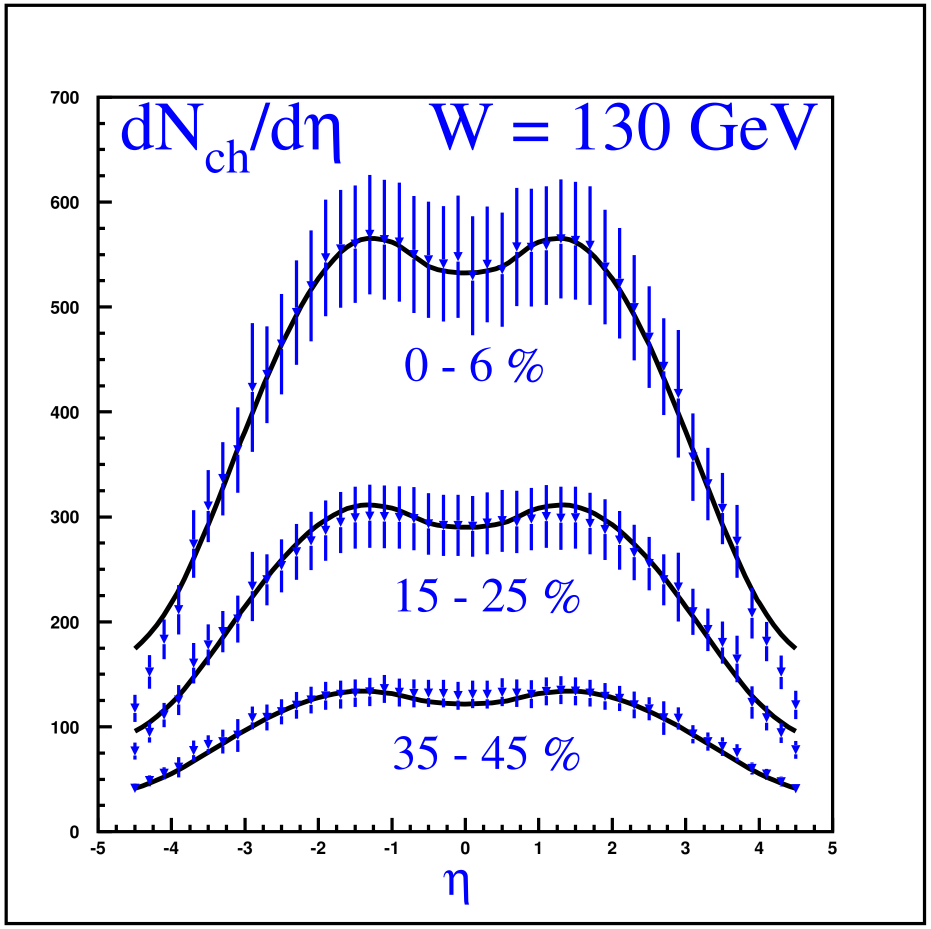
• Deviation from the WNM comes from the logarithm.

• Successfully reproduce multiplicities, their evolution with centrality and pseudorapidity distributions at RHIC: can RHIC data be explained by initial state effects?

• Elliptic flow partially explained by correlations inside the ladder (Kovchegov '02); also observed in string models (DPMJET) (Ranft here).



Kharzeev '01; data: PHOBOS '01



Kharzeev '01; data: PHOBOS '01

Transverse momentum distributions

• Transverse momentum distributions are expected to be affected by a medium: jet quenching.

