Mechanika Kwantowa dla doktorantów
zestaw 27 na dzień 8.06.2017 godz. 8:15

1. Continuation from the previous set (Ch. 8-5 in Feynman and Hibbs). Find the continuum limit od a periodic system of N "balls" connected with springs described by the following Lagrangian:

$$L = \frac{1}{2}\sum_{j=1}^{N} \dot{q}_j^2 - \frac{\nu^2}{2}\left\{\sum_{j=1}^{N-1}\left(q_{j+1}-q_j\right)^2 + \left(q_1 - q_N\right)^2\right\}.$$

   Show that it reduces to the scalar field theory. Using the variational principle for this field derive equation of motion.

2. Using the variational principle derive classical field equations for a particle moving in electromagnetic fields described by the following Lagrangian:

$$\begin{aligned} S &= \frac{m}{2}\int \dot{\vec{q}}(t)^2\, dt \\ &+ e\int\left\{\phi[\vec{q}(t),t] - \frac{1}{c}\,\dot{\vec{q}}\cdot\vec{A}[\vec{q}(t),t]\right\} d^3x\, dt \\ &+ \frac{1}{8\pi}\int\left\{\left|\vec{\nabla}\phi + \frac{1}{c}\frac{\partial\vec{A}}{\partial t}\right|^2 - \left|\vec{\nabla}\times\vec{A}\right|^2\right\} d^3x\, dt, \end{aligned}$$

   with the following degrees of freedom: $\phi(\vec{x},t)$, $\vec{A}(\vec{x},t)$ and $\vec{q}(t)$.

3. Prove the following identity:

$$\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\zeta_{3/2}(\alpha) = \int \frac{d^3p}{(2\pi\hbar)^3}\left(e^{\beta(\frac{p^2}{2m}-\mu)} - 1\right)^{-1}$$

   where

$$\sum_{\nu=1}^{\infty}\frac{\alpha^\nu}{\nu^s} = \zeta_s(\alpha)\,.$$

4. Prove:

$$\int_0^\infty \frac{dxdydz}{e^{x+y+z}-1} = \zeta_3(1).$$