# Mechanika Kwantowa dla doktorantów

zestaw 9 – 15.12.2016 at 8:15

1. Using path integral formalism calculate the propagator of a free particle moving on a circle of radius $r$. The problem consists in performing the sumation over all possible windings of a free motion. So the trajectory from $x_i \to x_f$ is characterized by the winding number $n$
$$x_f^{(n)} = x_i + s + nL$$
where $L = 2\pi r$ and $s$ is the shortest distance along the circle between $x_i$ and $x_f$. The summation over $n$ can be performed by means of a Poisson summation formula. Solution of this problem can be found in a book:
*Classical and Quantum Dynamics: From Classical Paths to Path Integrals*
by Walter Dittrich, Martin Reuter
(Springer)

2. Using known formula for a classical action of a harmonic oscillator in one dimension $S(b, a, T)$ calculate $\partial^2 S/\partial b \partial a$. Show that in focal points $\partial^2 S/\partial b \partial a$ has a singularity and when the particle crosses the focal point $\partial^2 S/\partial b \partial a$ changes its sign. Using the formula
$$E(b, a, T) = -\frac{\partial S(b, a, T)}{\partial T}$$
calculate the energy and $\partial E/\partial T = -\partial^2 S(b, a, T)/\partial T^2$. Next, calulate
$$D = -\frac{\partial^2 S/\partial b \partial a}{\partial^2 S/\partial T^2}$$
at the conjugate (focal) points. Suppose the system crosses the focal point, what happens to the sign of $D$?

3. For the harmonic oscillator the energy is given by
$$E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega^2 x^2.$$
Show that for fixed $E$ the time of motion from $a \to b$ is equal to
$$T = \frac{1}{\omega} \left[ \arcsin \frac{b}{x_T} - \arcsin \frac{a}{x_T} \right]$$
where
$$\pm x_T = \pm \sqrt{\frac{2E}{m\omega^2}}$$
are classical turning points. From this formula derive $\partial T/\partial E$. Note that this is the quantity responsible for the singularities of $S$.