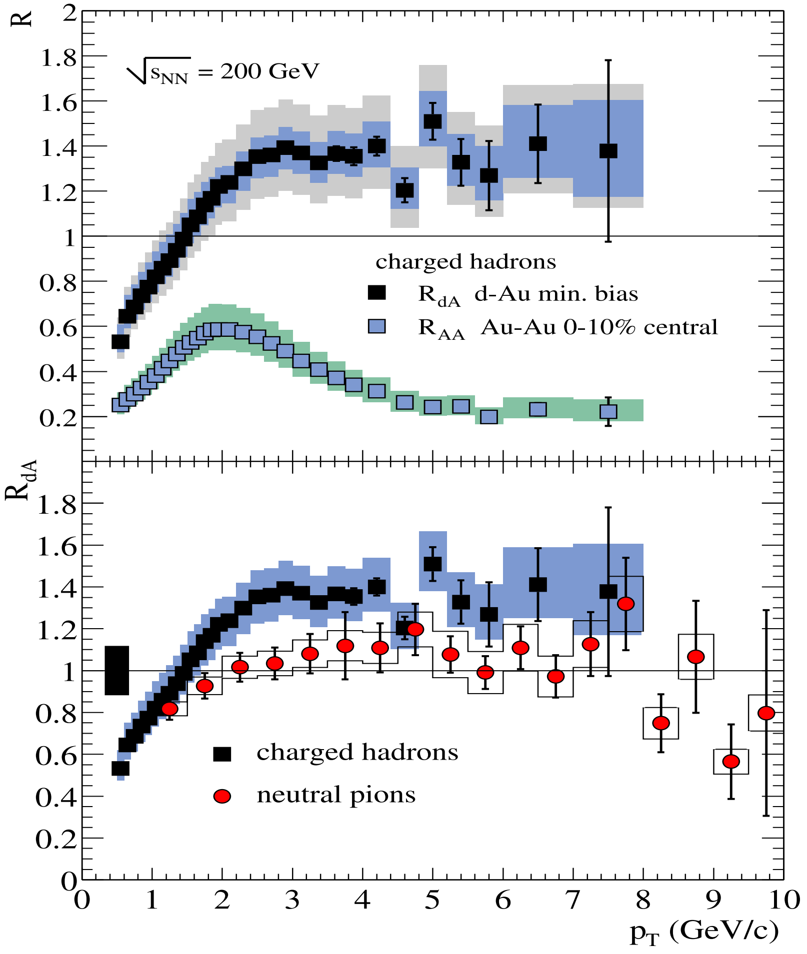
$$R_{pA} = \frac{\frac{dN_{pA}}{dyd^2p\,d^2b}}{A^{1/3}\frac{dN_{pp}}{dyd^2p\,d^2b}}\,, \qquad R_{AA} = \frac{\frac{dN_{AA}}{dyd^2p\,d^2b}}{A^{4/3}\frac{dN_{pp}}{dyd^2p\,d^2b}}\,. \tag{12}$$

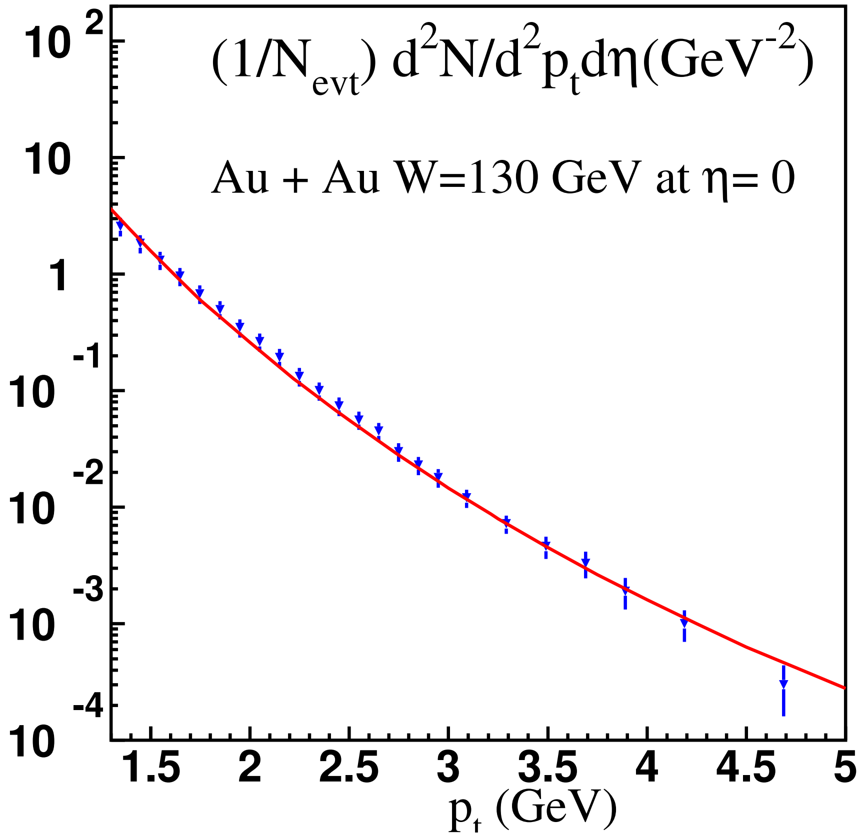
• It has been argued that the depletion observed at RHIC in AuAu collisions can be explained by initial state effects (Kharzeev '02).

• But then such depletion should also exist in dAu, contrary to data (PHENIX, STAR, PHOBOS, BRAHMS, '03).

