Advanced Quantum Mechanics

problem set number 15

27.1.2012 at 8:30 (romm 431a).

1. Find in the first order of time-dependent perurbation theory probability of a transition from energy eigenstate $n \to m$ in a potential $V(x,t) = V(x)f(t)$, where $f(t)$:

$$f(t) = \begin{cases} \frac{1}{2}e^{\gamma t}, & t < 0, \\ 1 - \frac{1}{2}e^{-\gamma t}, & 0 < t < \frac{T}{2}, \\ 1 - \frac{1}{2}e^{-\gamma(T-t)}, & \frac{T}{2} < t < T, \\ \frac{1}{2}e^{-\gamma(t-T)}, & T < t. \end{cases}$$

   Result:

$$P(n \to m) = \left(\frac{\gamma^2}{\gamma^2 + (E_m - E_n)^2}\right)^2 |V_{mn}|^2 \frac{4\sin^2 \frac{(E_m - E_n)T}{2\hbar}}{(E_m - E_n)^2}.$$

   Make a plot of function $f(t)$. Compare the result with a situation when the perturbation is momentairly switched on.

2. Under which conditions the transition probability for a harmonic potential (calculated in the first order of perturbation theory)

$$V(x,t) = 2\,V(x)\,\cos(\omega t)$$

   reads:

$$\frac{dP(n \to m)}{dt} = \frac{2\pi}{\hbar}|V_{mn}|^2 \left\{\delta(E_m - E_n - \hbar\omega) + \delta(E_m - E_n + \hbar\omega)\right\}.$$

3. Hamiltonian H_0 has two eigenstates $|1\rangle$ i $|2\rangle$ corresponding to energies E_1 i E_2. Initially the system was in state $|1\rangle$. At $t = 0$ perturbation described by a symmetric potential V ($V_{12} = V_{21}$) has been switched on. Calculate probability that at $t > 0$ the system is in state $|2\rangle$. Perform calculations exactly and in the first order of perturbation theory. When perturbation theory gives the correct answer? Repeat the calculation for a degenerate system: $E_1 = E_2$ and $V_{11} = V_{22}$.