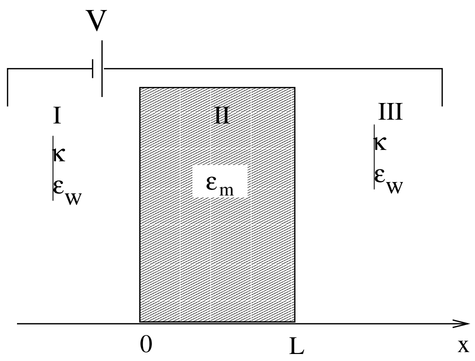
Problem 1: membrane potential

Consider a dielectric membrane in an electrolyte with an applied voltage V.



Solve the linearized Poisson-Boltzmann-equation $\frac{d^2}{dx^2}\phi(x) = \kappa^2(\phi(x) - \phi^{(0)})$ with boundary conditions $\phi^{(0)}(I) = \phi(-\infty) = 0$ $\phi^{(0)}(III) = \phi(\infty) = V$ and determine the voltage difference

$$\Delta V = \phi(L) - \phi(0)$$

Calculate the charge density $\rho_{mob}(x)$and the integrated charge on both sides of the membrane. What is the capacity of the membrane ?

Problem 2: activity

The chemical potential of an ion with charge Ze is given in terms of the activity a by $\mu = \mu^0 + kT \ln a$. Assume that the deviation from the ideal behaviour $\mu^{id} = \mu^0 + kT \ln c$ is due to electrostatic interactions only. Then for an ion with radius R Debye-Hückel theory gives

$$\mu - \mu^{id} = kT \ln a - kT \ln c = \Delta G_{solv} = -\frac{Z^2 e^2}{8\pi\epsilon} \frac{\kappa}{(1 + \kappa R)}$$

For a 1-1 electrolyte calculate the mean activity coefficient $\gamma^c_\pm = \sqrt{\gamma^c_+ \gamma^c_-} = \sqrt{\frac{a_+}{c_+}\frac{a_-}{c_-}}$ and discuss the limit of extremely dilute solutions (Debye Hückel limiting law)