Mechanika Kwantowa dla doktorantów

zestaw 3 na dzień 28.10.2013 godz. 10:30

1. Lagrange function for the harmonic oscillator reads:
$$L = \frac{m}{2}\dot{x}(t)^2 - \frac{m\omega^2}{2}x(t)^2.$$
Calculate the classical trajectory leading from point $(x_a, t_a) \to (x_b, t_b)$. Calculate classical action along this trajectory.

   HINT: After finding classical trajectory $\bar{x}(t)$, calculate the action integrating by parts and using equations of motion.

2. For certain values $\omega(t_b - t_a) = \omega T$ both classical trajectory and classical action exhibit singularities. Find conditions that make them both finite. Discuss the meaning of these conditions.

3. Find classical action for the harmonic oscillator with an external force $F(t)$. Take limit $\omega \to 0$ to obtain classical action for a particle moving in an external force. Finally take limit $F \to 0$ to obtain an action of a free particle.

4. One slit diffraction.

   Consider one dimensional problem of a particle moving from $x = 0$ at $t = 0$ to point (x, t) through a screen which the particle reaches at time $t_1 < t$. The screen has a hole of size $2b$ centered around x_0. (This process can be understood in the following way: at time t_1 we block all possible trajectories except those passing through a hole). Calculate the propagator for such a motion.

   In order to simplify calculactions replace the sharp hole by a "Gaussian hole"
$$G(x) = \begin{cases} 0 & \text{for} \quad x < x_0 - b \text{ or } x_0 + b < x \\ \\ 1 & \text{for} \quad x_0 - b \le x \le x_0 + b \end{cases} \quad \to \quad G(x) = e^{-(x-x_0)^2/2b^2}.$$
What is the width of the image of this "Gaussian hole" at time t? How does it compare with the classical expectation?

   Since the propagation before and after the slit is free, we can use the formula for a free particle propagator
$$K_0(b, a) = \sqrt{\frac{m}{2\pi i\hbar(t_b - t_a)}}\, e^{\frac{i}{\hbar}\frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}}$$
and the formula for "folding"the propagators
$$K(b, a) = \int dx_c\, K(b, c)K(c, a).$$
Here $a = (x_a, t_a)$ etc.

   This problem can be found in the book of Feynman and Hibbs "Quantum Mechanics and Path Integrals" chapt. 3.2.