

Mechanika Kwantowa dla doktorantów
zestaw 27 – 16.5.2012

1. Chapter 8-4 from Feynman and Hibbs. Lagrange function of a periodic system of N "balls" connected with springs is given by:

$$L = \frac{1}{2} \sum_{j=1}^N \dot{q}_j^2 - \frac{\nu^2}{2} \left\{ \sum_{j=1}^{N-1} (q_{j+1} - q_j)^2 + (q_1 - q_N)^2 \right\}.$$

Here $q_i(t)$ is one dimensional displacement of the i -th ball. Rewrite L in terms of normal coordinates Q_α . Assume that N is odd.

2. Assume that the system from the previous problem has been quantized. Find the wave function Φ_0 of the ground state. Calculate expectation values of the following operators

$$Q_\alpha, \quad Q_\alpha^*, \quad Q_\alpha^2, \quad Q_\alpha^{*2}, \quad Q_\alpha^* Q_\alpha$$

in this state. Here Q_α, Q_α^* are complex normal coordinates

3. Find the continuum limit of the system from the previous problems (Ch. 8-5 in Feynman and Hibbs). Show that it reduces to the scalar field theory.