Mechanika Kwantowa dla doktorantów
zestaw 25 na dzień 25.05.2017 godz. 8:15

1. Calculate trace of the density matrix for the forced harmonic oscillator. To this end use the formula derived in the winter semester, whch after continuation to the Eulidean metric, takes the following form (assume $\hbar = \omega = m = 1$):

$$K_f(x', x; T) = \sqrt{\frac{1}{2i\,\pi \sin T}}\, e^{-V} \longrightarrow \rho(x', x, \beta) = \sqrt{\frac{1}{2\pi \sinh \beta}}\, e^{-V_E} \tag{1}$$

where

$$\begin{aligned} V_E &= \frac{1}{2\sinh\beta} \left\{ \left(x^2 + x'^2\right) \cosh\beta - 2\, x x' \right. \\ & -2\, x \int\limits_0^\beta dt\, f(t)\, \sinh t - 2\, x' \int\limits_0^\beta ds\, f(s)\, \sinh\left(T - s\right) \\ & \left. -2 \int\limits_0^\beta dt\, f(t) \int\limits_0^t ds\, f(s)\, \sinh\left(\beta - t\right) \sinh s \right\}. \end{aligned} \tag{2}$$

The result should read

$$\mathrm{Tr}\,\rho = \frac{1}{2\sinh\frac{\beta}{2}} \exp\left\{ \frac{1}{2} \int\limits_0^\beta dt\, f(t) \int\limits_0^t ds\, f(s) \frac{e^{(t-s)-\beta} + e^{-(t-s)}}{1 - e^{-\beta}} \right\} \tag{3}$$

2. Find classical trajectory and the corresponding action for a particle moving in Eulidean time τ under the influence of the force:

$$f(\tau) = \delta\left(\tau - s\right) - \delta\left(\tau - t\right), \tag{4}$$

with the boundary conditions $r_{\mathrm{cl}}(0) = r_{\mathrm{cl}}(\beta) = 0$.