Mechanika Kwantowa dla doktorantów

zestaw 16 – 9.03.2017 at 8:15

room D-2-02

1. Hamiltonian describing a particle of spin 1 has the following form:

$$H = A\frac{1}{\hbar}s_z + 2C\frac{1}{\hbar^2}s_x^2,$$

where A i C are constants. Find the energy levels and the corresponding eigen wave functions. At $t = 0$ the particle is in the eigenstate of spin $s_z = +\hbar$. Find the expectation value of the spin operator

$$\langle s_i \rangle (t) = \psi^*(t)\hat{s}_i\psi(t)$$

where ψ is a time dependent wave function corresponding to the above initial condition.

2. Consider two posible discretizations of the integral

$$\sigma = \frac{e}{c}\int_{t_1}^{t_2} dt\,\frac{d\vec{r}}{dt}\cdot\vec{A}(t)$$

given as

$$\begin{aligned}\sigma_1 &= \sum_k (\vec{r}_{k+1}-\vec{r}_k)\cdot\vec{A}(\vec{r}_k, t_k),\\ \sigma_2 &= \sum_k (\vec{r}_{k+1}-\vec{r}_k)\cdot\vec{A}(\vec{r}_{k+1}, t_{k+1}).\end{aligned}$$

Calculate $\sigma_1 - \sigma_2$ in the continuum limit.

3. Two possible discretizations of σ that lead to the correct hamiltonian for the particle moving in the electromagnetic potential $\vec{A}$ read as follows

$$\begin{aligned}s_1 &= \sum_k (\vec{r}_{k+1}-\vec{r}_k)\cdot\frac{1}{2}\left(\vec{A}(\vec{r}_k, t_k)+\vec{A}(\vec{r}_{k+1}, t_{k+1})\right),\\ s_2 &= \sum_k (\vec{r}_{k+1}-\vec{r}_k)\cdot\vec{A}\left(\frac{1}{2}(\vec{r}_k+\vec{r}_{k+1}), \frac{1}{2}(t_k+t_{k+1})\right).\end{aligned}$$

Prove the equivalnce by computing the hamiltonian from the "one-step" time evolution

$$\psi(\vec{x}, t+\varepsilon) = \int d^3\vec{y}\,K(\vec{x},\vec{y},\varepsilon)\psi(\vec{y}, t)$$

with

$$
\begin{aligned}
K(\vec{x},\vec{y},\varepsilon) &= \left(\frac{m}{2\pi i\varepsilon\hbar}\right)^{3/2}\exp\left(\frac{i}{\hbar}\left[\frac{m(\vec{x}-\vec{y})^2}{2\varepsilon}+\frac{e}{c}(\vec{x}-\vec{y})\cdot\frac{1}{2}(\vec{A}(\vec{x})+A(\vec{y}))\right]\right)\\
&= \left(\frac{m}{2\pi i\varepsilon\hbar}\right)^{3/2}\exp\left(\frac{i}{\hbar}\left[\frac{m(\vec{x}-\vec{y})^2}{2\varepsilon}+\frac{e}{c}(\vec{x}-\vec{y})\cdot\vec{A}\left(\frac{\vec{x}+\vec{y}}{2}\right)\right]\right).
\end{aligned}
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