Mechanika Kwantowa dla doktorantów
zestaw 15 – 2.03.2017 at 8:15

1. Hamiltonian H_0 has two eigenstates $|1\rangle$ i $|2\rangle$ corresponding to energies E_1 i E_2. Initially the system was in state $|1\rangle$. At $t = 0$ perturbation described by a symmetric potential V ($V_{12} = V_{21}$) has been switched on. Calculate probability that at $t > 0$ the system is in state $|2\rangle$. Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a degenerate system: $E_1 = E_2$ and $V_{11} = V_{22}$.

2. Under which conditions the transition probability for a harmonic potential (calculated in the first order of perturbation theory)

$$V(x,t) = 2\,V(x)\,\cos(\omega t)$$

reads:

$$\frac{dP(n \to m)}{dt} = \frac{2\pi}{\hbar}|V_{mn}|^2 \left\{\delta(E_m - E_n - \hbar\omega) + \delta(E_m - E_n + \hbar\omega)\right\}.$$

3. Hamiltonian describing a particle of spin 1 has the following form:

$$H = A\frac{1}{\hbar}s_z + 2C\frac{1}{\hbar^2}s_x^2,$$

where A i C are constants. Find the energy levels and the corresponding eigen wave functions. At $t = 0$ the particle is in the eigenstate of spin $s_z = +\hbar$. Find the expectation value of the spin operator

$$\langle s_i \rangle\,(t) = \psi^*(t)\hat{s}_i\psi(t)$$

where ψ is a time dependent wave function corresponding to the above initial condition.