Life before QCD: S matrix theory

$$S = \mathbf{1} + i\,T$$

$$S_{fi} = \langle f\,|S|\,i\rangle = \delta_{fi} + i\,T_{fi}$$

$$T_{fi} = i\,(2\pi)^4\delta^{(4)}(p_f - p_i)\,A_{fi}$$

Postulates concerning S matrix:

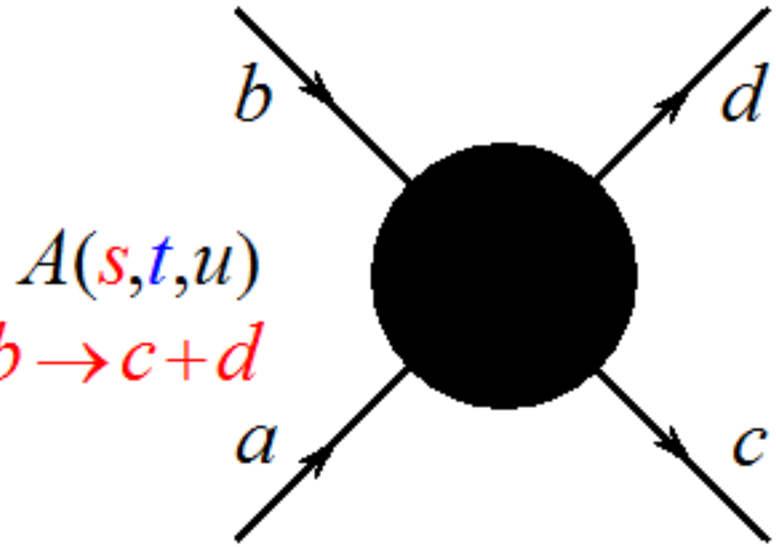
- Lorentz invariance
- unitarity
- analyticity
- crossing

Lorentz invariance

$$s = (p_a + p_b)^2$$

$$t = (p_a - p_c)^2$$

$$u = (p_a - p_d)^2$$



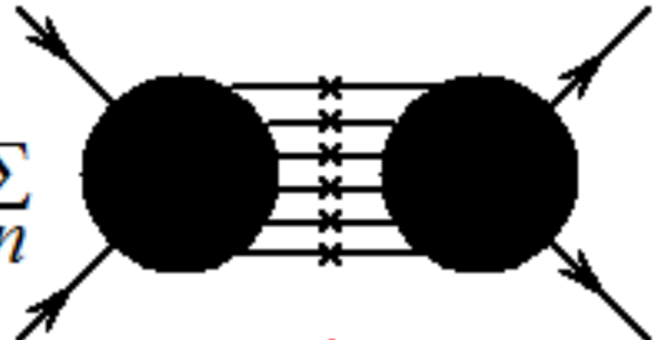
$$s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2 \sim 0$$

$$\rightarrow \quad s > 0,\ t, u < 0$$

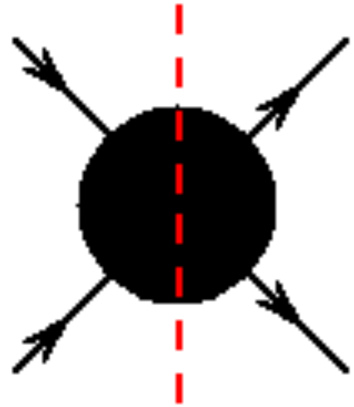
$$\cos\theta = 1 + 2\frac{t}{s}$$

Unitarity: optical theorem

$$\sigma_{tot} = \frac{1}{2s} \sum_n$$

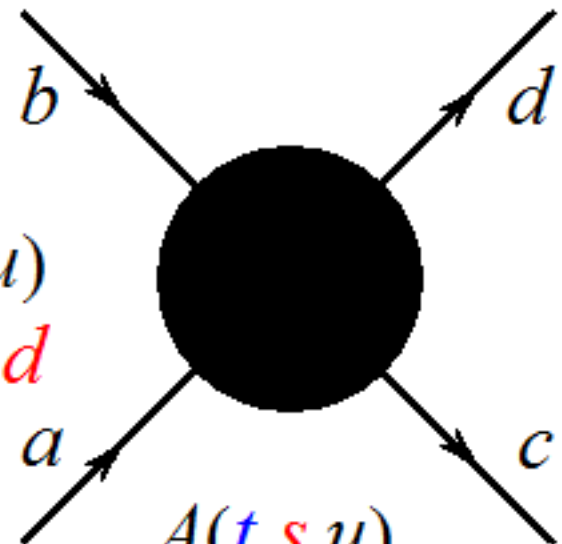


$$= \frac{1}{s}$$



$$\sigma_{tot} \underset{s\to\infty}{\simeq} \frac{1}{s}\,\mathrm{Im}\,A_{\mathrm{el}}(s, t = 0)$$

Crossing



$A(s,t,u)$

$a+b \rightarrow c+d$

$A(t,s,u)$

$a+\overline{c} \rightarrow \overline{b}+d$

Pomeranchuk Theorem

For asymptotically constant cross-sections (expected in the 60's) :

$$\sigma_{\text{tot}}(ab) \underset{s\to\infty}{\longrightarrow} \kappa_1$$

$$\sigma_{\text{tot}}(a\bar{b}) \underset{s\to\infty}{\longrightarrow} \kappa_2$$

one can prove using dispersion relation:

$$\kappa_1 - \kappa_2 = 0 \implies \sigma_{\text{tot}}(ab) - \sigma_{\text{tot}}(a\bar{b}) \underset{s\to\infty}{\longrightarrow} 0$$

