

$\bar{B} \rightarrow X_s \gamma$  — CURRENT STATUS\*

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Our current knowledge of  $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$  is briefly summarized, with particular attention to uncertain non-perturbative effects.

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**1. Introduction**

Weak radiative  $\bar{B}$ -meson decays ( $\bar{B} = \bar{B}^0$  or  $B^-$ ) are generated by the Flavour-Changing Neutral Current (FCNC)  $b \rightarrow s \gamma$  transition that arises at one loop only. The diagrams contain sums over all the up-type quark flavours that are highly non-degenerate in mass. The relevant Cabibbo–Kobayashi–Maskawa (CKM) factor  $|V_{ts}^* V_{tb}|$  is very close to  $|V_{cb}|$  that occurs in the leading  $b$ -quark decays. In effect, the usual Glashow–Iliopoulos–Maiani suppression mechanism for FCNC processes is not at work. Numerically, the inclusive branching ratio  $\mathcal{B}_{s\gamma} \equiv \mathcal{B}(\bar{B} \rightarrow X_s \gamma) \simeq 0.14 \frac{\alpha_{em}}{\pi}$  in the Standard Model (SM).

Contributions of potentially the same size arise in extensions of the SM. In supersymmetric models, for instance,  $\Delta \mathcal{B}_{s\gamma}^{\text{SUSY}} / \mathcal{B}_{s\gamma}^{\text{SM}}$  scales roughly like  $[100 \text{ GeV} / (\text{superpartner masses})]^2$ . Consequently, comparing measurements of  $\mathcal{B}_{s\gamma}$  with theory predictions at a few percent level gives us strong constraints on new physics, irrespectively of whether any deviations from the SM are observed.

Accurate determination of  $\mathcal{B}_{s\gamma}$  is challenging on both the experimental and theoretical sides. At the  $B$ -factories, one of the main difficulties is precise subtraction of the so-called continuum background, *i.e.* hard photons that originate from non- $B\bar{B}$  processes. Both the continuum and the  $\bar{B} \rightarrow X_c \gamma$  backgrounds become more severe towards lower photon energies. It is clearly reflected by the behaviour of errors in the most recent BELLE measurement [1] of the (isospin- and CP-averaged) branching ratio

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$$\mathcal{B}_{s\gamma}(E_\gamma > E_0) = \begin{cases} (3.02 \pm 0.10_{\text{stat}} \pm 0.11_{\text{syst}}) \times 10^{-4}, & \text{for } E_0 = 2.0 \text{ GeV}, \\ (3.21 \pm 0.11_{\text{stat}} \pm 0.16_{\text{syst}}) \times 10^{-4}, & \text{for } E_0 = 1.9 \text{ GeV}, \\ (3.36 \pm 0.13_{\text{stat}} \pm 0.25_{\text{syst}}) \times 10^{-4}, & \text{for } E_0 = 1.8 \text{ GeV}, \\ (3.45 \pm 0.15_{\text{stat}} \pm 0.40_{\text{syst}}) \times 10^{-4}, & \text{for } E_0 = 1.7 \text{ GeV}. \end{cases} \quad (1)$$

On the other hand, the theory prediction is based on an approximate equality of the hadronic and partonic decay widths

$$\Gamma(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > E_0} \simeq \Gamma(b \rightarrow X_s^{\text{parton}} \gamma)_{E_\gamma > E_0} \quad (2)$$

that breaks down when  $E_0$  is too close to the endpoint  $E_{\text{max}} = \frac{m_B^2 - m_K^{*2}}{2m_B} \simeq 2.56 \text{ GeV}$ , *i.e.* when  $E_{\text{max}} - E_0 \sim \Lambda \equiv \Lambda_{\text{QCD}}$ . It has become customary to use  $E_0 = 1.6 \text{ GeV} \simeq \frac{m_b}{3}$  for comparing theory with experiment. The SM prediction for this value of  $E_0$  reads [2]

$$\mathcal{B}_{s\gamma}(E_\gamma > 1.6 \text{ GeV}) = (3.15 \pm 0.23) \times 10^{-4}. \quad (3)$$

It includes the  $\mathcal{O}(\alpha_s^2)$  QCD corrections and the leading electroweak ones. Both the known and unknown non-perturbative effects have been taken into account in estimating the central value and the uncertainty.

