Mechanika Kwantowa dla doktorantów

zestaw 3 – 27.10.2016

8:00 sharp!

1. One slit diffraction - continuation.

   Consider one dimensional problem of a particle moving from $x = 0$ at $t = 0$ to point (x, t) through a screen, which the particle reaches at time $\tau_0 < t = \tau + \tau_0$. The screen has a hole of size $2b$ centered around x_0. (This process can be understood in the following way: at time τ_0 we block all possible trajectories except those passing through a hole). Calculate the propagator for such a motion.

   In order to simplify the calculactions replace the sharp hole by a "Gaussian hole":

$$G(x) = \begin{cases} 0 & \text{for} \quad x < x_0 - b \text{ or } x_0 + b < x \\ \\ 1 & \text{for} \qquad x_0 - b \leq x \leq x_0 + b \end{cases} \quad \to \; G(x) = e^{-(x-x_0)^2/2b^2}.$$

   What is the width of the image of this "Gaussian hole" at time t? How does it compare with the classical expectation?

   Since the propagation before and after the slit is free, we can use the formula for a free particle propagator

$$K_0(b, a) = \sqrt{\frac{m}{2\pi i\hbar(t_b - t_a)}}\, e^{\frac{i}{\hbar}\frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}}$$

   and the formula for "folding"the propagators

$$K(b, a) = \int dx_c\, K(b, c) G(x_c) K(c, a).$$

   Here $a = (x_a, t_a)$ etc.

   This problem can be found in the book of Feynman and Hibbs "Quantum Mechanics and Path Integrals" chapt. 3.2.

2. Two slit diffraction from the previous set.

3. Prove the following identities:

$$\begin{aligned} \begin{pmatrix} N+1 \\ \mu \end{pmatrix} &= \begin{pmatrix} N \\ \mu \end{pmatrix} + \begin{pmatrix} N \\ \mu - 1 \end{pmatrix}, \\ \begin{pmatrix} N+N' \\ \mu - \mu' \end{pmatrix} &= \sum_{\mu' \leq \nu \leq \mu} \begin{pmatrix} N \\ \mu - \nu \end{pmatrix} \begin{pmatrix} N' \\ \nu - \mu' \end{pmatrix}. \end{aligned}$$

4. Prove the virial theorem for a set of massive particles $\{i\}$ moving under the influence of forces $\vec{F}_i$:

$$\langle T \rangle = -\frac{1}{2} \left\langle \sum_i \vec{r}_i \cdot \vec{F}_i \right\rangle$$

where T is the kinetic energy of the system and the average is defined with respect to time

$$\langle X \rangle = \frac{1}{T} \int_0^T dt \, X(t).$$