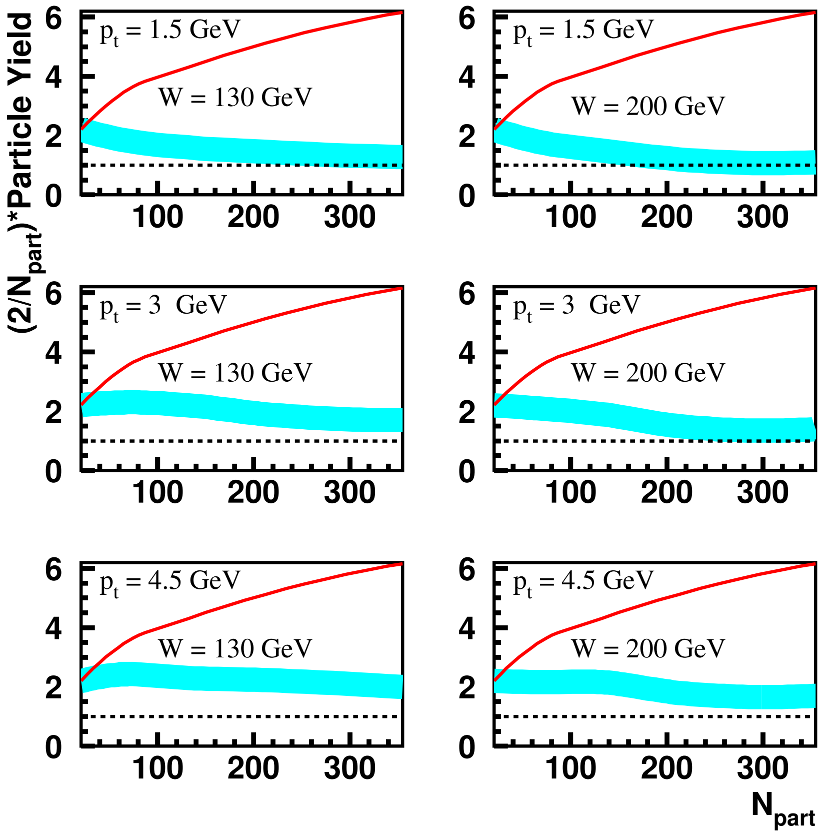
• Depletion/enhancement of the p_T spectra depends on the behavior of the distribution f above the saturation scale (Kharzeev '02, Baier '03).

PHENIX '03

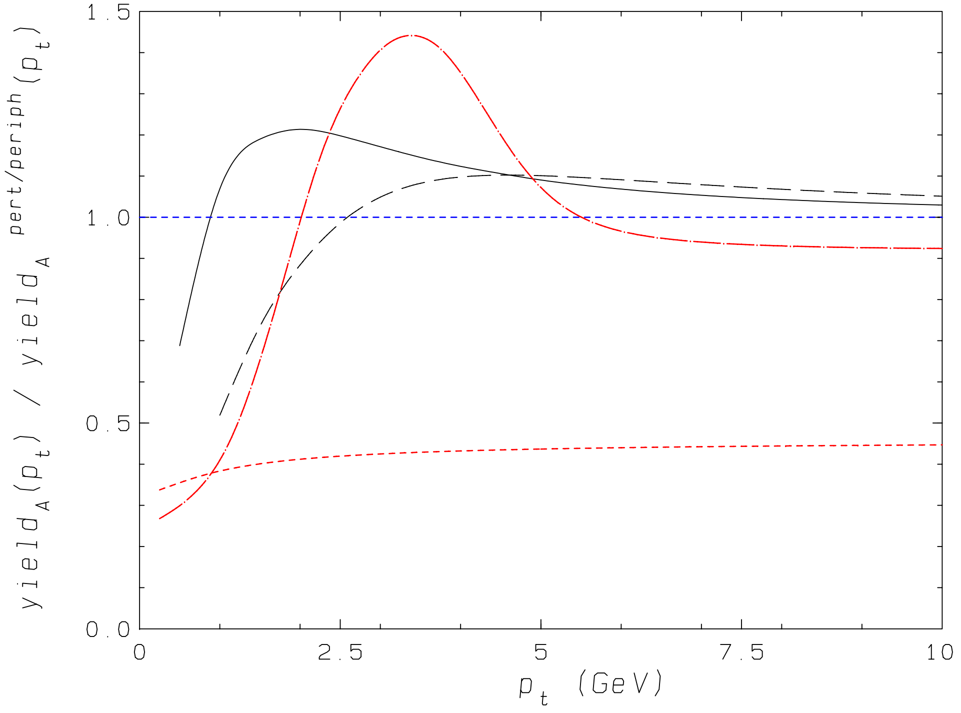

R
$\sqrt{s_{NN}}$ = 200 GeV
charged hadrons
R_{dA} d-Au min. bias
R_{AA} Au-Au 0-10% central
R_{dA}
charged hadrons
neutral pions
p_T (GeV/c)