The currently available experimental world averages read

$$\mathcal{B}_{s\gamma}(E_\gamma > 1.6 \text{ GeV}) = \begin{cases} (3.52 \pm 0.23_{\text{exp}} \pm 0.09_{\text{model}}) \times 10^{-4} & [3], \\ (3.50 \pm 0.14_{\text{exp}} \pm 0.10_{\text{model}}) \times 10^{-4} & [4]. \end{cases} \quad (4)$$

They have been obtained by extrapolation from measurements at higher  $E_0$  using models of the photon energy spectrum and fitting their parameters to data. The BELLE result in Eq. (1) is more recent than the above averages. Its preliminary version [5] has been used in Eq. (4) instead, together with older measurements of CLEO, BELLE and BABAR [6, 7]<sup>1</sup>. The SM prediction (3) and the averages (4) are consistent at the  $1.2\sigma$  level.

Theoretical analyses of  $\bar{B} \rightarrow X_s \gamma$  employ the formalism of an effective theory that arises after decoupling of the  $W$ -boson and all the heavier particles. The relevant flavour-changing interactions are given by dimension-five and -six local operators<sup>2</sup>

$$\begin{aligned} O_{1,2} &= (\bar{s}\Gamma_i c)(\bar{c}\Gamma'_i b), & O_{3,4,5,6} &= (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma'_i q), \\ O_7 &= \frac{em_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & O_8 &= \frac{gm_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a. \end{aligned} \quad (5)$$

<sup>1</sup> The average in Ref. [3] has a larger error because it includes results at  $E_0 \geq 1.8 \text{ GeV}$  from the older measurements [6, 7] only, ignoring the more precise ones from Ref. [5].

<sup>2</sup> The specific matrices  $\Gamma_i$  and  $\Gamma'_i$  can be found in Ref. [8]. Additional operators may arise beyond SM.

One begins with perturbatively calculating their Wilson coefficients  $C_i$  at the renormalization scale  $\mu_0 \sim (M_W, m_t)$ . Next, renormalization group is used for the evolution of  $C_i$  down to the scale  $\mu_b \sim m_b/2$ . These two steps were finalized a few years ago including  $\mathcal{O}(\alpha_s^2)$  effects in the SM [9]. The last step amounts to evaluating the inclusive decay width  $\Gamma(\bar{B} \rightarrow X_s \gamma)$  that is generated by  $Q_i$  at the scale  $\mu_b$ . Non-perturbative effects show up at this last stage only.

The final states in  $\bar{B} \rightarrow X_s \gamma$  are required to be charmless, *i.e.* to contain no charmed ( $C \neq 0$ ) hadrons. Various ways of energetic photon production ( $E_\gamma \gtrsim \frac{m_b}{3}$ ) in  $\bar{B}$ -meson decays to such states are displayed in Fig. 1. The first and second rows describe situations without and with long-distance charm loops, respectively. The meson in its rest frame is shown as a grey (brown) ball of diameter  $\sim \Lambda^{-1}$ , while the heavy  $b$ -quark is localized in the center at distances of order  $m_b^{-1}$  (black blob). Decay products are depicted diagrammatically, and their hadronization is implicitly assumed.

