Mechanika Kwantowa dla doktorantów
zestaw 4 – 3.11.2016 at 8:15

1. Startując z 2 równań telegrafistów:

$$\frac{\partial P_{\pm}(x,t)}{\partial t} = a\left(P_{\pm}(x,t) - P_{\mp}(x,t)\right) \mp v\frac{\partial P_{\pm}(x,t)}{\partial x}$$

   wyprowadzić 2 równania 2 rzędu każde, jedno na P_+ drugie na P_-.

2. Show that for large N

$$\binom{N}{\mu}\left(\frac{1}{2}\right)^N \approx \sqrt{\frac{2}{\pi N}}e^{-\frac{j^2}{2N}}.$$

   Here $j = \mu - \nu$ and $N = \mu + \nu$ (see lecture notes on random walks).

3. Lagrange'a function for the harmonic oscillator reads:

$$L = \frac{m}{2}\dot{x}(t)^2 - \frac{m\omega^2}{2}x(t)^2.$$

   Calculate classical trajectory leading from point $(x_a, t_a) \to (x_b, t_b)$. Calculate classical action along this trajectory.

   HINT: After finding classical trajectory $\bar{x}(t)$, calculate the action integrating by parts and using equations of motion.

4. For certain values $\omega(t_b - t_a) = \omega T$ both classical trajectory and classical action exhibit singularities. Find conditions that make them both finite. Discuss meaning of these conditions.

5. Find classical action for the harmonic oscillator with external force $F(t)$. Take limit $\omega \to 0$ to obtain classical action for a particle moving in external force. Finally take limit $F \to 0$ to obtain action of a free particle.