Mechanika Kwantowa dla doktorantów

zestaw 13 na dzień 25.1.2012 środa godz. 9:00

(w pok. 438, sala do ustalenia)

1. Calculate energy levels for the particle in the potential

$$V(x) = -\frac{V_0}{\cosh^2 \frac{x}{a}}$$

for arbitary V_0 and a. Next, calculate energy levels in the semiclassical (WKB) approximation. Compare with the exact result.

2. Using results from the lecture that in semiclassical approximation

$$\oint \sum_r p_r dq_r = 2\pi\hbar \left(n + \frac{m}{4}\right) \tag{1}$$

where m is a number of focal points pased with each circling of one closed orbit, find energy levels of a particle moving in Coulomb potential. Use the fact that $m = 4$ (Euler 1744).

HINT:

To calculate the inegral in (1) consider motion in $x - y$ plane and introduce polar coordinates (r, φ). Rewrite the Lagrangian, calculate generalized momenta, calculate the Hamiltonian. Assume classical trajectory with $\dot{r}(t) = 0$. Use Hamilton equations to eliminate r and find the energy (the result should coincide with the exact one for the hydrogen atom). Discuss whether restricting onself o the circular trajectory leads to oversimplification.

3. Prove that

$$\int_{-T/2}^{T/2} d\tau_1 \int_{\tau_1}^{T/2} d\tau_2 \ldots \int_{\tau_{n-1}}^{T/2} d\tau_n = \frac{1}{n!} T^n.$$

This integral was needed to calculate the propagator for many instanton transition from $-a \to \pm a$.