Mechanika Kwantowa dla doktorantów

zestaw12-14 na dzień 25.2.2014 godz. 13:15

sala 431a

1. Consider Euclidean motion (in an inverted potential) of a given energy $E < 0$, leading from $x_1 \to x_2$ ($x_1 < x_2$) in time T. As a potential take $V(x) = \kappa(x^2 - a^2)^2$ with $\kappa = 1/8a^2$. For one instanton-like motion (without turning) it is clear that as $T \to \infty$ then $x_1 \to -a$, $x_2 \to a$ and $E \to 0$. Show that in this limit

$$E = -8a^2 e^{-T}.$$

HINT. Use the classical formula for T. In the limit $E \to 0$ the integral will contain a singular part that can be divided into a finite and still singular part by subtracting and adding $((x-a)^2 + 2E)^{-1/2}$. In the finite part one can immediately set $E = 0$. The other part can be calculated exactly for finite, but small E.

2. Find in the first order of time-dependent perurbation theory probability of a transition from energy eigenstate $n \to m$ in a potential $V(x,t) = V(x)f(t)$, where $f(t)$:

$$f(t) = \begin{cases} \frac{1}{2}e^{\gamma t}, & t < 0, \\ 1 - \frac{1}{2}e^{-\gamma t}, & 0 < t < \frac{T}{2}, \\ 1 - \frac{1}{2}e^{-\gamma(T-t)}, & \frac{T}{2} < t < T, \\ \frac{1}{2}e^{-\gamma(t-T)}, & T < t. \end{cases}$$

Result:

$$P(n \to m) = \left(\frac{\gamma^2}{\gamma^2 + (E_m - E_n)^2}\right)^2 |V_{mn}|^2 \frac{4\sin^2\frac{(E_m-E_n)T}{2\hbar}}{(E_m - E_n)^2}.$$

Make a plot of function $f(t)$. Compare the result with a situation when the perturbation is momentairly switched on.

3. Under which conditions the transition probability for a harmonic potential (calculated in the first order of perturbation theory)

$$V(x,t) = 2\,V(x)\,\cos(\omega t)$$

reads:

$$\frac{dP(n \to m)}{dt} = \frac{2\pi}{\hbar}|V_{mn}|^2 \left\{\delta(E_m - E_n - \hbar\omega) + \delta(E_m - E_n + \hbar\omega)\right\}.$$