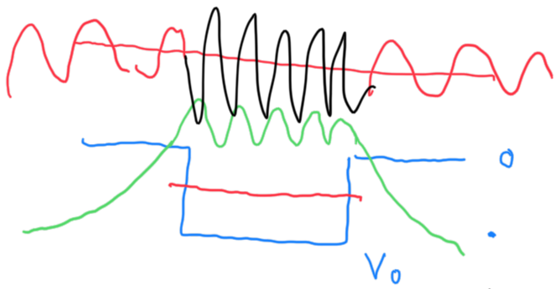
Wykład 7 Mechanika Kwantowa 13.4.2021

ROZPRASZANIE W 1-DIM.

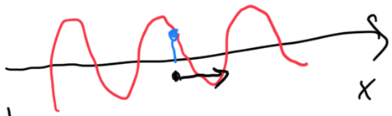


$$k = \frac{\sqrt{2m(E+V_0)}}{\hbar}$$

$$\omega = \frac{E}{\hbar}$$

$$\psi(x,t) = A e^{-i\omega t \pm ikx}$$

Stała faza

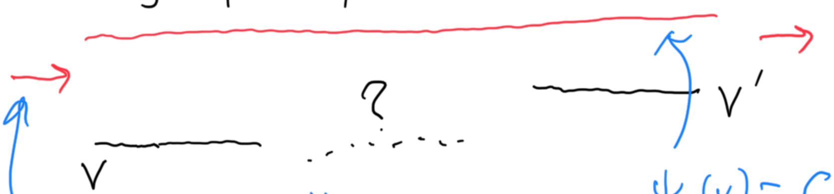


$+$: $-\omega t + kx = \text{const}$

$x = \frac{\omega}{k} \cdot t + \frac{\text{const}}{k}$ fala poruszająca się w prawo

$-$: $x = -\frac{\omega}{k} t + \frac{\text{const}}{k}$ -||- -||- -||- w lewo

Mamy potencjał



$\psi_p(x) = A e^{ikx}$ | $\psi_t(x) = C e^{ik'x}$

$\psi_o(x) = B e^{-ikx}$ | $\psi'_p(x) = D e^{-ik'x}$

R. Schrödingera

$$\psi^* i\hbar \frac{\partial \psi}{\partial t} = -\psi^* \frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi\psi^*$$

$$\psi (-i\hbar) \frac{\partial \psi^*}{\partial t} = -\psi \frac{\hbar^2}{2m} \frac{\partial^2 \psi^*}{\partial x^2} + V\psi^*\psi$$

odejmujemy te równania:

$$\left(\psi^* \frac{\partial \psi}{\partial t} + \frac{\partial \psi^*}{\partial t} \psi\right) = i \frac{\hbar}{2m} \left(\psi^* \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial x^2} \psi\right)$$

$$\frac{\partial}{\partial t}\underbrace{(\psi^*\psi)}_{P(x,t)} = \frac{i\hbar}{2m}\underbrace{\frac{\partial}{\partial x}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)}_{\frac{\partial}{\partial x}\left(-S(x,t)\right)}$$

$$S = -\frac{i\hbar}{2m}\left(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi\right)$$ ← Prąd gęstości prawdopodobieństwa

Równanie ciągłości:

$$\boxed{\frac{\partial P}{\partial t} + \frac{\partial S}{\partial x} = 0} \qquad \int_{-\infty}^{+\infty} dx$$

$$\underbrace{\frac{\partial}{\partial t}\int_{-\infty}^{+\infty} P(x,t)\,dx}_{=0} + \underbrace{S(+\infty) - S(-\infty)}_{} = 0$$

Jeżeli f. falowe są zlokalizowane $S(+\infty) - S(-\infty) = 0$

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Dla stanów rozproszeniowych

$x \ll 0$

$$S = -\frac{i\hbar}{m}\left(|A|^2 ik - |B|^2 ik\right) = v\left(|A|^2 - |B|^2\right)$$

$x \gg 0$ $\quad S = -\frac{i\hbar}{m}|C|^2 ik' = v'|C|^2$

$$S(+\infty) - S(-\infty) = v'|C|^2 - v|A^2| + v|B|^2 = 0$$

$$\boxed{v|A|^2 = v|B|^2 + v'|C|^2}$$

To równanie się upraszcza dla pot. sym

$V = V' \rightarrow v = v'$

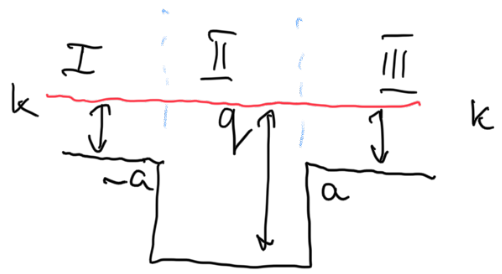
$$|A|^2 = |B|^2 + |C|^2$$

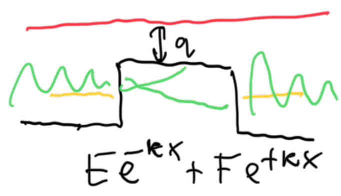
Współczynnik transmisji: $T = \frac{|C|^2}{|A|^2}$

odbicia: $R = \frac{|B|^2}{|A|^2}$

$T + R = 1$

Rozpraszanie na studni potencjału.





$$\left.\begin{aligned} \psi_I(-a) &= \psi_{II}(-a) \\ \psi_I'(-a) &= \psi_{II}'(-a) \end{aligned}\right\} \longrightarrow M_1 \begin{bmatrix} A \\ B \end{bmatrix} = M_2 \begin{bmatrix} E \\ F \end{bmatrix}$$

$$\left.\begin{aligned} \psi_{II}(a) &= \psi_{III}(a) \\ \psi_{II}'(a) &= \psi_{III}'(a) \end{aligned}\right\} \longrightarrow M_3 \begin{bmatrix} E \\ F \end{bmatrix} = M_4 \begin{bmatrix} C \\ D \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \underbrace{M_1^{-1} M_2 M_3^{-1} M_4}_{M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}} \begin{bmatrix} C \\ D \end{bmatrix}$$

