Mechanika Kwantowa dla doktorantów

zestaw 3 na dzień 17.03.2016 godz. 14:00

sala B-2-01

1. Na poprzednich ćwiczeniach obliczyliśmy

$$R_{\text{cont.}} = \frac{\det'\left(-\frac{d^2}{d\tau^2} + V''(\bar{x}(\tau))\right)}{\det\left(-\frac{d^2}{d\tau^2} + \omega^2\right)} = \frac{1}{12}. \tag{1}$$

Z drugiej strony na wykładzie wyprowadziliśmy ogólny wzór

$$R_{\text{cont.}} = \frac{1}{2A^2} \tag{2}$$

gdzie A iest współczynikiem przy rozwinięciu asymptotycznym unormowanego modu zerowego

$$y_1(\tau)|_{\tau\to\infty} = Ae^{-\omega\tau},$$

który

$$y_1(\tau) = \mathcal{N}\,\frac{d\bar{x}(\tau)}{d\tau}.$$

Znaleźć A i sprawdzić równość (1) i (2).

2. In order to obtain $\lambda_1(T)$ defined at the lecture $-$ *i.e.*the lowest eigenvalue that goes to 0 in the limit of $T \to \infty$ $-$ we have to solve

$$(-\partial_t^2 + V''(\bar{x}))\, y_T(t) \;=\; \lambda_1(T)\, y_T(t) \tag{3}$$

for *finite* (but large !) T. Thus $\lambda_1(T)$ must be *small* because for $T \to \infty$ it is zero. First, we convert (3) into an integral equation:

$$y_T(t) = y_{\lambda=0}(t) - \lambda_1(T)\,\frac{1}{2A^2\omega}\int\limits_{-T/2}^{t} dt'\{\tilde{y}_1(t)\, y_1(t') - y_1(t)\,\tilde{y}_1(t')\}\, y_T(t') \tag{4}$$

Prove that indeed (4) satisfies (3). Note that

$$y_T\left(-\tfrac{1}{2}T\right) = y_{\lambda=0}\left(-\tfrac{1}{2}T\right) = 0 \qquad \text{and} \qquad (-\partial_t^2 + V''(\bar{x}))\, y_{\lambda=0} = 0\,. \tag{5}$$

In order to check that $y_T(t)$ of (4) is a solution of (3) employ the familiar equations

$$\partial_t^2 y_1(t) = V''(\bar{x}) y_1(t) \qquad \text{and} \qquad \partial_t^2 \tilde{y}_1(t) = V''(\bar{x})\tilde{y}_1(t)\,, \tag{6}$$

and note that

$$\int\limits_{-T/2}^{t} dt'\,\{\tilde{y}_1(t)\, y_1(t') - y_1(t)\,\tilde{y}_1(t')\}\, y_T(t') =$$

$$= \int\limits_{-T/2}^{T/2} dt'\,\theta(t-t')\,\{\tilde{y}_1(t)\, y_1(t') - y_1(t)\,\tilde{y}_1(t')\}\, y_T(t')\,. \tag{7}$$

(remember that $\partial_t\theta(t-t') = \delta(t-t')$, $\partial_t^2\theta(t-t') = \delta'(t-t')$ and $\int dt'\delta'(t-t')f(t') = \partial_t f(t)$.)