Mechanika Kwantowa dla doktorantów

list of problems for the oral exam

Problems with a (*) are considered to be more difficult. To pass the exam you have to discuss at least one (*) problem of your choice.

1. Show starting from the Schroedinger equation that the propagator

$$K(b,a) = \langle x_b | \, e^{-i(t_b - t_a)\hat{H}} \, | x_b \rangle \tag{1}$$

can be written in the path integral form with $T = t_b - t_a = t_N - t_0 = N\epsilon$ in the following form

$$K(b,a) = \lim_{\epsilon \to 0} \int dx_1 \dots dx_{N-1} \left(\frac{m}{2i\epsilon\hbar\pi} \right)^{\frac{1}{2}N} e^{\frac{i\epsilon}{\hbar} \sum_{j=0}^{N-1} L_j} \,. \tag{2}$$

where

$$L_j = \tfrac{1}{2} m \left(\frac{x_{j+1} - x_j}{\epsilon} \right)^2 - V(x_j) \,.$$

2. Having the expression (2) for the propagator, show that the wave function

$$\psi(x, t_x) = \int dy \, K(x, y; t_x - t_y) \psi(y, t_y) \tag{3}$$

satisfies the Schroedinger equation. To this end expand both sides of (3) for small time difference $t_x - t_y = \epsilon$.

3. Interpret the double slit diffraction experiment in terms of the path intrgral approach to the quantum mechanics. Assume that the slits can be approximated by the Gaussians

$$\Theta(x - (a - \delta))\Theta((a + \delta) - x) \sim \sqrt{\frac{1}{2\pi\delta}} \exp\left(-\frac{(x-a)^2}{2\delta} \right) . \tag{4}$$

4. Discuss the analogy between the random walks and the Schroedinger equation.

5. One dimensional random walk in time interval $t = N\epsilon$. One step in space is Δ. What is the probability to reach point $x = j\Delta$ in time t? What is the corresponding probability density? Using the properties of the binomial coefficients derive the diffusion equation for the probability density and the Smoluchowski equation.

6. Discuss the Metropolis algorithm to calculate the ground state energy of the quantum system in Euclidean time. How can one calculate the modulus of the wave function?

7. Prove virial theorem.
8. (*) Construct the random walk that leads to the Dirac equation in 2 dimensions. How can one calculate probability distribution to reach certain point in space in time t?
9. Discuss the derivation the van Vleck formula for the "quantum" prefactor of the propagator and its physical meaning.
10. Derive the propagator for a free harmonic oscillator. Discuss the situation when $\sin(\omega T) = 0$.
11. Derive the propagator for a forced harmonic oscillator.
12. (*) Discuss the semiclassical approximation to K with the help of singlevaluedness of K. What is the role of the caustic points?
13. (*) Discuss the difference between two approaches to semiclassical approximation based on fixed time and fixed energy actions.
14. Discuss the energy splitting of the ground state energy for a double well potential. To this end use the dilute instantom gas approximation.
15. (*) How can one calculate the instanton correction factor $\tilde{K}$ for general double well potential?
16. What is the energy spectrum in a periodic potential?
17. Discuss the consequences of the invariance of the integrational measure for path integrals under the transformation $x(t) \to x(t) + \eta(t)$. What is the discretized form of the momenum operator and of the kinetic energy operator?
18. Costruct peturbation thery (both stationary and time-dependent) in the path integral formalism. Derive Fermi golden rule.
19. Discuss Bohr-Aharonov effect.
20. Lippmann-Schwinger equation: formulation, $\pm i\varepsilon$ prescription and boundary conditions, Green's functions.
21. Scattering amplitude, cross-section, Bohr approximation.
22. Formulation of the perturbative expansion of Lippmann-Schwinger equation in terms of the transition operator T, optical theorem.
23. Eikonal approximation.
24. Partial waves. Separation of variables in the configuration space and in the momentum space.

25. Definition of S operator, phase shifts, generic dapendence on k, Argand plot.

26. Schroedinger's cat: formulation of the problem in terms of the coherent states, difference between the quantum superpositon and statistical mixture.

27. Bell's inequality.

28. Statistical physics: density matrix in terms of the path integral. Simple examples: free particle, harmonic oscillator.

29. Polaron: perturbative and variational approaches.

30. (*) Polaron in the path integral approach.

31. (*) Density matrix for identical particles. Bose-Einstein condensation in grand canonical ensamble formulation.