Mechanika Kwantowa dla doktorantów

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sala 431a

1. Find in the first order of time-dependent perurbation theory probability of a transition from energy eigenstate $n \to m$ in a potential $V(x,t) = V(x)f(t)$, where $f(t)$:

$$f(t) = \begin{cases} \frac{1}{2}e^{\gamma t}, & t < 0, \\ 1 - \frac{1}{2}e^{-\gamma t}, & 0 < t < \frac{T}{2}, \\ 1 - \frac{1}{2}e^{-\gamma(T-t)}, & \frac{T}{2} < t < T, \\ \frac{1}{2}e^{-\gamma(t-T)}, & T < t. \end{cases}$$

   Result:

$$P(n \to m) = \left(\frac{\gamma^2}{\gamma^2 + (E_m - E_n)^2}\right)^2 |V_{mn}|^2 \frac{4\sin^2 \frac{(E_m - E_n)T}{2\hbar}}{(E_m - E_n)^2}.$$

   Make a plot of function $f(t)$. Compare the result with a situation when the perturbation is momentairly switched on.

2. Show that:

$$\lim_{T\to\infty} \frac{1}{T} \frac{\sin^2\left(\frac{\omega T}{2}\right)}{\left(\frac{\omega}{2}\right)^2} = 2\pi\delta(\omega)$$

3. Hamiltonian H_0 has two eigenstates $|1\rangle$ i $|2\rangle$ corresponding to energies $E_1 = E_2$. Initially the system was in state $|1\rangle$. At $t = 0$ perturbation described by a symmetric potential V ($V_{12} = V_{21}$, V_{11} and V_{22} arbitrary) has been switched on. Calculate probability that at $t > 0$ the system is in state $|2\rangle$. Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a nondegenerate system: $E_1 \neq E_2$.