Mechanika Kwantowa dla doktorantów

zestaw 8 – 8.12.2016 at 8:15

1. Landau Levels. Consider motion of a charged particle in a constant external magnetic field B parallel to the z axis. Use the following vector potential $\vec{A} = \frac{1}{2}(-By, Bx, 0)$. Rewrite the hamiltonian as a sum of two harmonic oscillators and a free motion in the z direction plus a perturbation. Calculate unperturbed energies and compare with the energy obtained last time for a potential $\vec{A} = (0, Bx, 0)$. Use perturbation theory and calculate full energy of Landau levels. Show that the first order perturbation theory gives the exact result.

2. Comparing expression for the propagator of the harmonic oscillator written in the energy representation

$$K(x_2, t_2; x_1, t_1) = \sum_{n=1}^{\infty} \psi_n(x_2)\psi_n^*(x_1) \exp\left(-\frac{i}{\hbar}E_n(t_2 - t_1)\right)$$

with the explicit formula derived in the path integral formalism

$$K(x_2, t_2; x_1, t_1) = \sqrt{\frac{m\omega}{2\pi i\hbar \sin \omega t}} e^{\frac{i}{\hbar}S_{\text{cl}}},$$

find the first three eigenenergies of the oscillator and the corresponding wave functions. To this end use the following variables $\xi = \sqrt{m\omega/\hbar}\, x$ and $\psi_n(x) = (m\omega/\pi\hbar)^{1/4}\phi_n(\xi)$. Rewrite trigonometric functions in terms of $e^{\pm i\omega T}$and expand K in powers of exponents:

$$K \sim e^{-i\omega T/2}(a_0 + a_1 e^{-i\omega T} + a_2 e^{-i2\omega T} + \ldots).$$

3. Using path integral formalism calculate the propagator of aftrr particle moving on a circle of radius r. The problem consits in performing the sumation over all possible windings of a free motion. So the trajectory from $x_i \to x_f$ is characterized by the winding number n

$$x_f^{(n)} = x_i + s + nL$$

where $L = 2\pi r$ and s is the shortest distance along the circle betwee x_i and x_f. The summation over n can be performed by means of a Poisson summation formula.
Solution of this problem can be found in a book:
Classical and Quantum Dynamics: From Classical Paths to Path Integrals
by Walter Dittrich, Martin Reuter
(Springer)