Mechanika Kwantowa dla doktorantów

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1. Chapter 8-4 from Feynman and Hibbs. Lagrange function of a periodic system of N "balls" connected with springs is given by:

$$L = \frac{1}{2}\sum_{j=1}^{N} \dot{q}_j^2 - \frac{\nu^2}{2}\left\{\sum_{j=1}^{N-1} (q_{j+1} - q_j)^2 + (q_1 - q_N)^2\right\}.$$

   Here $q_i(t)$ is one dimensional displacement of the $i-$th ball. Rewrite L in terms of normal coordinates Q_α. Assume that N is odd.

2. Assume that the system from the previous problem has been quantized. Find the wave function Φ_0 of the ground state. Calculate expectation values of the following operators

$$Q_\alpha, \quad Q^\star_\alpha, \quad Q^2_\alpha, \quad Q^{\star\,2}_\alpha, \; Q^\star_\alpha Q_\alpha$$

   in this state. Here $Q_\alpha, \quad Q^\star_\alpha$ are complex normal coordinates

3. Find the continuum limit od the system from the previous problems (Ch. 8-5 in Feynman and Hibbs). Show that it reduces to the scalar field theory.

4. *Generalized* ζ function is defined as

$$\zeta(s,a) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^s}. \qquad (1)$$

   Prove that

$$\zeta(s,a) = \frac{1}{\Gamma(s)} \int_0^\infty dx\, x^{s-1} \frac{e^{-ax}}{1-e^{-x}} \qquad (2)$$

   which is convergent for $s > 1$.

   Prove another useful integral representation of ζ function

$$\zeta(s,a) = -\frac{\Gamma(1-s)}{2\pi i} \int_{C_H} dz\, z^{s-1} \frac{e^{az}}{1-e^{z}} \qquad (3)$$

   where C_H starts at $-\infty$ above the cut (which we choose to be on the negative real axis), encircles the origin clockwise and goes back to $-\infty$ below the cut. This formula is useful to study the structure of singularities of ζ in complex s plane.