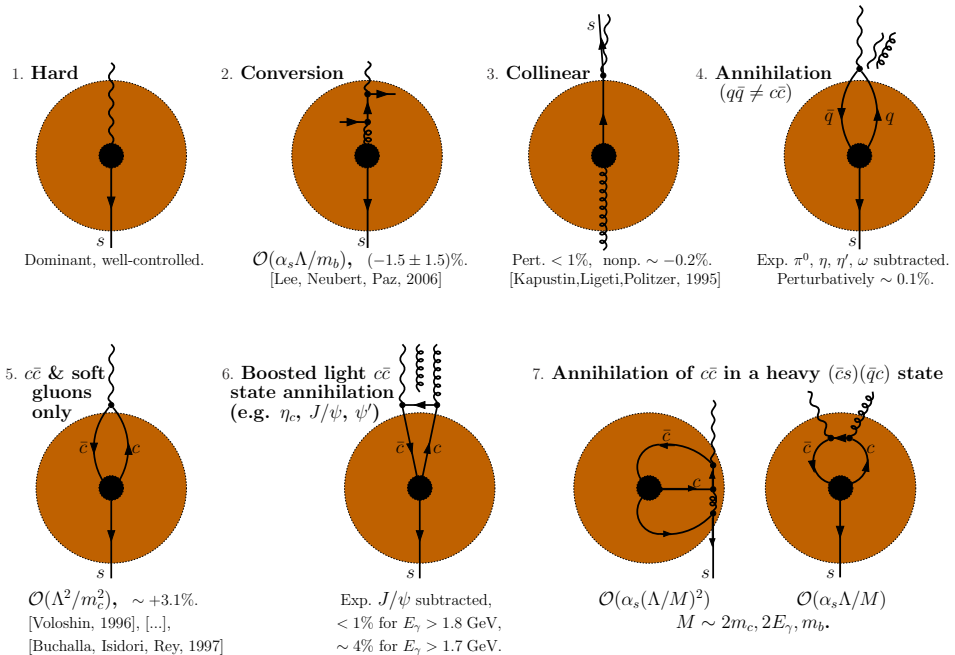


Fig. 1. Energetic photon production ( $E_\gamma \gtrsim \frac{m_b}{3}$ ) in charmless decays of the  $\bar{B}$  meson. The first and second rows describe situations without and with long-distance charm loops, respectively (see the text).

## 2. Contributions without long-distance charm loops

### 2.1. Hard

In the first diagram in Fig. 1, the photon is emitted directly from the hard process of the  $b$ -quark decay. This means that the photon emission vertex and the  $b$ -quark annihilation vertex either coincide in position space (as in  $Q_7$  in Eq. (5)) or get connected by a (chain of) high-virtuality parton propagator(s) (virtuality  $\sim m_b^2$  or larger). In the latter case, Operator Product Expansion (OPE) can be performed at the amplitude level to make the two vertices coincide. Next, in analogy to the semileptonic  $B$ -meson decays [10], the optical theorem and another OPE are applied to show that

$$\Gamma(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > E_0}^{\text{hard}} = \Gamma(b \rightarrow X_s^{\text{parton}} \gamma)_{E_\gamma > E_0}^{\text{hard}} + \mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right), \quad (6)$$

so long as  $(m_b - 2E_0) \sim m_b$ . The  $\mathcal{O}(\Lambda^2/m_b^2)$  corrections are expressed [11] in terms of local operator matrix elements between the  $\bar{B}$ -meson states at rest. These matrix elements can be extracted from  $B$ - $B^*$  mass splitting and/or from properties of the semileptonic decay spectra (see *e.g.* Ref. [12]). Numerically, the  $\mathcal{O}(\Lambda^2/m_b^2)$  corrections amount to around  $-3\%$  of the decay rate.

Other-than-hard contributions to  $\Gamma(b \rightarrow X_s^{\text{parton}} \gamma)$  arise only inside perturbative corrections of order  $\mathcal{O}(\alpha_s)$  and higher. In the actual calculations of these corrections, all the momentum regions are included, *i.e.* hard contributions are not singled out. Consequently, the remaining non-perturbative effects to be discussed below should be understood as containing a subtraction of the corresponding perturbative terms (if present).

### 2.2. Conversion

In the second diagram of Fig. 1, the  $b$ -quark decays in a hard way into quarks and gluons only. Next, one of the decay products scatters radiatively with remnants of the  $\bar{B}$ -meson in a non-soft manner, *i.e.* with momentum transfer much larger than  $\Lambda$ . Such a situation should be distinguished from collinear photon emission in jet fragmentation where only soft interactions are sufficient (see the next subsection). Here, contributions to the decay rate are suppressed with respect to the leading hard  $b \rightarrow s\gamma$  one by  $\alpha_s$  (due to non-soft scattering) and by  $\Lambda/m_b$  (due to dilution of the target). The analysis of Ref. [13] confirms that no other suppression factors occur.

