

Advanced Quantum Mechanics
 problem set number 17
 29.2.2012 at 10:15 (antresola).

1. W podręczniku Feynmana-Hibbsa jest wyprowadzona tożsamość:

$$\left\langle \frac{\delta F}{\delta x(s)} \right\rangle_S = -\frac{i}{\hbar} \left\langle F \frac{\delta S}{\delta x(s)} \right\rangle_S,$$

która w wersji dyskretnej ma postać

$$\left\langle \frac{\delta F}{\delta x_k} \right\rangle_S = -\frac{i}{\hbar} \left\langle F \frac{\delta S}{\delta x_k} \right\rangle_S,$$

gdzie

$$\langle F \rangle_S = \int [\mathcal{D}(x(t))] F[x(t)] e^{\frac{i}{\hbar} S[x(t)]}.$$

Korzystając z tej tożsamości wykazać, że dla cząstki w 3 wymiarach:

$$\langle (x_{k+1} - x_k)^2 \rangle = \langle (y_{k+1} - y_k)^2 \rangle = \langle (z_{k+1} - z_k)^2 \rangle = \frac{i\epsilon\hbar}{m},$$

$$\langle (x_{k+1} - x_k)(y_{k+1} - y_k) \rangle = \langle (z_{k+1} - z_k)(y_{k+1} - y_k) \rangle = \langle (x_{k+1} - x_k)(z_{k+1} - z_k) \rangle = 0,$$

gdzie opuściliśmy indeks S w wyrażeniach $\langle \dots \rangle_S$.

2. Derive commutation relation, *i.e.* calculate the following transition amplitude:

$$\langle \chi | m \frac{1}{\epsilon} ((x_{k+1} - x_k)x_k - x_{k+1}(x_{k+1} - x_k)) | \psi \rangle_S,$$

where

$$\langle \chi | F | \psi \rangle_S = \iint dx_1 dx_2 \chi^*(x_2) \left[\int_{x_1}^{x_2} [\mathcal{D}(x(t))] F[x(t)] e^{\frac{i}{\hbar} S[x(t)]} \right] \psi(x_1).$$

HINT. Since $x_k = x(t_k)$ and $x_{k+1} = x(t_k + \epsilon)$ it is enough to consider evolution of the wave function by one step of ϵ in time (only for expressions involving $x_{k+1}x_k$).