Mechanika Kwantowa dla doktorantów

zestaw 25 – 25.4.2012

1. Consider scattering off an infinte "hard ball":
$$V(r) = \begin{cases} 0 & \text{dla} \quad R < r \\ & \\ \infty & \text{dla} \quad r < R \end{cases} .$$
Calculate phase shifts from the conditon $A_l(R) = 0$ where:
$$A_l(r) = e^{i\delta_l(k)} \left[j_l(kr) \, \cos\delta_l(k) - y_l(kr) \, \sin\delta_l(k) \right].$$
Find low energy behaviour of δ_l. Calculate the cross-section for the lowest partial wave $l = 0$. As you will see the cross-section is not geometrical i.e. $\sigma \neq \pi R^2$.

2. Sum up higher partial waves
$$\sigma_{\rm tot} = \frac{4\pi}{k^2} \sum_l (2l+1) \sin^2\delta_l(k)$$
up to a maximal clasically allowed $l_{\rm max} \sim kR$. To this end use
$$\sin^2\delta_l(k) = \frac{\tan^2\delta_l(k)}{1 + \tan^2\delta_l(k)}$$
and the formula for $\tan\delta_l(k)$ in terms of spherical Bessel functions. Then use asymptotic form of Bessel functions. The resulting cross-section is still not geometrical ($\sigma_{\rm tot} = 2\pi R^2$). Try to interpret this result (Sakurai, Advanced Quantum Mechanics, Chapt.7.6).

3. Using expansion for the wave function from the previous problem set
$$\psi = \sum_l (2l+1) i^l A_l(r) P_l(\cos\vartheta)$$
and conditions
$$\beta_l = \left. \frac{r}{A_l} \frac{dA_l}{dr} \right|_{r=R},$$
$$\tan\delta_l = \frac{kR\, j_l'(kR) - \beta_l\, j_l(kR)}{kR\, y_l'(kR) - \beta_l\, y_l(kR)}$$
derive general formula (in terms of spherical Bessel functions) for the scattering length for the finite spherical well ($V_0 > 0$):
$$V(r) = \begin{cases} 0 & \text{dla} \quad a < R \\ & \\ -V_0 & \text{dla} \quad r < R \end{cases} .$$
In particular calculate $\tan\delta_0$. Discuss two limits $k \to 0$ and $k \to \infty$. How δ_0 depends on V_0?