For cross-sections growing with energy one prove that:

$$\frac{\sigma_{\text{tot}}(ab)}{\sigma_{\text{tot}}(a\bar{b})} \underset{s\to\infty}{\longrightarrow} 1$$

Froissart-Martin Bound

$$\sigma_{\text{tot}} \leq c \ln^2 s \quad c \geq \frac{\pi}{m_\pi^2} \sim 60\,\text{mb}$$

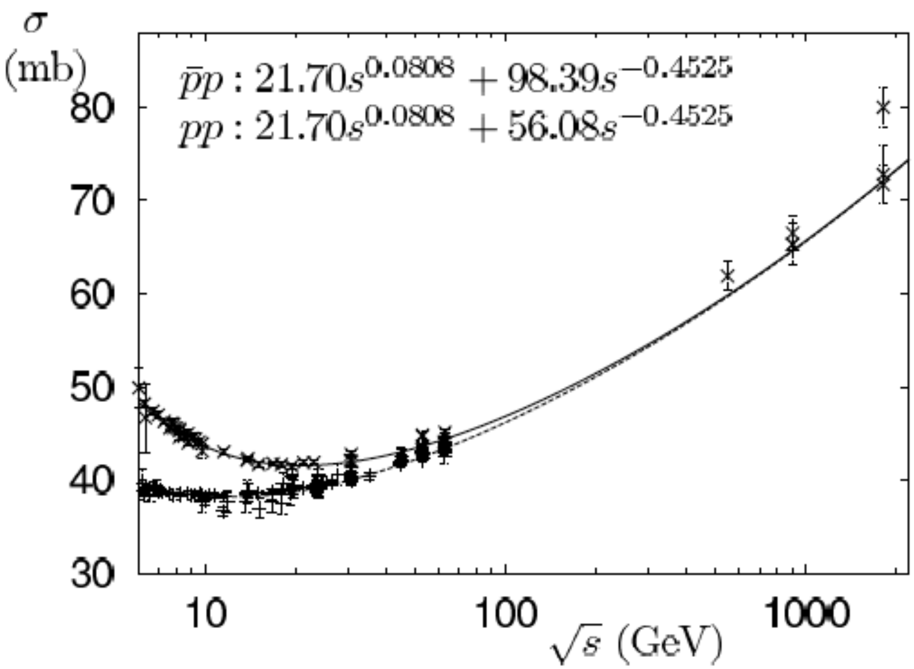
for the LHC energies ~ 4 barns
while exp. ~ 120 mb

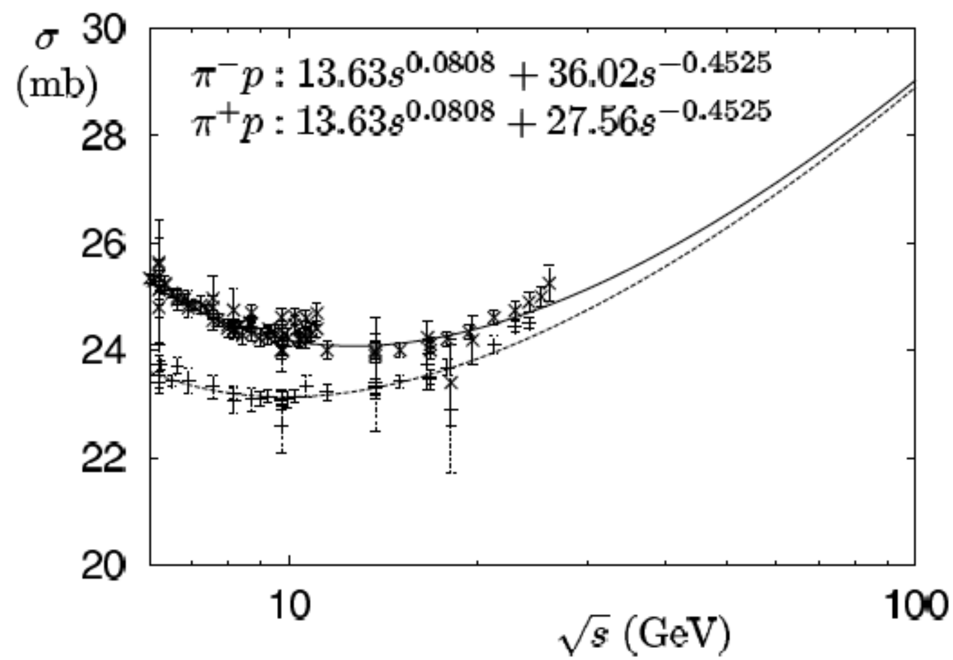
Total cross-section

If the amplitude is predominantly imaginary:

$$\sigma_{tot} \underset{s\to\infty}{\simeq} \frac{1}{s} \operatorname{Im} A_{\rm el}(s, t=0) \sim s^{\alpha(0)-1}$$

$$\alpha(0) = 1.08$$





P. Landshoff, Acta Phys. Pol. B39 (2008) 2063 – pre LHC