

Mechanika Kwantowa dla doktorantów  
zestaw 16 na dzień 7.3.2013 czwartek 8:30  
sala na antresoli

1. Find in the first order of time-dependent perturbation theory probability of a transition from energy eigenstate  $n \rightarrow m$  in a potential  $V(x, t) = V(x)f(t)$ , where  $f(t)$ :

$$f(t) = \begin{cases} \frac{1}{2}e^{\gamma t}, & t < 0, \\ 1 - \frac{1}{2}e^{-\gamma t}, & 0 < t < \frac{T}{2}, \\ 1 - \frac{1}{2}e^{-\gamma(T-t)}, & \frac{T}{2} < t < T, \\ \frac{1}{2}e^{-\gamma(t-T)}, & T < t. \end{cases}$$

Result:

$$P(n \rightarrow m) = \left( \frac{\gamma^2}{\gamma^2 + (E_m - E_n)^2} \right)^2 |V_{mn}|^2 \frac{4 \sin^2 \frac{(E_m - E_n)T}{2\hbar}}{(E_m - E_n)^2}.$$

Make a plot of function  $f(t)$ . Compare the result with a situation when the perturbation is momentarily switched on.

2. Under which conditions the transition probability for a harmonic potential (calculated in the first order of perturbation theory)

$$V(x, t) = 2V(x) \cos(\omega t)$$

reads:

$$\frac{dP(n \rightarrow m)}{dt} = \frac{2\pi}{\hbar} |V_{mn}|^2 \{ \delta(E_m - E_n - \hbar\omega) + \delta(E_m - E_n + \hbar\omega) \}.$$

3. Hamiltonian  $H_0$  has two eigenstates  $|1\rangle$  i  $|2\rangle$  corresponding to energies  $E_1$  i  $E_2$ . Initially the system was in state  $|1\rangle$ . At  $t = 0$  perturbation described by a symmetric potential  $V$  ( $V_{12} = V_{21}$ ) has been switched on. Calculate probability that at  $t > 0$  the system is in state  $|2\rangle$ . Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a degenerate system:  $E_1 = E_2$ .