Mechanika Kwantowa dla doktorantów

zestaw 5 – 10.11.2016 at 8:15

1. Startując z 2 równań telegrafistów:

$$\frac{\partial P_{\pm}(x,t)}{\partial t} = a\left(P_{\pm}(x,t) - P_{\mp}(x,t)\right) \mp v\frac{\partial P_{\pm}(x,t)}{\partial x}$$

   wyprowadzić 2 równania 2 rzędu każde, jedno na P_+ drugie na P_-.

2. Show that for large N

$$\binom{N}{\mu}\left(\frac{1}{2}\right)^N \approx \sqrt{\frac{2}{\pi N}}e^{-\frac{j^2}{2N}}.$$

   Here $j = \mu - \nu$ and $N = \mu + \nu$ (see lecture notes on random walks).

3. (Continuation) Find classical action for the harmonic oscillator with external force $F(t)$. Take limit $\omega \to 0$ to obtain classical action for a particle moving in external force. Finally take limit $F \to 0$ to obtain action of a free particle.

4. Following the steps discussed at the lecture find $n = 2, 3, 4$ moments of a random walk probability distribution:

$$\mu_n(t) = \langle (\Delta x(t))^n \rangle$$

   where

$$\Delta x(t) = v\int_0^t d\tau\,(-)^{N(\tau)}$$

   with probability of $N(\tau)$ reversals equal r given by

$$\mathcal{P}(N(\tau) = r) = e^{-at}\frac{(at)^r}{r!}.$$

   The averaging over the number of reversals is understood in the following way:

$$\langle F(N(\tau)\rangle = \sum_{r=0}^{\infty} F(r)e^{-at}\frac{(at)^r}{r!}.$$

5. Show that the Laplace transform of the moments calculated in the previous problem is given by

$$\int_0^\infty dt\, e^{-st}\,\frac{\mu_m(t)}{m!\,v^m} = \begin{cases} \frac{1}{s^{(m+1)/2}}\frac{1}{(s+2a)^{(m+1)/2}} & \text{for odd } m \\ \frac{1}{s^{m/2+1}}\frac{1}{(s+2a)^{m/2}} & \text{for even } m\,. \end{cases}$$