Mechanika Kwantowa dla doktorantów

zestaw 7 – 24.11.2016 at 8:15

1. Particle of mass m and charge e is moving in an external magnetic field B parallel to the z axis. Lagrangian for this case reads (why?):

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{eB}{2c}(x\dot{y} - y\dot{x}).$$

   Calculate propagator K for a motion from $\vec{a} = (x_a, y_a, z_a)$ to $\vec{b} = (x_b, y_b, z_b)$ in time T.

   HINT: Introduce new variables $x'(t)$ and $y'(t)$ that correspond to the rotation in the $x-y$ plane with constant angular velocity α. Choose α in such a way that motion in $x-y$ plane factorizes into two independent harmonic oscillators (and a free motion in z direction). In this case it is trivial to write the propagator. Then perform the inverse transformation and express the propagator in the original variables.

2. In 3 dimensions van Vleck formula takes the following form:

$$F = \text{const.} \det\left(-\frac{\partial^2 S_{cl}}{\partial a_i \partial b_j}\right)$$

   where the particle moves from point $\vec{a} = (a_1 = x_a, a_2 = y_a, a_3 = z_a)$ to point $\vec{b} = (b_1 = x_b, b_2 = y_b, b_3 = z_b)$. Find the constant by explicit calculation of F from the known action of a particle moving in an external magnetic field along the z axis (problem 1) and compare with the explicit form of the prefactor from the same problem.