Mechanika Kwantowa dla doktorantów

zestaw 18 na dzień 23.03.2017 godz. 8:15

1. Prove by substituting to the spherical Bessel equation that

$$j_l(z) = \frac{1}{2}\frac{1}{i^l}\int_{-1}^{1} dt\, e^{izt} P_l(z).$$

To this end use the identity

$$P_l(z) = \frac{1}{2^l l!}\frac{d^l}{dz^l}(z^2-1)^l.$$

2. Hard ball potential – continuation.

We will show that the total cross-section for all $l-$sectors reads:

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l(k)$$

where $\delta_l(k)$ denote phase shifts. For a hard ball potential from the previous set calculate the cross-section for the lowest partial wave $l=0$ and for low energy k. As you will see the cross-section is not geometrical i.e. $\sigma \neq \pi R^2$.

Sum up higher partial waves up to a maximal clasically allowed $l_{\max} \sim ka$. To this end use

$$\sin^2\delta_l(k) = \frac{\tan^2\delta_l(k)}{1+\tan^2\delta_l(k)}$$

and the formula for $\tan\delta_l(k)$ in terms of spherical Bessel functions. Then use asymptotic form of Bessel functions. The resulting cross-section is still not geometrical ($\sigma_{\text{tot}} = 2\pi a^2$). Try to interpret this result (Sakurai, Advanced Quantum Mechanics, Chapt.7.6).

3. Consider Yukawa potential:

$$V(r) = V_0\frac{e^{-\mu r}}{\mu r},$$

where $1/\mu$ denotes interaction range. Calculate scattering amplitude in Born approximation $f^{(1)}(\theta)$, where θ is a scattering angle. Calculate differential cross-section $d\sigma/d\Omega$ and take limit $\mu \to 0$ (Coulomb potential).