Kharzeev '02; data: PHENIX '02


(2/N_part)*Particle Yield
p_t = 1.5 GeV
W = 130 GeV
p_t = 1.5 GeV
W = 200 GeV
p_t = 3 GeV
W = 130 GeV
p_t = 3 GeV
W = 200 GeV
p_t = 4.5 GeV
W = 130 GeV
p_t = 4.5 GeV
W = 200 GeV
N_part
0
2
4
6
100
200
300

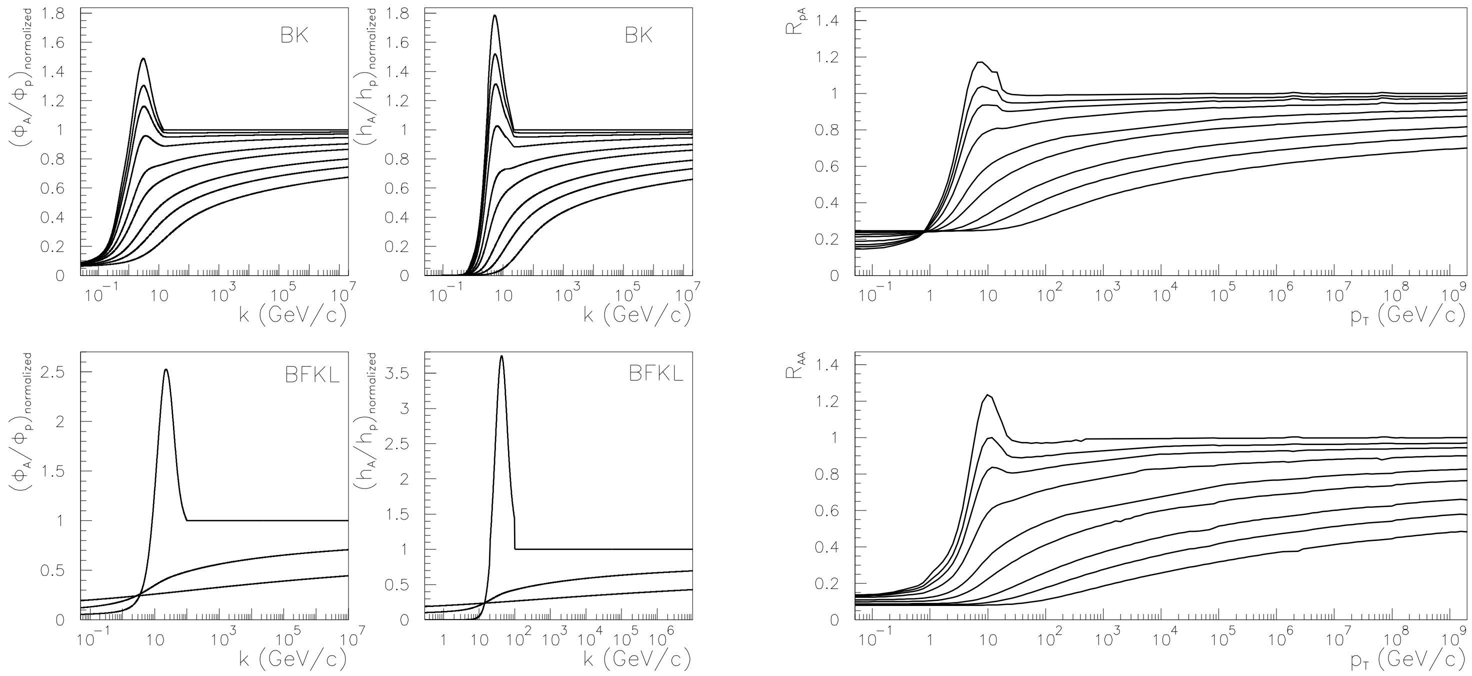
Kharzeev '02



Baier '03

• Enhancement exists in most initial conditions (Baier '03, Jalilian-Marian '03).

• Evolution (even BFKL) erases enhancement very efficiently and leads to suppression (Albacete '03), contrary to expectations (Kharzeev '03).



5. Conclusions

- A minireview of some aspects of saturation physics has been presented.

- Saturation offers a link between small x and heavy ion physics, and between old preQCD ideas and new QCD realizations: formulation and implications of unitarity in QCD.

- In model building, it still acts more like a guidance than as a concrete, defined framework.

- While RHIC data for AuAu are described (but still room for final state effects), dAu rules out an initial state suppression.

- Maybe RHIC is at the border of the applicability of saturation ideas (Mueller '02).

- Anyway, much is being learned about QCD in these studies: LHC?