# Mechanika Kwantowa dla doktorantów
zestaw 22 na dzień 27.04.2017 godz. 8:15

1. The Lippmann-Schwinger formalism can be also applied to a *one-dimensional* transmission-reflection problem with a finite range potential, $V(x) \neq 0$ for $-a < x < a$ only.

   - Suppose we have an incident wave coming from the left $\langle x | \phi \rangle = e^{ikx}/\sqrt{2\pi}$. How must we handle the singular $1/(E - H_0)$ operator if we want to have a transmitted wave for $x > a$ and a reflected one (+ the original wave) for $x < -a$? Is the the $E \to E + i\varepsilon$ prescription still correct? Obtain expression for the appriopriate Green's function and write integral equation for $\left\langle x | \psi^{(?)} \right\rangle$.
   - Consider special case of an attractive ($\gamma > 0$) potential
   $$V = -\left(\frac{\gamma \hbar^2}{2m}\right)\delta(x).$$
   Solve the integral equation to obtain the transmission and reflection amplitudes.
   - The above potential has only one bound state (find it). Show that the transmission and reflection amplitudes have bound-state poles at the expected positions where $k$ is regarded as complex.

2. Prove that
$$|z\rangle = e^{-|z|^2/2} \sum_{n=0} \frac{z^n}{\sqrt{n!}} |n\rangle$$
is a normalized eigen-state of the annihilation operator $\hat{a}$. To this end calculate the product of two eigenstates $\langle z' | z\rangle$ and find a norm subsituting $z' = z$. Prove completness relation:
$$\mathbf{1} = \frac{1}{\pi} \int d^2 z \, |z\rangle \langle z| \, .$$

3. For a Schrödinger's cat state
$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(e^{-i\pi/4} |z\rangle + e^{i\pi/4} |-z\rangle\right)$$
calculate probability distribution in the configuration space
$$P_\psi(\xi) = |\langle \xi | \psi\rangle|^2 \, .$$
Remember that
$$\psi_z(\xi) = \langle \xi | \psi\rangle = C \exp\left(-\frac{1}{2}(\xi - \sqrt{2}z)^2\right).$$
For the final answer choose $z = \rho$ or $z = i\rho$ where $\rho$ is a real number.

4. Unit vector $\vec{n}_\theta$ in $x - z$ plane has the following form:

$$\vec{n}_\theta = \cos\theta\,\vec{n}_z + \sin\theta\,\vec{n}_x$$

where $\vec{n}_{x,z}$ are unit vectors along $x$ an $z$ axis respectively. Find egenstates of the spin operator

$$S_\theta = \vec{n}_\theta \cdot \vec{S}$$

where $\vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$ is spin 1/2 operator.

The system is in the state with $S_z = +1/2$ (in units where $\hbar = 1$). We perform the measurement of $S_\theta$ on this state. What are the possible results of this meaurement and what are the corresponding probabilites. Next we measure $S_z$. What is the probability to recover the initial value $S_z = +1/2$.