Definicja:

$$\begin{bmatrix} C \\ B \end{bmatrix} = S \begin{bmatrix} A \\ D \end{bmatrix}$$

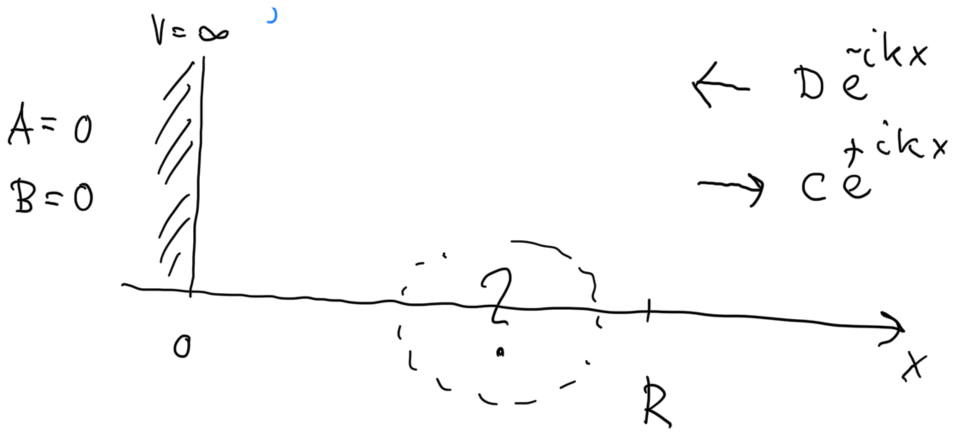
A, D mamy pod kontrolą

C, B ← szukamy

$$S = \frac{1}{M_{11}} \begin{bmatrix} 1 & -M_{12} \\ M_{12} & \det M \end{bmatrix} \leftarrow \text{zadanie}$$

↑ macierz rozpraszania S

PRZESUNIĘCIE FAZOWE



1) BRAK POTENCJAŁU: Rozw. $\phi(x) = \sin(kx)$

$$= \frac{1}{2i}\left(e^{ikx} - e^{-ikx}\right) \rightarrow D = -\frac{1}{2i} \quad C = \frac{1}{2i}$$

2) WŁĄCZAMY POTENCJAŁ

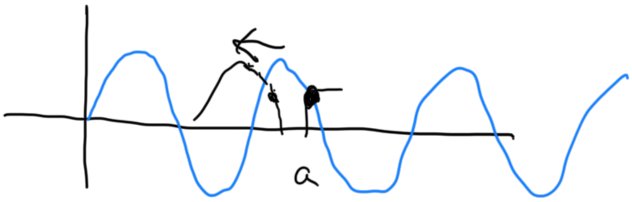
$$|D|^2 = |C|^2 \qquad C \rightarrow \frac{1}{2i} e^{2i\delta(k)}$$

Dla $x \gg R$

$$\phi(x) = \frac{1}{2i}\left(\underset{\substack{\uparrow \\ \text{odbita}}}{e^{ikx+2i\delta(k)}} - \underset{\substack{\uparrow \\ \text{padająca}}}{e^{-ikx}}\right)$$

$$= \frac{1}{2i} e^{i\delta(k)} \left(e^{ikx+i\delta(k)} - e^{-ikx-i\delta(k)}\right)$$

$$= e^{i\delta(k)} \sin\left(kx + \delta(k)\right)$$



$$\sin(kx_1 + \delta) = \sin(\underset{kx_0}{\underset{\shortparallel}{a}}) \rightarrow x_0 = \frac{a}{k}$$

$a \quad \delta(k)$

$\rightarrow x_1 = \frac{\cdot\cdot}{k} - \frac{\delta(k)}{k}$

jeżeli $\delta(k) > 0$ $\quad x_1 < x_0 \quad \leftarrow$ potencjał przyciągający

jeżeli $\delta(k) < 0$ $\quad x_1 > x_0 \quad \leftarrow$ potencjał odpychający

FALA ROZPROSZONA

$$\psi(x) = \phi(x) + \psi_{ROZPR}(x)$$

$$\psi_{ROZPR}(x) = \psi(x) - \phi(x) =$$

$$= \frac{1}{2i}\left(e^{ikx+2i\delta} - \cancel{e^{-ikx}}\right) - \frac{1}{2i}\left(e^{ikx} - \cancel{e^{-ikx}}\right)$$

$$= \frac{1}{2i} e^{ikx}\left(e^{2i\delta} - 1\right) =$$

$$= \frac{1}{2i} e^{ikx} e^{i\delta}\left(e^{i\delta} - e^{-i\delta}\right) =$$

$$= e^{ikx} e^{i\delta} \sin\delta = A_{ROZPR}\, e^{ikx}$$

$$A_{ROZPR} = e^{i\delta(k)} \sin\delta(k)$$

PRĄD PRAWDOP. W PRZYPADKU TRÓJWYMIAROWYM

$$\vec{S} = -\frac{i\hbar}{2m}\left(\psi^* \vec{\nabla}\psi - (\vec{\nabla}\psi^*)\psi\right)$$

$$\frac{\partial P}{\partial t} + \vec{\nabla}\cdot\vec{S} = 0$$

$$\int d^3x\, \vec{\nabla}\cdot\vec{S} = \int_{\partial V} d\vec{a}\cdot\vec{S} \rightarrow 0$$