The considered amplitudes should efficiently interfere with the leading hard  $b \rightarrow s\gamma$  ones to make the effect relevant, *i.e.* both the photon and the  $s$ -quark should move roughly back-to-back with energies close to  $m_b/2$ .

For this reason, we can restrict ourselves to the gluon-to-photon case<sup>3</sup> that resembles Compton scattering, and think of the scattered quark as soft on both external lines. This quark may either be a valence quark or any of the sea quarks. Thus, we deal with a process that may be viewed as gluon-to-photon conversion in the QCD medium.

In Ref. [13], the Vacuum Insertion Approximation (VIA) has been used to find that conversion gives a correction  $\Delta\mathcal{B}_{s\gamma}/\mathcal{B}_{s\gamma} \in [-3, -0.3]\%$  to the isospin averaged branching ratio

$$\mathcal{B}_{s\gamma} = \frac{\Gamma(\bar{B}^0 \rightarrow X_s \gamma) + \Gamma(B^- \rightarrow X_s \gamma)}{\Gamma_{\text{tot}}(\bar{B}^0) + \Gamma_{\text{tot}}(B^\pm)} \equiv \frac{\Gamma_{s\gamma}^0 + \Gamma_{s\gamma}^-}{\Gamma_{\text{tot}}^0 + \Gamma_{\text{tot}}^-}, \quad (7)$$

and generates a sizeable isospin asymmetry  $\Delta_{0-} = (\Gamma_{s\gamma}^0 - \Gamma_{s\gamma}^-)/(\Gamma_{s\gamma}^0 + \Gamma_{s\gamma}^-)$ . However, assuming a larger uncertainty of  $\sim 5\%$  in  $\Delta\mathcal{B}_{s\gamma}/\mathcal{B}_{s\gamma}$  has been recommended in view of the fact that VIA is a very rough approximation.

It is interesting to note that instead of the VIA one may consider the  $\text{SU}(3)_{\text{flavor}}$  limit that amounts to neglecting differences between light quark masses as well as electromagnetic corrections to the meson wave-functions. In this limit,  $\Gamma_{s\gamma}^0 \simeq \Gamma_{(\text{no conversion})} + Q_d \Delta\Gamma_1 + (Q_u + Q_s) \Delta\Gamma_2$  and  $\Gamma_{s\gamma}^- \simeq \Gamma_{(\text{no conversion})} + Q_u \Delta\Gamma_1 + (Q_d + Q_s) \Delta\Gamma_2$ , which follows from the fact that interference of the gluon-to-photon conversion with the leading hard amplitude is linear in the quark charges. Next, using  $Q_u + Q_d + Q_s = 0$ , one immediately finds

$$\Delta\mathcal{B}_{s\gamma}/\mathcal{B}_{s\gamma} \simeq \frac{Q_d + Q_u}{Q_d - Q_u} \Delta_{0-} = -\frac{1}{3} \Delta_{0-} = (+0.2 \pm 1.9_{\text{stat}} \pm 0.3_{\text{sys}} \pm 0.8_{\text{ident}}) \%, \quad (8)$$

where the BABAR measurement [7] of  $\Delta_{0-}$  for  $E_\gamma > 1.9 \text{ GeV}$  has been used in the last step. It seems reasonable to assume that the valence quark effects in  $\Delta\mathcal{B}_{s\gamma}/\mathcal{B}_{s\gamma}$  dominate over the  $\text{SU}(3)_{\text{flavor}}$ -violating ones. Thus, if the more precise future measurements of  $\Delta_{0-}$  remain consistent with zero, our control over uncertain contributions to  $\mathcal{B}_{s\gamma}$  may significantly improve.

### 2.3. Collinear

In the third diagram of Fig. 1, a collinear photon is emitted in the process of hadronization. We require  $|\vec{p}|_\gamma > 1.6 \text{ GeV} \sim m_b/3$ . The maximal parton three-momentum in the  $b$ -quark decay is  $m_b/2$ , but a typical one is much lower. Thus, there is rather little phase-space for a collinear emission<sup>4</sup>.

<sup>3</sup> This very case has been displayed in the second diagram of Fig. 1. The analogous quark-to-photon transition could hardly give enough interference.

