Mechanika Kwantowa dla doktorantów

zestaw 11 na dzień 17.12.2012 poniedziałek

godz. 10:30, sala 128

1. Przedyskutować zależność od cechowania wzoru na propagator cząstki w jednorodnym polu magnetycznym.

2. Consider Euclidean motion (in an inverted potential) of a given energy $E < 0$, leading from $x_1 \to x_2$ ($x_1 < x_2$) in time T. As a potential take $V(x) = \kappa(x^2 - a^2)^2$ with $\kappa = 1/8a^2$. For one instanton-like motion (without turning) it is clear that as $T \to \infty$ then $x_1 \to -a$, $x_2 \to a$ and $E \to 0$. Show that in this limit

$$E = -8a^2 e^{-T}.$$

   HINT. Use classical formula for T. In the limit $E \to 0$ the integral will contain a singular part that can be divided into a finite and still singular part by subtracting and adding $((x-a)^2 + 2E)^{-1/2}$. In the finite part one can immediately set $E = 0$. The other part can be calculated exactly for finite, but small E.

3. Prove that

$$\int_{-T/2}^{T/2} d\tau_1 \int_{\tau_1}^{T/2} d\tau_2 \dots \int_{\tau_{n-1}}^{T/2} d\tau_n = \frac{1}{n!} T^n.$$

   This integral was needed to calculate the propagator for many instanton transition from $-a \to \pm a$.

4. Using results from the lecture that in semiclassical approximation

$$\oint \sum_r p_r dq_r = 2\pi\hbar \left(n + \frac{m}{4}\right) \qquad (1)$$

   where m is a number of focal points pased with each circling of one closed orbit, find energy levels of a particle moving in a Coulomb potential. Use the fact that $m = 4$ (Euler 1744).

   HINT:

   To calculate the inegral in (1) consider motion in $x - y$ plane and introduce polar coordinates (r, φ). Rewrite the Lagrangian, calculate generalized momenta, calculate the Hamiltonian. Assume classical trajectory with $\dot{r}(t) = 0$. Use Hamilton equations to eliminate r and find the energy (the result should coincide with the exact one for the hydrogen atom). Discuss whether restricting onself o the circular trajectory leads to oversimplification.