Mechanika Kwantowa dla doktorantów

zestaw 4 na dzień 29.10.2012 poniedziałek

godz. 10:30, sala 128

1. Find classical trajectory $(x_a, t_a) \to (x_b, t_b)$ and classical action for the harmonic oscillator with external force $F(t)$. Take limit $\omega \to 0$ to obtain classical action for a particle moving in external force. Finally take limit $F \to 0$ to obtain action of a free particle.

   HINT.

   In order to find a trajectory one has to solve inhomogenous differencial equation constructing the appriopriate Green's function. In order to calculate the action, integrate kinetic term by parts and use equation of motion.

2. One slit diffraction.

   Consider one dimensional problem of a particle moving from $x = 0$ at $t = 0$ to point (x, t) through a screen which the particle reaches at time $t_1 < t$. The screen has a hole of size $2b$ centered around x_0. (This process can be understood in the following way: at time t_1 we block all possible trajectories except those passing through a hole). Calculate the propagator for such a motion.

   In order to simplify calculactions replace the sharp hole by a "Gaussian hole"

$$G(x) = \begin{cases} 0 & \text{for} \quad x < x_0 - b \text{ or } x_0 + b < x \\ \\ 1 & \text{for} \quad x_0 - b \le x \le x_0 + b \end{cases} \quad \to \; G(x) = e^{-(x-x_0)^2/2b^2}.$$

   What is the width of the image of this "Gaussian hole" at time t? How does it compare with the classical expectation?

   Since the propagation before and after the slit is free, we can use the formula for a free particle propagator

$$K_0(b, a) = \sqrt{\frac{m}{2\pi i\hbar(t_b - t_a)}}\, e^{\frac{i}{\hbar}\frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}}$$

   and the formula for "folding"the propagators

$$K(b, a) = \int dx_c\, K(b, c) K(c, a).$$

   Here $a = (x_a, t_a)$ etc.

   This problem can be found in the book of Feynman and Hibbs "Quantum Mechanics and Path Integrals" chapt. 3.2.