<sup>4</sup> The phase-space could be further reduced in the future by performing the  $E_0$ -extrapolation solely for the hard contributions. The remaining ones could be subtracted from the experimental data at higher  $E_0$  in advance.

Moreover, charmless hadronic  $B$ -decays are parametrically suppressed either by  $\alpha_s$  (for  $Q_8$ ), or by the small Wilson coefficients ( $\lesssim 0.07$  for  $Q_{3,4,5,6}$ ), or by the CKM angles (for the  $u$ -quark analogues of  $Q_{1,2}$ ).

Perturbatively, ( $n > 2$ )-body decays that involve operators other than  $Q_7$  are responsible for  $4 \div 6\%$  of  $\Gamma(b \rightarrow X_s^{\text{parton}} \gamma)$  for  $E_0 \in [1.6, 2.0]$  GeV. A small fraction of them involves collinear configurations. In  $b \rightarrow sg\gamma$ , a collinear logarithm ( $\ln m_b/m_s \simeq \ln 50$ ) arises at  $\mathcal{O}(\alpha_s)$  only in the 88-term, where “ $kj$ -term” stands for a product of amplitudes generated by  $Q_k$  and  $Q_j$ . For  $E_\gamma > 1.6$  GeV, the decay rate changes by less than 1% or 0.2% when this logarithm is respectively set to zero or modified according to Ref. [14]. Fragmentation functions have been used in that paper to determine the collinear ( $\ln m_b$ )-terms. Such small numerical effects imply that uncertainties in  $B_{s\gamma}$  that are due to non-perturbative collinear effects can be safely neglected at present.

#### 2.4. Annihilation of light quarks ( $q\bar{q} \neq c\bar{c}$ )

The fourth diagram in Fig. 1 describes radiative annihilation of light quark pairs that have been produced in the  $\bar{B}$ -meson decay. Perturbatively, such effects amount to only around 0.1% of the decay rate. In reality, they are very much enhanced for several lightest  $q\bar{q}$  mesons ( $\pi^0$ ,  $\eta$ ,  $\eta'$ ,  $\omega$ ,  $\rho$ ) because of the limited number (or lack) of alternative hadronic decay channels. However, photons originating from radiative decays of  $\pi^0$  and  $\eta$  are vetoed on the experimental side. Radiative decays of other light mesons are simulated and treated as background, too. Thus, it is mandatory to assume that all the non-perturbatively enhanced radiative  $q\bar{q}$  annihilation processes are removed from the signal. On the other hand, the perturbative contributions are so tiny that retaining them in the theory calculations does not hurt.

### 3. Contributions with long-distance charm loops

Radiative annihilation of intermediate  $c\bar{c}$  states requires much more care, which is signaled by the presence of large perturbative charm loop contributions. It is sufficient to mention that changing the charm quark mass from its measured value  $m_c(m_c) \simeq 1.28$  GeV [15] to  $m_c = m_b$  results in a suppression of  $\Gamma(b \rightarrow X_s^{\text{parton}} \gamma)$  by around 35% (!). Thus, it is important to verify what fraction of  $\mathcal{B}_{s\gamma}$  originates from long-distance  $c\bar{c}$  loops, *i.e.* from those intermediate  $c\bar{c}$  states that are not properly accounted for in the perturbative approach.

We do not need to worry about purely hadronic annihilation of intermediate  $c\bar{c}$  states because such processes become kinematically allowed in  $\bar{B} \rightarrow X_s \gamma$  only for  $E_\gamma < (m_B^2 - (m_{\eta_c} + m_K)^2)/(2m_B) \simeq 1.5$  GeV.

### 3.1. A loop with soft gluons only

Let us first consider those  $c\bar{c}$  contributions that are not suppressed by  $\alpha_s$ , *i.e.* no hard gluons are present. They are represented by the fifth diagram in Fig. 1 where soft-gluon dressing is implicitly assumed. Bringing the two charm quarks close to their mass shell in the intermediate state implies that the momentum  $p_g$  absorbed by the soft gluon field in the  $c\bar{c}$  annihilation process must satisfy  $(p_g + p_\gamma) \sim 4m_c^2$ . Thus, for  $m_c^2 \gg m_b \Lambda$ , the on-shell-like configuration is kinematically inaccessible, which implies that the charm quark loop is a short-distance one in this limit. Its interaction with the soft gluon field can be expressed in terms of a series of local operators.

