Advanced Quantum Mechanics

problem set number 16

24.2.2012 at 8:30 (room 431a).

1. Hamiltonian H_0 has two eigenstates $|1\rangle$ i $|2\rangle$ corresponding to energies E_1 i E_2. Initially the system was in state $|1\rangle$. At $t = 0$ perturbation described by a symmetric potential V ($V_{12} = V_{21}$) has been switched on. Calculate probability that at $t > 0$ the system is in state $|2\rangle$. Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a degenerate system: $E_1 = E_2$ and $V_{11} = V_{22}$.

2. W podręczniku Feynmana-Hibbsa jest wyprowadzona tożsamość:

$$\left\langle \frac{\delta F}{\delta x(s)} \right\rangle_S = -\frac{i}{\hbar} \left\langle F \frac{\delta S}{\delta x(s)} \right\rangle_S ,$$

która w wersji dyskretnej ma postać

$$\left\langle \frac{\delta F}{\delta x_k} \right\rangle_S = -\frac{i}{\hbar} \left\langle F \frac{\delta S}{\delta x_k} \right\rangle_S ,$$

gdzie

$$\langle F \rangle_S = \int [\mathcal{D}(x(t)]\ F[x(t)]\, e^{\frac{i}{\hbar} S[x(t)]}.$$

Korzystając z tej tożsamości wykazać, że dla cząstki w 3 wymiarach:

$$< (x_{k+1} - x_k)^2 >=< (y_{k+1} - y_k)^2 >=< (z_{k+1} - z_k)^2 >= \frac{i\epsilon\hbar}{m},$$

$$< (x_{k+1}-x_k)(y_{k+1}-y_k) >=< (z_{k+1}-z_k)(y_{k+1}-y_k) >=< (x_{k+1}-x_k)(z_{k+1}-z_k) >= 0,$$

gdzie opuściliśmy indeks S w wyrażeniach $\langle \ldots \rangle_S$.

3. Derive commutation relation, *i.e.* calculate the following transition amplitude:

$$\langle \chi |\, m \frac{1}{\epsilon} \left((x_{k+1} - x_k) x_k - x_{k+1}(x_{k+1} - x_k) \right) |\psi\rangle_S ,$$

where

$$\langle \chi \,| F |\, \psi \rangle_S = \iint dx_1 dx_2\, \chi^*(x_2) \left[\int_{x_1}^{x_2} [\mathcal{D}(x(t)]\ F[x(t)]\, e^{\frac{i}{\hbar} S[x(t)]} \right] \psi(x_1).$$

HINT. Since $x_k = x(t_k)$ and $x_{k+1} = x(t_k + \varepsilon)$ it is enough to consider evolution of the wave function by one step of ε in time (only for expressions involving $x_{k+1} x_k$).