Mechanika Kwantowa dla doktorantów
zestaw 20 na dzień 6.04.2017 godz. 8:15

1. Find possible poles of f_0 for a problem from a previous set for finite spherical well.

2. Find energies of the bound states in spherical well from the first problem for $l = 0$. Depending on R and V_0 there is only a finite number of such states. Suppose that we tune V_0 in a continuous way. Then the energies of the bound states change (how?), and when V_0 increases new bound states appear for some discrete values of V_0^n ($n = 1, 2, 3 \ldots$). Calculate V_0^n end the energy of the corresponding bound state. Show that V_0^n correspond to the singularities of f_0 for scattering energy $E \to 0$. Try to interpret this result.

3. Prove addition theorem for spherical harmonics:

$$\sum_m Y_l^m(\vec{n}_1) Y_l^{m\,*}(\vec{n}_2) = \frac{2l+1}{4\pi} P_l(\vec{n}_1 \cdot \vec{n}_2).$$

4. Consider scattering on the repelling potential ($\gamma > 0$)

$$\frac{2m}{\hbar^2} V(r) = \gamma \delta(r - R).$$

   Write Schrödinger equation and then for $l = 0$ solve it on the left and on the right of the potential and glue the solutions appropriately. Then calculate δ_0. Consider the case when γ is very large. Next, show that for any γ but small kR function $\cot \delta_0$ exhibits resonant behaviour in scattering energy E (namely $\cot \delta_0 = -c(E - E_r)$, which tends to zero for $E = E_r$). Find position of these resonances and their widths.