In reality, the inequality  $m_c^2 > m_b \Lambda$  is barely satisfied. Consequently, one should worry about all orders of the OPE. Moreover, as the relevant QCD interactions are soft, one should treat  $\alpha_s$  as a quantity of order unity despite using the language of Feynman diagrams. As follows from Ref. [16], the  $c\bar{c}$  loop with a single external soft gluon<sup>5</sup> gives a correction to  $\mathcal{B}_{s\gamma}$  that can be written as

$$\frac{\Lambda^2}{m_c^2} \sum_{n=0}^{\infty} b_n \left( \frac{\Lambda m_b}{m_c^2} \right)^n. \quad (9)$$

Each  $\Lambda^n$  above should be understood as matrix element of a local operator between  $\bar{B}$ -meson states at rest. The numerical coefficients  $b_n$  are found by Taylor-expanding the  $c\bar{c}$  loop diagram in the soft gluon external momentum. Larger numbers of gluons bring higher powers of  $\Lambda^2/m_c^2$ .

The leading  $n = 0$  term in Eq. (9) is calculable because it involves the same hadronic matrix element as the  $\mathcal{O}(\Lambda^2/m_b^2)$  correction in Section 2.1. Matrix elements entering at  $n \geq 1$  remain unknown. However, the coefficients  $b_n$  are found to decrease rapidly with  $n$  [16]. Consequently, the  $n = 0$  term is believed to provide a good approximation to the whole series. The corresponding correction to  $\mathcal{B}_{s\gamma}$  amounts to around +3.1%.

### 3.2. Boosted light $c\bar{c}$ state annihilation

If a hard gluon is attached to the  $c\bar{c}$  loop, we get a suppression by  $\alpha_s$  but need to deal with charm quarks close to their mass shell. Consequently, techniques from the previous section become inapplicable. Now the charm quarks can form a light  $c\bar{c}$  meson like  $J/\psi$  (see the sixth diagram in Fig. 1) whose radiative decay modes are enhanced due to its tiny hadronic decay width. The measured inclusive branching ratios  $\mathcal{B}(\bar{B} \rightarrow X_s J/\psi) \simeq (1.094 \pm 0.032)\%$  [17] and  $\mathcal{B}(J/\psi \rightarrow gg\gamma) \simeq (9.2 \pm 1.0)\%$  [18] illustrate the relevance of the intermediate  $J/\psi$  state.

<sup>5</sup> The loop with no external gluons vanishes for the on-shell photon.

According to the current conventions, photons originating from radiative  $J/\psi$  decays are treated as background in  $\bar{B} \rightarrow X_s \gamma$  measurements. The high photon energy cut suppresses this background by orders of magnitude with respect to what the above-mentioned total branching ratios might suggest. For the BELLE results quoted in Eq. (1), the intermediate  $J/\psi$  background is as large as 4% and 1% of the signal for  $E_0 = 1.7$  GeV and 1.8 GeV, respectively [19]. It has been calculated using Monte Carlo simulations based on the measured inclusive  $\mathcal{B}(\bar{B} \rightarrow X_s J/\psi)$  spectra and on the rich PDG [18] collection of  $J/\psi$  exclusive radiative decay modes.

The same prescription should actually be applied to all the  $c\bar{c}$  resonances that lay below the  $D\bar{D}$  production threshold<sup>6</sup>. The corresponding background subtraction (with respect to that for  $J/\psi$ ) is expected to be about 6 times smaller for  $\psi'$  and many times smaller for the remaining (much wider) resonances. With the present uncertainties, the only non-negligible subtraction is the already included  $J/\psi$  one.

