Mechanika Kwantowa dla doktorantów

zestaw 14 – 19.1.2017 at 8:15

1. Show the following relations for the harmonic oscillator:

$$\begin{aligned} < x(t) > &= \overline{x}(t) < 1 > \\ < x(t)x(s) > &= [\overline{x}(t)\overline{x}(s) + g(s,t)] < 1 > \\ < x(t)x(s)x(u) > &= [\overline{x}(t)\overline{x}(s)\overline{x}(u) + \overline{x}(t)g(s,u) + \overline{x}(s)g(t,u) + \overline{x}(u)g(t,s)] < 1 > \end{aligned}$$

   where $\overline{x}(t)$ denotes the classical trajecory, and $g(t,s)$ reads as follows:

$$g(t,s) = \frac{i\hbar}{m\omega \sin\omega T} \sin\omega(t_2 - t)\sin\omega(s - t_1), \quad s < t$$

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   HINT

   Use the following formula

$$\left\langle \exp\frac{i}{\hbar}\int dt\, f(t)x(t) \right\rangle_S = \exp\frac{i}{\hbar}(S'_{cl} - S_{cl}) < 1 >_S$$

   and substitute known expressions for classical actions for the harmonic oscillator (S_{cl}) and for the harmonic oscillator with external force $f(t)$ (S'_{cl}). Show that the difference

$$S'_{cl} - S_{cl} = \frac{i}{2\hbar}\int\int f(t)f(s)g(t,s)dtds + \int f(t)\overline{x}(t)dt. \qquad (1)$$

   Differnciating (1) with respect to f one arrives at the formulae above.

2. Find in the first order of time-dependent perurbation theory probability of a transition from energy eigenstate $n \to m$ in a potential $V(x,t) = V(x)f(t)$, where $f(t)$:

$$f(t) = \begin{cases} \frac{1}{2}e^{\gamma t}, & t < 0, \\ 1 - \frac{1}{2}e^{-\gamma t}, & 0 < t < \frac{T}{2}, \\ 1 - \frac{1}{2}e^{-\gamma(T-t)}, & \frac{T}{2} < t < T, \\ \frac{1}{2}e^{-\gamma(t-T)}, & T < t. \end{cases}$$

   Result:

$$P(n \to m) = \left(\frac{\gamma^2}{\gamma^2 + (E_m - E_n)^2}\right)^2 |V_{mn}|^2 \frac{4\sin^2\frac{(E_m - E_n)T}{2\hbar}}{(E_m - E_n)^2}.$$

   Make a plot of function $f(t)$. Compare the result with a situation when the perturbation is momentairly switched on.

3. Hamiltonian H_0 has two eigenstates $|1\rangle$ i $|2\rangle$ corresponding to energies E_1 i E_2. Initially the system was in state $|1\rangle$. At $t = 0$ perturbation described by a symmetric potential V ($V_{12} = V_{21}$) has been switched on. Calculate probability that at $t > 0$ the system is in state $|2\rangle$. Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a degenerate system: $E_1 = E_2$ and $V_{11} = V_{22}$.