The last question to be posed in this subsection is whether anything should be accordingly subtracted from the perturbative calculation of  $\Gamma(b \rightarrow X_s^{\text{parton}} \gamma)$ . All the  $b \rightarrow sg\gamma$  diagrams with charm loops at  $\mathcal{O}(\alpha_s)$  and  $\mathcal{O}(\alpha_s^2 \beta_0)$  contribute to this width at the levels of 3.6%, 3.3% and 2.9% for  $E_0 = 1.6, 1.7$  and 1.8 GeV, respectively. Only small fractions of these numbers should correspond to narrow resonances because the diagrams are calculated in fixed order, without any Coulomb ladder resummation [20]. Thus, including the perturbative contributions with no subtraction at all seems to be the right choice.

### 3.3. Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state

The last two diagrams in Fig. 1 are supposed to represent radiative annihilation of  $c\bar{c}$  pairs in heavy  $(\bar{c}s)(\bar{q}c)$  states, *i.e.* in all the states lay above the  $D\bar{D}$  production threshold. Here, we include not only  $c\bar{c}$ -meson-like states, but also rescattering of heavy  $C \neq 0$  hadrons, or charm quark annihilation that takes place before hadronization. Contributions from such intermediate states are largely accounted for by the perturbative results and/or those already discussed in Section 3.1. However, non-perturbative rescattering effects in particular exclusive modes may be sizeable. For instance, branching ratios [17]  $\mathcal{B}(B^- \rightarrow D_{sJ}(2457)^- D^*(2007)^0) \simeq 1.2\%$  or  $\mathcal{B}(B^0 \rightarrow D^*(2010)^+ \bar{D}^*(2007)^0 K^-) \simeq 1.2\%$  are not very small, while the available kinetic energy remains below 1 GeV.

Fortunately, we have already exhausted the case with no hard gluons in Section 3.1. Thus, the non-perturbative contributions that we are now after must be suppressed by at least one power of  $\alpha_s$ . Such a suppression

<sup>6</sup> New measurements of inclusive radiative  $J/\psi$  and  $\psi'$  decay spectra have recently become available [18].

of the annihilation channels is not surprising because the considered states can decay into open charm without any hard gluon emission/exchange. In fact, when these states are really “long-distance” ones,  $\alpha_s$  is not the only dumping factor. The  $c$  and  $\bar{c}$  wavefunctions are then diluted over distances of order at least  $\Lambda^{-1}$ , so the annihilation probability must be correspondingly suppressed. By analogy to the  $B$ -meson decay constant that scales like  $(\Lambda/m_b)^{3/2}$  due to the meson size, we may conclude that  $c\bar{c}$  annihilation probability for the “long-distance” states is down by  $(\Lambda/M)^{3/2}$ , where  $M$  is any of the hard scales in the problem  $(2m_c, 2E_\gamma, m_b)$ . To be conservative, let us skip the power “3/2” and assume for error estimates that the considered unknown non-perturbative corrections to  $\mathcal{B}_{s\gamma}$  are of order  $\mathcal{O}(\alpha_s \Lambda/M)$ .

An uncertainty due to such corrections at the 5% level has been assumed in the SM prediction in Eq. (3), just because  $\alpha_s(m_b) \simeq 0.2$ , and  $\Lambda/M$  is not much smaller. This error has been added in quadrature to the other ones, so it should be interpreted as a “theoretical  $1\sigma$ ” rather than a strict upper bound on the size of such effects. Actually, the uncertainty might have been overestimated because, as shown in Fig. 1, the  $\mathcal{O}(\alpha_s \Lambda/M)$  correction corresponds to the  $b \rightarrow sg\gamma$  channel that poorly interferes with the leading term; the corresponding perturbative interference has no peak near  $E_\gamma^{\max} \simeq m_b/2$ . On the other hand, the hard  $c\bar{c}s \rightarrow s\gamma$  subprocess that might lead to effective interference is suppressed by an additional power of  $\Lambda/M$  due to wave-function dilution. Nevertheless, any more optimistic error estimate would need to be based on a detailed analysis that seems to be rather difficult.

#### 4. Final remarks

In this short status summary, the main stress has been put on the rarely discussed issues which, however, are responsible for the main theory uncertainty in  $\mathcal{B}_{s\gamma}$ . As far as the perturbative calculations are concerned, the reports in Ref. [21] remain still up-to-date. Several recent analyses of the photon energy spectrum near endpoint can be found in Ref. [22]. They are essential for the  $E_0$  extrapolation that has been mentioned in Section 1.

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